Ramsey Taxation

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Ricardian Equivalence Theorem

Holding current and future government spending constant, a change in current taxes with an equal and opposite change in the present value of future taxes leaves the equilibrium interest rate and the consumptions of individuals unchanged

- This theorem suggests that under certain conditions the timing of taxes does not matter
- What matters is the present value of tax liabilities
- Key: consumers realize that a tax break today is not free:taxes tomorrow will be higher, so they save the tax break

Optimal taxation of capital and labor-Household

The household problem: Let preferences be given by:

$$\sum_{t=0}^{\infty} \beta^t U(c_t, n_t)$$

The budget constraint of the household is:

$$\sum_{t=0}^{\infty} p_t[c_t + k_{t+1}] = \sum_{t=0}^{\infty} p_t[(1 - \tau_{lt})\, w_t n_t + k_t R_{k,t}]$$

The gross after tax return on capital has the form

$$R_{kt} = 1 + (1 - \tau_{kt})(\upsilon_t - \delta) \qquad \text{for} \quad \text{all} \quad t \geq 0$$

Optimal taxation of capital and labor-Firm

The firm's problem is:

$$\max_{k_t, n_t} A_t F(k_t, n_t) - k_t \upsilon_t - w_t n_t$$

Feasibility is given by:

$$g_t + c_t + k_{t+1} = A_t F(k_t, n_t) + (1-\delta)k_t$$

The government budget constraint is given by:

$$\sum_{t=0}^{\infty} p_t g_t = \sum_{t=0}^{\infty} p_t [\tau_{lt} w_t n_t + \tau_{kt} k_t (\upsilon_t - \delta)]$$

Optimal taxation of capital and labor

Definition: A competitive equilibrium with taxes where the government finances the purchases g_t is an allocation c_t, n_t, k_{t+1} a price system p_t, w_t, R_{kt} , and taxes τ_{lt}, τ_{kt} such that:

1. c_t, n_t, k_{t+1} is feasible for g_t and initial condition k_0
2. c_t, n_t, k_{t+1} maximizes the household utility given prices, p_t, w_t, R_{kt} , taxes τ_{lt}, τ_{kt} , and initial condition initial condition k_0 ,
3. firms maximize profits given prices w_t, υ_t ,
4. the government budget constraint holds.

Optimal taxation of capital and labor-FOC

FOC

The first order conditions for the household problem, using λ_h for the multiplier on the household budget constraint is:

$$c_t : \beta^t U_{ct} = \lambda_h p_t$$

$$n_t : \beta^t U_{nt} = \lambda_h p_t w_t (1 - \tau_{lt})$$

$$k_{t+1} : [-p_t + p_{t+1} R_{kt+1}]\lambda_h = 0$$

Optimal taxation of capital and labor

for all $t \geq 0$. These equations imply

$$\frac{-U_{nt}}{U_{ct}} = w_t(1 - \tau_{lt})$$

$$\frac{\beta U_{ct+1}}{U_{ct}} = \frac{p_{t+1}}{p_t} = \frac{1}{1 + r_{t+1}}$$

$$1 + r_{t+1} = R_{kt+1}$$

The firms order conditions for the firm problem are:

$$n_t : A_t F_n(k_t, n_t) - w_t = 0$$

$$k_t : A_t F_k(k_t, n_t) - \upsilon_t = 0$$

Implementability

We now summarize what type of allocation constitute an equilibrium with taxes that finances the exogenous stream of government purchases.

$$\sum_{t=0}^{\infty} \beta^t [U_{ct} c_t + U_{nt} n_t] = U_{C0} K_0 R_{k_0}$$

1. rewrite the budget constraint of households, by grouping the terms on k_{t+1} as:

$$\sum_{t=0}^{\infty} p_t \left(c_t - (1-\tau_{lt}) w_t n_t\right) = \sum_{t=0}^{\infty} p_t k_t R_{kt} - \sum_{t=0}^{\infty} p_t k_{t+1}$$

2. use the FOC in the household problem with respect to k_{t+1}

$$\sum_{t=0}^{\infty} p_t \left(c_t - (1-\tau_{lt}) w_t n_t\right) = p_0 k_0 R_{k0} + \sum_{t=0}^{\infty} p_{t+1} k_{t+1} [R_{k,t+1} - \frac{p_t}{p_t + 1}]$$

Implementability

1. use the FOC for the household with respect to c_t and replace p_t by $\frac{\beta^t U_{ct}}{\lambda_h}$

$$\sum_{t=0}^{\infty} \beta^t [U_{ct} c_t - (1-\tau_{lt}) w_t n_t] = p_0 \lambda_h k_0 R_{k0}$$

2. use the FOC for the household with respect to n_t and replace $p_t w_t (1-\tau_{lt})$ by $\frac{-\beta_t U_{nt}}{\lambda_h}$

$$\sum_{t=0}^{\infty} \beta^t [U_{ct} c_t + U_{nt} n_t] = p_0 \lambda_h k_0 R_{k0}$$

3. use $\frac{U_{c0}}{p_0} = \lambda_h$

$$\sum_{t=0}^{\infty} \beta^t [U_{ct} c_t + U_{nt} n_t] = U_{C0} k_0 R_{k0}$$

Ramsey Problem

- The Ramsey problem is to find the policy τ_{lt}, τ_{kt} that produce a competitive equilibrium with taxes with the highest utility for the agents.
- In other words,the Ramsey problem is to find the â€œoptimal" taxes to finance a given stream of government purchases, subject to the fact that agents behave competitively for that taxes.
- Given our characterization of an equilibrium with taxes, the Ramsey problem can be solved by finding the allocation that maximizes the utility of the agents subject to feasibility for all $t \geq 0$ and subject to the implementability constraint.

Analysis of the Ramsey Problem

Let λ be the Lagrange multiplier of the implementability constraint, then the problem is:

$$\max_{c_t, n_t, k_{t+1}, \tau_{k0}} \sum_{t=0}^{\infty} \beta^t (U(c_t, n_t) + \lambda [c_t U_c(c_t, n_t) + n_t U_n(c_t, n_t)]$$

$$-\lambda U_c(c_0, n_0) k_0 \left(1 + (1 - \tau_{k0})(F_{k0} - \delta)\right)$$

subject to $\tau_{k0} \leq \tau_k^m$ and

$$g_t + c_t + k_{t+1} = A_t F(k_t, n_t) + (1 - \delta) k_t$$

for all $t \geq 0$ given k_0.

Analysis of the Ramsey Problem

This problem can be written as:

$$\max_{c_t, n_t, k_{t+1}, \tau_{k0}} \sum_{t=0}^{\infty} \beta^t W(c_t, n_t; \lambda)$$

$$-\lambda U_c(c_0, n_0) K_0 \left(1 + (1 - \tau_{k0})(F_{k0} - \delta)\right)$$

subject to

$$g_t + c_t + k_{t+1} = A_t F(k_t, n_t) + (1 - \delta) k_t$$

for all $t \geq 0$ given k_0 where

$$W(c, n; \lambda) = U(c, n) + \lambda [c U_c(c, n) + n U_n(c, n)]$$

Analysis of the Ramsey Problem

Writing in this form, the problem is similar to the plannerâ€™s problem for the neoclassical growth model, except that:

1. the period utility function $U(c, n)$ has been replaced by $w(c, n; \lambda)$
2. the first term in the discounted utility is different from the other terms. The function W has the interpretation of the social value of c, n , since $U(c, n)$ is the private value and $\lambda[cU_c(c, n) + nU_n(c, n)]$ are the taxes corresponding to period t allocation, in terms of period t utils.

The FOC for this problem are:

$$1 = \frac{\beta W_{c,t+1}}{W_{c,t}}[F_{k,t+1} + (1-\delta)]$$

and

$$-\frac{W_{nt}}{W_{ct}} = F_{nt}$$

Proposition (Chamley-Judd): No taxation of capital in steady state. Assume that $g_t \to \overline{g}$ and $A_t \to \overline{A}$ as $t \to \infty$. In a steady state of the Ramsey problem corresponding to $\overline{g}, \overline{A}$, there is no taxation of capital, i.e. $\bar{\tau}_k = 0$

Proof. The foc for the Ramsey problem in a steady state implies:

$$1 = \beta[F_{kt} + (1 - \delta)]$$

since

$$W_{c,t} = W_{c,t+1}$$

The marginal rate of substitution for agent is

$$1 = \beta[F_{k,t+1} + (1 - \delta)]$$

since

$$U_{ct} = U_{C,t+1}$$

and hence $\overline{\tau_k} = 0$ QED.

Analysis of the Ramsey Problem

The marginal rate of substitution between consumption in t and $t+T$ and the marginal rate is given by

$$MRS_{t,t+T} \equiv \frac{\beta^{t+T} U_{ct+T}}{\beta^{t} U_{ct}}$$

The marginal rate of transformation between consumption at t and $t+T$, given by:

$$MRT_{t,t+T} \equiv [(1-\delta+F_{kt+1})(1-\delta+F_{kt+2})\ldots(1-\delta+F_{kt+T})]^{-1}$$

$$\frac{MRS_{t,t+T}}{MRT_{t,t+T}} = [\frac{\beta^{T} U_{ct+T}}{U_{ct}}][(1-\delta+F_{k,t+1})(1-\delta+F_{k,t+2})\ldots(1-\delta+F_{k,t+T})]$$

$$= [\frac{\beta^{T} U_{ct+T}}{U_{ct}}]\,[(1+(1-\tau_{k,t+1})(F_{k,t+1}-\delta))\ldots(1+(1-\tau_{k,t+T})(F_{k,t+T}-$$

Analysis of the Ramsey Problem

$$= [\frac{\beta^T U_{ct+T}}{U_{ct}}] [(1 + (1 - \tau_{k,t+1})(F_{k,t+1} - \delta)) \ldots (1 + (1 - \tau_{k,t+T})(F_{k,t+T} -$$

$$\times \frac{1 - \delta + F_{k,t+1}}{1 + (1 - \tau_{k,t+1})(F_{k,t+1} - \delta)} \cdots \frac{1 - \delta + F_{k,t+T}}{1 + (1 - \tau_{k,t+T})(F_{k,t+T} - \delta)}$$

which in equilibrium gives

$$\frac{MRS_{t,t+T}}{MRT_{t,t+T}} = \frac{1 - \delta + F_{k,t+1}}{1 + (1 - \tau_{k,t+1})(F_{k,t+1} - \delta)} \cdots \frac{1 - \delta + F_{k,t+T}}{1 + (1 - \tau_{k,t+T})(F_{kt+T} - \delta)}$$

At steady state

$$F_{kt} = \overline{\upsilon}$$

$$\rho = (1 - \overline{\tau}_K)(\overline{\upsilon} - \delta)$$

Analysis of the Ramsey Problem

$$\frac{MRS_{t,t+T}}{MRT_{t,t+T}} = \left(\frac{(1+\overline{\upsilon}-\delta)}{1+(1-\overline{\tau}_K)(\overline{\upsilon}-\delta)}\right)^T$$

$$\left(\frac{(1+\frac{\rho}{1-\overline{\tau}_K}}{1+\rho}\right)^T$$

$$= \left(1+\frac{\rho\overline{\tau}_K}{(1+\rho)(1-\overline{\tau}_K)}\right)^T$$

a constant positive tax rate on capital implies that the tax wedge between marginal rate of substitution between consumption between t and $t+T$ and marginal rate of transformation of consumption between t and $t+T$ grows exponentially with T . Thus, setting capital taxes to zero avoid this growing wedge.

Analysis of the Ramsey Problem

Proposition 2. If preferences are of the form

$$U(c,n) = \frac{c^{1-\sigma} n^{\varphi}}{1-\sigma}$$

Then the solution of the Ramsey problem has zero tax rates starting at $t \geq 2$. Proof. Direct computation gives:

$$W_{ct} = U_{ct} + \lambda(U_{ct} + c_t U_{cct} + n_t U_{nct})$$

$$U_c = c^{-\sigma} n^{\varphi}$$

$$cU_{cc} = -\sigma c^{-\sigma} n^{\varphi}$$

$$nU_{cn} = \varphi c^{-\sigma} n^{\varphi}$$

Thus the foc of the Ramsey problem gives

$$1 = \frac{\beta W_{c,t+1}}{W_{ct,}}[1 - \delta + F_{k,t+1}] = \frac{\beta U_{c,t+1}}{U_{c,t}}[1 - \delta + F_{k,t+1}]$$

for $t \geq 1$, which implies that $\tau_{k,t} = 0$ for $t \geq 2$.

Tax on Labor

Proposition 3. Let preferences be as follows

$$U(c,n) = \frac{c^{1-\sigma} n^{\varphi}}{1-\sigma} or$$

$$U(c,n) = \frac{c^{1-\sigma}}{1-\sigma} - A\frac{n^{\varphi}}{1-\varphi}$$

for $\sigma \geq 0$. Then the tax rate on labor τ_{lt} is constant from $t \geq 1$.
Proof. Consider first the case where the utility function is given as in the first line. Direct computation gives

$$U_n = \varphi \frac{c^{-\sigma}}{1-\sigma} n^{\varphi-1}$$

$$nU_{nn} = (\varphi-1)\varphi \frac{c^{-\sigma}}{1-\sigma} n^{\varphi-1}$$

$$cU_{cn} = -\sigma\varphi \frac{c^{-\sigma}}{1-\sigma} n^{\varphi-1}$$

Tax on Labor

$$U_c = c^{-\sigma} n^{\varphi}$$

$$cU_{cc} = -\sigma c^{-\sigma} n^{\varphi}$$

$$nU_{cn} = \varphi c^{-\sigma} n^{\varphi}$$

thus

$$W_{ct} = U_{ct} + \lambda(U_{ct} + c_t U_{cct} - n_t U_{nct} = U_{ct}(1 + \lambda - \lambda\sigma - \lambda\varphi)$$

$$W_{nt} = U_{nt} + \lambda(U_{nt} + c_t U_{cnt} - n_t U_{nnt} = U_{nt}(1 + \lambda - \lambda\sigma + \lambda(\varphi - 1))$$

The foc of the Ramsey problem gives

$$\frac{-W_{nt}}{W_{ct}} = F_{nt}$$

Replacing the expressions for W_{nt} and W_{ct}

$$\frac{-U_{nt}}{U_{ct}} . \frac{1+\lambda-\lambda\sigma+\lambda(\varphi-1)}{1+\lambda-\lambda\sigma-\lambda\varphi} = F_{nt}$$

and hence the optimal labor tax rate solves:

$$1-\tau_{lt} = \frac{1+\lambda-\lambda\sigma+\lambda(\varphi-1)}{1+\lambda-\lambda\sigma-\lambda\varphi}$$

for $t \geq 1$. Therefore τ_{lt} is constant.

Following analogous steps when the utility function is as in the second line we also conclude that the tax rate on labor is constant. QED.

Consider the budget constraint of a household subject to a net income tax rates and investment tax credits as follows:

$$\sum_{t=0}^{\infty} p_t(i_t + c_t) = \sum_{t=0}^{\infty} p_t[(n_t w_t + \upsilon_t k_t) - \tau_t\,(n_t w_t + \upsilon_t k_t - i_t)]$$

where the law of motion of capital is

$$K_{t+1} = i_t + (1-\delta)k_t$$

Alternative Tax Scheme

Using the law of motion of capital we obtain

$$\sum_{t=0}^{\infty} p_t([k_{t+1} - k_t(1-\delta)][1-\tau_t] + c_t) = \sum_{t=0}^{\infty} p_t[(n_t w_t + \upsilon_t k_t)(1-\tau_t)]$$

The terms with k_{t+1} are

$$k_{t+1}\left[p_t\left(1-\tau_t\right) - p_{t+1}\left(1-\tau_{t+1}\right)\left((1-\delta+\upsilon_{t+1}\right)\right]$$

so setting them to zero we obtain

$$1 + r_{t+1} = \frac{1-\tau_{t+1}}{1-\tau_t}(1-\delta+\upsilon_{t+1})$$

Thus, if income tax rates are constant, $\tau_{t+1} = \tau_t$, we obtain

$$r_{t+1} + \delta = \upsilon_{t+1}$$

which is the same expression that is obtained if net capital income is taxed at the rate $\tau_{t+1} = 0$. A constant income tax rate together with an investment tax credit are equivalent to a constant labor income tax and a zero tax on capital income.