1 A Neoclassical Growth Model with External Finance

Consider a simple neoclassical growth model. Suppose the economy has access to foreign finance at a fixed real interest rate $\bar{r}$.

1. Setup the problem.

2. Find the steady state allocation and discuss your results. Does the economy borrow a positive value in the long run?

3. Think deeply on how does the transition occurs. You may find it counter-intuitive at the beginning.

2 (Optional) A Neoclassical Growth Model with Foreign Direct Investment (FDI)

Consider a simple neoclassical growth model. Suppose you can have foreign direct investment but the return is at the marginal rate for capital.

1. Setup the problem.

2. Find the steady state allocation and discuss your results. Does the economy receive a positive value of FDI in the long run?

3. Think deeply on how does the transition occurs.
3 Representative Agent model Analysis by Simulation

Consider the standard representative agent model with labor supply decision. There is a representative household who solves an infinite-period consumption, labor and investment choices such that

$$\max_{\{c_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t^{1-\sigma} - 1}{1 - \sigma} - \gamma \frac{l_t^{1+\phi}}{1 + \phi} \right)$$

subject to

$$c_t + i_t = w_t l_t + v_t k_t + \Pi_t$$
$$k_{t+1} = (1 - \delta) k_t + i_t$$

for all $t = 0, 1, 2, \ldots$ and $\beta < 1$.

There is a representative firm which rents capital and employs workers to maximize its profit:

$$\max \Pi_t = Ak_t^\alpha l_t^{1-\alpha} - w_t l_t - v_t k_t$$

Markets clear such that $l_t^d = l_t^s$ and $k_t^d = k_t^s$. The final good is the numéraire good with price one. (If needed, you can take $\sigma = 1$ (i.e. log utility))

1. Write down the FOCs and the Euler equation. Solve for the steady state equilibrium.

2. Explain how does your S.S. results (all important macro variables like $y, c, k, i$) depend on $A, \beta, \delta$ and $k_0$.

Suppose the economy is in the steady state at $t = 0$. Analyze the following situations using Dynare (The code provided for you). Specifically explain what would happen to each variable over time using supply-demand analysis in different markets. You can take $\beta = 0.95, \delta = 0.1$. For $\sigma$ try $\sigma = 0.5, 1, 2$

3. An unexpected negative productivity shock happens at time $t = 2$.

4. An expected positive productivity shock happens at time $t = 2$. 

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5. Vary $\sigma$ from 0 to 10 and see how does your responses to $c_t$ vary. Explain intuitively.

4. **Intertemporal Consumption Choice: Uncertainty**

Consider a two-period Intertemporal consumption decision model, where the household’s utility function is given by:

$$U(c_1; c_2) = E[\log (c_1) + \beta \log (c_2)]$$

where $c_1$ and $c_2$ denote period one and two consumptions, respectively, and $\beta < 1$ is the discount factor. The income in period 1 is $y$. But in period 2 the income is $y(1 + \varepsilon)$ with probability $\frac{1}{2}$ and it is $y(1 - \varepsilon)$ with probability $\frac{1}{2}$ where $\varepsilon > 0$ and it is small relative to 1. Household saves (or borrows) $b$ in period 1 and receives $b(1 + r)$ in period 2.

1. Setup the household maximization problem and write down the FOCs accurately.

2. Solve the optimum household choice for $c_1, c_2$ and saving $b$ for a given $r$. How does your answer vary with $\varepsilon, r$ and $y$. Explain the economic intuition.

3. Solve the general equilibrium problem and find the market rate of return $r$. How does your answer vary with $\varepsilon$ and $y$. Explain the economic intuition.

4. (optional) Now repeat the above exercises for the utility function: $U(c_1; c_2) = E \left[ \frac{c_1^{1-\eta}}{1-\eta} + \beta \frac{c_2^{1-\eta}}{1-\eta} \right]$.

5. **(Optional) Simulating an Infinite-Period Model**

Consider the infinite-period model of consumption choice, where the social planner solves:

$$\max_{\{c_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \log c_t$$

subject to

$$c_t + k_{t+1} = A k_t^\alpha + (1 - \delta) k_t$$

for all $t = 0, 1, 2, ....$ Again $\beta < 1$.

1. Write down the Euler Equation.
2. Solve for the steady state allocation.

3. Write a Matlab code to find the optimum \( c_0^* \) for a given \( k_0 \).

4. Plot the time series of \( c_t \) and \( k_t \).

5. Log-Linearize the model and solve for \( k_t \) and \( c_t \).

6. How does the speed of convergence depend on \( \beta \).

6 (Optional) Simulating a Representative Agent model with No Labor-Leisure Decision

Consider the infinite-period model of consumption choice, where the social planner solves:

\[
\max_{\{c_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \log c_t
\]

subject to

\[
c_t + k_{t+1} = Ak_t^\delta + (1 - \delta) k_t
\]

for all \( t = 0, 1, 2, \ldots \). Again \( \beta < 1 \).

In this problem we’d like to do the comparative statics using Dynare.

1. Calibrate your model: Replace the parameters of the model with some sensible numbers.

2. Setup the Dynare code to solve the problem.

3. Suppose the initial capital drops by 10%. Determine the new steady state and plot the transition paths for \( y_t, k_t, i_t, c_t, v_t \). Explain how the results make sense.

4. Suppose the parameter \( A \) rises by 10%. Determine the new steady state and plot the transition paths for \( y_t, k_t, i_t, c_t, v_t \). Explain how the results make sense.