Welfare Gains from Trade: Weak Linkages and Complementarity of Intermediate Goods

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Abstract

In this paper, we propose a theoretical model of international trade that shows the welfare gains from trade are much higher than what Arkolakis et al. (2012) find. We use the idea of weak linkages and the complementarity between the intermediate goods as in Jones (2011). The idea is simple. Production requires many different intermediate goods to get together, and if the firm cannot purchase any of them from home nor from foreign, the production process fails. Same is true if any of these linkages are low productive and the firm cannot import it from a high productive sector from abroad. We propose a multi-industry, multi-country trade model, and show that the complementarity of the intermediate goods results in much larger welfare gains than the current literature measures, using the same trade data. Also, similar to Ossa (2015), we show that the variation of trade elasticities among industries dramatically increases the welfare gains from trade; using Ossa's mechanism alongside the assumption of complementarity magnify the estimated welfare gains from trade.

Keywords: Welfare gains from trade, Complementarity, Weak links, Trade elasticity, Multi-industry general equilibrium model.

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1. Introduction

How large are the welfare gains from international trade? In an influential paper, Arkolakis et al. (2012) show that the standard classical quantitative trade models in the literature have a similar finding to this question: "Not Much". They show that welfare gains from trade in these models can be calculated using the estimates from the trade elasticity and the share of expenditure on domestic goods. As an example, it can be shown that for the United States, the gains from trade would be ranging from 0.7% to 1.4%. Recent papers try to solve this counterintuitive result by introducing new mechanisms and models, representing other features of the economic environments, and predicting larger welfare gains from trade .

In this paper, we introduce a new mechanism to attack the issue by arguing that the complementarity between the intermediate inputs in the production process leads to much larger welfare gains from trade. Our main idea is that while goods in their usage as final and consumption products can be substitutable to each other with an elasticity greater than one, they cannot easily be substituted if they are to be used as intermediate inputs and the elasticity of substitution among intermediate goods would be less than one. Therefore, if the trade is shut down, the endowments in the economy will move to the industries with low productivity which are the weak links of the economy in the production chain and this mechanism would lead to much lower TFP and consequently lower welfare. Thus by moving toward autarky, there would be massive welfare loss due to the complementarity of the intermediate goods in the production process; and the models in international trade would lose a large welfare effect if they ignore the role of such complementarity. This mechanism is similar to the one which is used in Jones (2011) to explain the enormous differences in income across countries in the development literature. The idea is that if a firm is not able to import some varieties, then its whole production line would stop working; leading to a severe value-added loss.

In our model there are many countries all of which produce the variety of goods, using labor and aggregate intermediate good. We assume that there are some sectors in the economy which purchase these varieties in each of the industries from various countries, combine them with a CES function with the elasticity of substitution specific to each of the sectors and sell their aggregate products to the final goods' and intermediate goods' aggregators. The Final goods' aggregator, combines these products with a CES function at the upper level with the elasticity of substitution equal or greater than one and sells the aggregate consumption good to the household. On the other hand, the Intermediate goods' aggregator also purchases the same products, but it aggregates them with an elasticity of substitution less than one and sells this aggregate intermediate good to the domestic varieties' producers.

We find that welfare gains from trade critically depends on the level of complementarity between intermediate inputs; the more complements the inputs are, the higher welfare an economy gain from trade. Also we show that this gain is sensitive to the share of intermediate inputs in the output; the higher tha share of intermediate inputs, the higher gain from trade we get.

Similar to Ossa (2015), our model also shows that the variation of trade elasticities among industries greatly increases the welfare gains from trade, where trade is much more important in some industries. Using the mechanism proposed in Ossa (2015) alongside the assumption of complementarity of intermediate goods magnifies the estimated welfare gains from trade, where Ossa (2015) argues that trade is critical in some industries and we add this point to Ossa (2015) that it may be essential to the economy as a whole too, if the intermediate goods are complements of each other in the production process.

Similar to the works of Alvarez and Lucas (2007), Krugman and Venables (1995), Eaton and Kortum (2002), Di Giovanni and Levchenko (2013), Caliendo and Parro (2015), Balistreri (2011), and Ossa (2015) our model takes intermediate goods into account. The main feature that distinguishes our paper from other quantitative trade models which involve trade in intermediates goods is that we assume the elasticity of substitution among intermediate goods is less than one in contrast to the elasticity of substitution among final goods being greater than one. We show that this complementarity induces vast welfare gains from trade which are missing in the literature.

To make this point, we propose a multi-country, multi-industry general equilibrium model that takes into account intermediate goods for production as complements to each other. Our trade model is a simple multi-industry version of Armington (1969) as in Ossa (2015). As it is shown by Costinot and Rodriguez-Clare (2014), multi-industry international trade models such as Anderson and Yotov (2010), Donaldson (2016), Caliendo and Parro (2015), Costinot et al. (2012), Hsieh and Ossa (2016), Levchenko and Zhang (2016), Ossa (2015), Shikher (2012a) and Shikher (2012b) result in larger welfare gains from trade comparing to models with only one sector. The reason is that the complementarity between industries introduced by a Cobb-Douglas form results in a larger welfare loss if the prices in some industries increase due to the inability or weakness of the country in producing in that sectors.

Costinot and Rodriguez-Clare (2014) , in their seminal paper, have surveyed a large literature regarding the gains from trade in numbers. They have also investigated the role of non-unity elasticities of substitution among sectors. They consider models in which the sectors have upper-level elasticities of substitution other than one, varying from 0 to infinity and show that the gains from trade are high when this elasticity is less than one. In contrast, as Jones (2011) also asserts, a better assumption is that the intermediate inputs are mostly complements with each other while the final consumptions goods are mostly imperfect substitutes. Therefore, we assume a two-tier aggregate function for the intermediate goods, where at both levels, the functions are general CES functions with the elasticity of substitution equal or greater than one for final goods' upper level and with the elasticity of substitution less than one for the intermediate inputs. In this framework, inability to produce or low productivity in some sectors would lead to much more significant welfare loss in moving toward autarky than what is calculated in Costinot and Rodriguez-Clare (2014) for multi-sector models. We also show that trade helps in fortifying the weak links of the economy by decreasing the domestic goods share in total expenditure in the industries with low productivity, the mechanism which is missed in Ossa (2015).

Although Costinot and Rodriguez-Clare (2014) show that there are larger welfare gains from trade if the upper aggregate function is a CES of the final goods with an elasticity of substitution less than one, instead of Cobb-Douglas, they argue that "all elasticities are equal to one until shown to be otherwise". In this paper, we argue that unlike final goods, the CES assumption with the elasticity of substitution less than one is not far from reality as Jones (2011) argues that "Complementarity and linkages often go together as in Hirschman (1958). This is in part because complementarity naturally arises when one considers intermediate goods; electricity, transportation, and raw materials are all essential inputs into the fsproduction." He shows that these assumptions bring us closer to understanding large income differences across countries, and in this paper, we show that they also bring us to understanding welfares that various nations gain from trading in intermediate goods.

In the importance of the effect of the complementarity of intermediate goods in the economy, Atalay (2017) in business cycle literature, using input prices and input choices data for U.S. industries, shows that the intermediate inputs produced by different upstream industries are complement to each other, and the estimates of elasticity of substitution are always significantly less than one. By this complementarity, he shows that sectoral and industry-specific shocks are important and are accounting for more than half of the variation in the aggregate output growth, due to the input-output linkages. Also, there is a vast literature on the capital-skill complementarity; and some papers such as Parro (2013) and Burstein et al. (2013) study the role of this complementarity on trade's outcomes. Parro (2013) and Burstein et al. (2013) show that since capital is more substitutable for unskilled labor, compared to skilled labor, growing imports in capital equipment in most of the countries would result in increasing demand for skilled labor and hence increases skill premium across countries. They show that trade increases the real wage of both skilled and unskilled labor, with disproportionally larger gains for skilled labor when allowing for capital-skill complementarity.

The remainder of the paper is as follows: In section 2, we will introduce the main idea of the complementarity and industries' linkages with a closed economy model which is a static and modified version of the model used in Jones (2011). In section 3, we develop a multi-industry Armington (1969) model of trade as in Ossa (2015) and solve the model and investigate the welfare gains from trade change when the intermediate goods are assumed to be the complement of each other. In section 4, we introduce numerical solutions of the model. In section 5, the results for the welfare gains from trade are shown by using simple examples.

2. Closed Economy Model

To investigate the role of the complementarity of intermediate goods in the welfare gains from the international trade, it will be useful to introduce the model in the more straightforward context of the closed economy before turning to the study of the open economies.

2.1. Economic Environment

The closed economy model used in this paper is similar to the model used in Jones (2011). There are S number of goods indexed by s, produced in the economy; each has a Cobb-Douglas production function:

$$Q_s = A_s \left(\frac{L_s}{\gamma}\right)^{\gamma} \left(\frac{X_s}{1-\gamma}\right)^{1-\gamma}$$

where γ is between zero and one, L_s is the amount of labor force used to produce good s, X_s is the aggregate intermediate good used to produce s and A_s is an exogenously given productivity level. In this production function, $1-\gamma$ measures the importance of linkages in the economy. If $\gamma = 1$, the productivity of labor force in each sector depends only on A_s and is independent of the rest of the economy.

Each of the goods in the economy can be used for one of the two following purposes: as a final good c_s or as an intermediate input z_s. Therefore,

$$c_s + z_s = Q_s$$

To be more convenient and similar to Jones (2011), instead of specifying a utility function over the continuum of final consumption uses, we define a single representative final good producer who produces the aggregate final consumption good, which represents the GDP in the economy:

$$Y = \left(\sum_{s=1}^{S} c_s^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

, where S is the number of goods (industries) produced in the economy. These final goods aggregate up with an elasticity of substitution greater than or equal to one which is specified by $\sigma \geq 1$.

While the final good producer combines the S goods with an elasticity of substitution greater than one in order to produce GDP, intermediate inputs are combined with an elasticity of substitution less than one by a representative intermediate good producer and this is the point in which "weak links" enter our model as in Jones (2011):

$$X = \left(\sum_{s=1}^{S} z_s^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$$

, where η is the elasticity of substitution between intermediate goods and $0 < \eta < 1$.

This aggregate intermediate good is what is used in producing the goods; hence the resource constraint holds as:

$$\sum_{s=1}^{S} X_s \leq X$$

So, varieties that are used as intermediate goods involve substantial complementarity, but when these same varieties combine to produce final consumption, there is more substitutability.

An example in Jones (2011) illustrates this: "computer services are today nearly an essential input into semiconductor design, banking, and health care. But computers are much more substitutable when used for final consumption—for entertainment, we can play computer games or watch television or ride bikes in the park. In order to produce within a firm, there are a number of complementary steps that must be taken. In final consumption (e.g., in utility), however, there appears to be a reasonably high degree of substitution across goods."

Finally, the resource constraints for labor force is:

$$\sum_{s=1}^{S} L_s \leq \bar{L}$$

2.2. Competitive Equilibrium

In this section, we investigate optimization problems of different sectors in the closed economy model and then introduce equilibrium conditions.

2.2.1. Optimization Problems

Letting final output good as numeraire, the optimization problems of various sectors in economy are like as follows:

Final Sector Problem: Taking the prices of the consumption varieties, p_s, as given, a representative firm in the perfectly competitive market solves:

$$\max_{c_s} \left(\sum_{s=1}^{S} c_s^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - \sum_{s=1}^{S} p_s c_s$$

Intermediate Sector Problem: Taking the prices of the intermediate varieties, p_s, and the price of the aggregate intermediate good, q, as given, a representative firm in the perfectly competitive market solves:

$$\max_{z_s} q \left(\sum_{s=1}^{S} z_s^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} - \sum_{s=1}^{S} p_s z_s$$

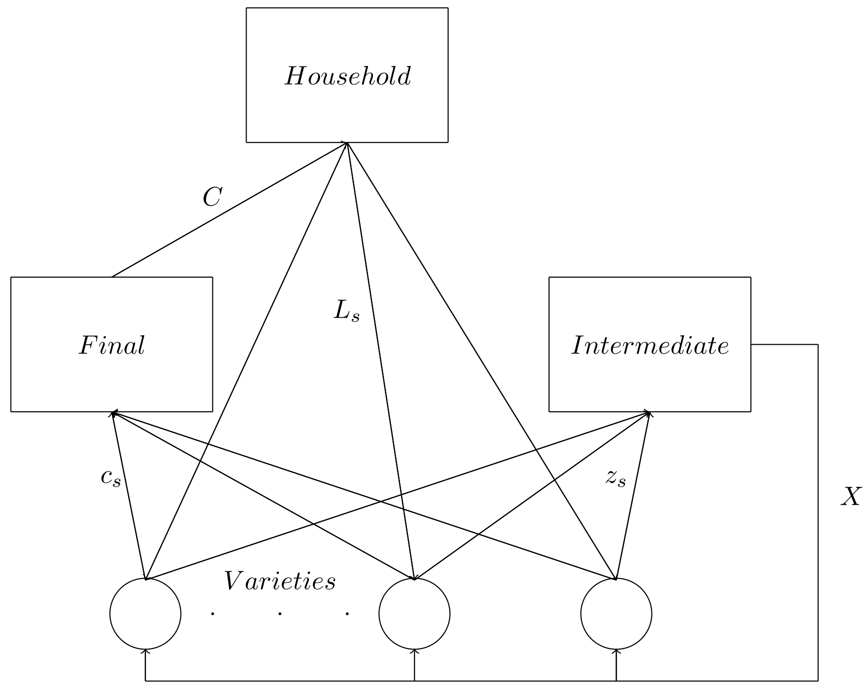

Figure 1: Schematic of the Closed Economy Model

Variety s's Problem: Taking its product's price, p_s, labor force's wage, w, and the price of the aggregate intermediate good, q, as given, a representative firm in the perfectly competitive market solves:

$$\max_{X_s, L_s} p_s A_s \left(\frac{L_s}{\gamma}\right)^{\gamma} \left(\frac{X_s}{1-\gamma}\right)^{1-\gamma} - wL_s - qX_s$$

Household Problem: In this economy, the household does not have any optimization problem. He has an inelastic labor supply and uses the income to purchase the aggregation of final consumption goods. So his budget constraint would be:

$$C = w\bar{L}$$

The schematic of the closed economy model has been shown in figure 1.

2.2.2. Defining the Competitive Equilibrium

Definition: *A competitive equilibrium in this economy consists of the quantities $Y, X, C, Q_s, L_s, X_s, c_s, z_s$ and prices p_s, q, w such that:*

1) Household problem holds, 2) Representative final sector producer's profit is maximized, 3) Representative intermediate good producer's profit is maximized, 4) Variety s producers for all $s \in \{1,2,...,S\}$ maximize their profits, 5) All markets clear; therefore $\sum_{s=1}^{S} L_s = \bar{L}$, $c_s + z_s = Q_s$ for all $s \in \{1,2,...,S\}$, and $\sum_{s=1}^{S} X_s = X$

So, the competitive equilibrium involves 11 endogenous variables and specifies 11 equations to pin them down. The market for final output clears by Walras' Law ($C = Y$).

2.3. Solving for the Competitive Equilibrium

By addressing the model for the closed economy, the aggregate production of the GDP and consequently, the consumption of the household in the economy are obtained as the following proposition:

PROPOSITION 1: (The Competitive Equilibrium, Closed Economy): GDP in the competitive equilibrium is:

$$Y = \left(B_\sigma^\gamma B_\eta^{1-\gamma}\right)^{\frac{1}{\gamma}} \bar{L} \tag{1}$$

where

$$B_\eta = \left(\sum_{s=1}^{S} A_s^{\eta-1}\right)^{\frac{1}{\eta-1}} \tag{2}$$

and B_σ is defined in a way analogous to B_η.

Proof: See Appendix.

This proposition shows how low productivity in a sector affects output through TFP and shows up as a reduction in TFP at the macro level; specifically, when the productivity of one of the sectors is very low, it pushes the whole TFP toward zero due to the term B_η, coming from the aggregator intermediate goods, while this is not true for the aggregator of final goods. This is a new feature above and beyond standard trade models. Also, the proposition shows that there is a multiplier effect in the presence of intermediate goods; an increase in productivity of different sectors raises output by more than one to one; since higher output leads to more intermediate goods, which boosts output, and so on. In other words, one percent increase in the productivity of all the sectors increases total output by 1_γ which is greater than one. Finally, it shows that the more share of intermediate goods results in a higher scaling effect 1_γ.

It is worth noting that B_η and B_σ, as the CES combinations of various sectors' productivity, are power means of the productivity of these sectors. The domain of $\eta - 1$ is $(-1, 0]$. Suppose, $\eta - 1 \to -1$, in this case, the equilibrium allocation depends on the harmonic mean of the productivity of sectors and if $\eta - 1 \to 0$, the equilibrium allocation depends on the geometric mean of them. So, B_η is between harmonic and geometric means of the productivity of various sectors, and the more the complementarity is among the sectors, the more B_η tends toward the harmonic mean, and so the impact of the weakest link in the economy on the reduction of TFP would be larger. Analogously, the domain of $\sigma - 1$ is $(0, +\infty)$. Let $\sigma - 1 \to 0$, in such a case; equilibrium depends on the geometric mean of the productivity of various sectors. If $\sigma - 1 \to 1$, then B_σ tends toward the arithmetic mean of the productivity of sectors and if $\sigma - 1 \to \infty$, B_σ tends toward the maximum of the productivity of them, like a "superstar" production function in Rosen (1981). So, B_σ varies from the geometric mean of the productivity of varieties to the arithmetic mean and the maximum value of them.

So weak links in the economy do not have as much effect in the final usages of the goods produced. In contrast, due to the complementarity of the intermediate goods, any distortion associated with misallocation and also low productivity in some sectors of the economy would spread, with more intensity, to other sectors and to the economy as a whole and would lead to a much lower TFP and consequently the lower aggregate product of the economy. However, the competition in the economy partly modifies and alleviates this effect by allocating more resources to the sectors with low productivity. An example similar to an argument from Jones (2011) may lead us to a clear understanding of the subject: "If the transportation sector has especially low productivity that would otherwise be very costly to the economy, the equilibrium allocation can put extra [labor force] in that sector to help offset its low productivity and prevent this sector from becoming a bottleneck. Of course, this must be balanced by the desire to give this sector a low amount of resources in an effort to substitute away from transportation on the consumption side. This can be seen in the math: the equilibrium solution for allocating [labor force] is:"

$$\frac{L_s}{\bar{L}} = \gamma \left(\frac{A_s}{B_\sigma} \right)^{\sigma - 1} + (1 - \gamma) \left(\frac{A_s}{B_\eta} \right)^{\eta - 1} \tag{3}$$

Because $\eta - 1 < 0$ while $\sigma - 1 > 0$, low productivity in producing variety s increases the desired labor force allocated to that sector according to the second term in the right side of the equality which is due to the complementarity effect, but reduces the assigned labor force according to the first term which represents the substitution effect.

3. Open Economy Model

In this section, we generalize the model to the open economy version which consists of N countries. These countries trade in varieties consisting of both usages as intermediate goods and final goods.

3.1. Economic Environment in Open Economy Model

In this economy, each of the countries has a fixed endowment of labor. Let total labor endowment be $L = \left(\bar{L}_1, ..., \bar{L}_n\right)$, where $\bar{L}_i$ is the entire endowment of the labor force in the country i. Labor is costlessly mobile across sectors within a country but is immobile across countries. Similar to the closed economy model, variety goods indexed by s in the country i are produced by a Cobb-Douglas production function in perfectly competitive markets:

$$Q_{is} = A_s \left(\frac{L_{is}}{\gamma}\right)^{\gamma} \left(\frac{X_{is}}{1-\gamma}\right)^{1-\gamma} \tag{4}$$

Preferences and the production function's parameter, γ, and also the elasticity of substitution for the final goods and intermediate goods, which are represented by σ and η respectively, are common to all countries.

The international trade model used here is a multi-country, multi-industry Armington (1969) type model in final and intermediate goods as in Ossa (2015). The multi-industry assumption has a crucial role in our model as in Jones (2011) since the model's mechanism is based on the complementarity of different intermediate goods and different elasticity of substitution for the consumption and intermediate goods.

Trade barriers for country i's exports to country j for the variety s, denoted by $\tau_{ijs} \geq 1$, are assumed to be of the Samuelson's iceberg costs type, in the sense that τ_{ijs} units must leave country i for one unit to arrive in country j, and the equality is for $i = j$, that the country consumes its own productions. Therefore, defining M_{ijs} as the amount of the variety s produced in the country i and sold in the country j, we have:

$$Q_{is} = \sum_{i=1}^{N} \tau_{ijs} M_{ijs} \tag{5}$$

For each of the varieties s, the available good in the country j is a CES aggregates of that variety produced in different countries including j itself and exported to the country j:

$$M_{js} = \left(\sum_{i=1}^{N} M_{ijs}^{\frac{\mu_s - 1}{\mu_s}}\right)^{\frac{\mu_s}{\mu_s - 1}} \tag{6}$$

where $\mu_s > 1$ is the elasticity of substitution between different countries production for the industry s and M_{js} is the available good s in the country j which could be used either as an intermediate good z_{js} or a consumption good c_{js}:

$$M_{js} = c_{js} + z_{js} \tag{7}$$

Similar to the closed economy model, there is a representative final good producer in country j as an aggregator for the final good consumption:

$$Y_j = \left(\sum_{s=1}^{S} c_{js}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{8}$$

where the final goods available in the country j aggregate up with an elasticity of substitution greater than one ($\sigma > 1$). In contrast, similar to the closed economy model, intermediate inputs are combined with an elasticity of substitution less than one, $\eta < 1$, to produce the aggregate intermediate good by a representative producer.

$$X_j = \left(\sum_{s=1}^{S} z_{js}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{9}$$

Finally, the resource constraints for the intermediate goods and labor force are respectively as below:

$$\sum_{s=1}^{S} X_{js} \leq X_j \tag{10}$$

and for all j we have:

$$\sum_{s=1}^{S} L_{js} \leq \bar{L}_j \tag{11}$$

3.2. The Competitive Equilibrium in Open Economy Model

In this section, we investigate optimization problems of different sectors in the open economy model and then introduce equilibrium conditions.

3.2.1. Optimization Problems

The optimization problems of various sectors in the economy are as follows:

Domestic Variety s's Problem: Similar to the closed economy problem, the variety s producer in the country i takes the price of variety s of country i in country j, p_{ijs}, as given, in addition to the price of labor and the aggregate inputs in country i: w_i and q_i. Unlike the closed economy, the representative firm bears extra costs due to the iceberg costs of exporting its products to the country j in which τ_{ijs} is taken as known by the firm. Since the production function is CRS, this producer faces the following problem for each country j:

$$\max_{X_{ijs}, L_{ijs}} p_{ijs} A_{is} \left(\frac{L_{ijs}}{\gamma} \right)^{\gamma} \left(\frac{X_{ijs}}{1-\gamma} \right)^{1-\gamma} - w_i L_{ijs} - q_i X_{ijs} - \left(1 - \frac{1}{\tau_{ijs}} \right) p_{ijs} A_{is} L_{ijs}^{\gamma} X_{ijs}^{1-\gamma}$$

Traded Variety s's Problem: There is an extra sector in the open economy model compared to the closed economy which aggregates the productions of various countries in the industry s with the elasticity of substitution $\mu_s > 1$. By taking p_{js} and p_{ijs} as given, it solves:

$$\max_{M_{ijs}} p_{js} \left(\sum_{i=1}^{N} M_{ijs}^{\frac{\mu_s-1}{\mu_s}} \right)^{\frac{\mu_s}{\mu_s-1}} - \sum_{i=1}^{N} p_{ijs} M_{ijs}$$

Final Sector Problem: Taking p_{js} as given, a representative firm in the perfectly competitive market purchases various industries consumption goods from the aggregators of various countries productions which are available in country j and aggregates them with the elasticity of substitution $\sigma > 1$ and sells the aggregate consumption good to the household for price P_j^F. So it solves:

$$\max_{c_{js}} P_j^F \left(\sum_{s=1}^{S} c_{js}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - \sum_{s=1}^{S} p_{js} c_{js}$$

Intermediate Sector Problem: Taking p_{js} as given, a representative firm in the perfectly competitive market purchases various industries intermediate goods from the aggregators of different countries productions which are available in country j and aggregates them with the elasticity of substitution $\eta < 1$ and sells the aggregate intermediate good to different varieties active in the country for price q_j. So it solves:

$$\max_{z_{js}} q_j \left(\sum_{s=1}^{S} z_{js}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} - \sum_{s=1}^{S} p_{js} z_{js}$$

Household Problem: The household's problem is similar to the closed economy model and for the household in country j the budget constraint would be:

$$P_j^F C_j = w_j \bar{L}_j$$

The schematic of the open economy model has been shown in figure 2.

3.2.2. Defining the Competitive Equilibrium

Definition: *A competitive open economy equilibrium consists of the quantities $Y_j, X_j, C_j, Q_{js}, L_{js}, X_{js}, c_{js}, z_{js}$ and prices $p_{ijs}, p_{js}, q_j, w_j$ such that in each country j:*
1) Household problem holds, 2) Representative final sector producer's profit is maximized, 3) Representative intermediate good producer's profit is maximized, 4) Traded Variety s aggregators maximize profit, 5) Variety s producers for all $s \in \{1, 2, ..., S\}$ maximize their profits, 6) All markets clear; therefore $\sum_{s=1}^{S} L_{js} = \bar{L}_j$, $c_{js} + z_{js} = Q_{js}$ for all $s \in \{1, 2, ..., S\}$, and $\sum_{s=1}^{S} X_{js} = X_j$.

3.3. Solving the Competitive Equilibrium

The competitive equilibrium involves 14 endogenous variables and specifies 14 equations to pin them down. The market for final output clears by Walras' Law ($C_j = Y_j$). By solving the model for the open economy, the aggregate production of the GDP and consequently, the consumption of the household in each country are obtained as in the following proposition:

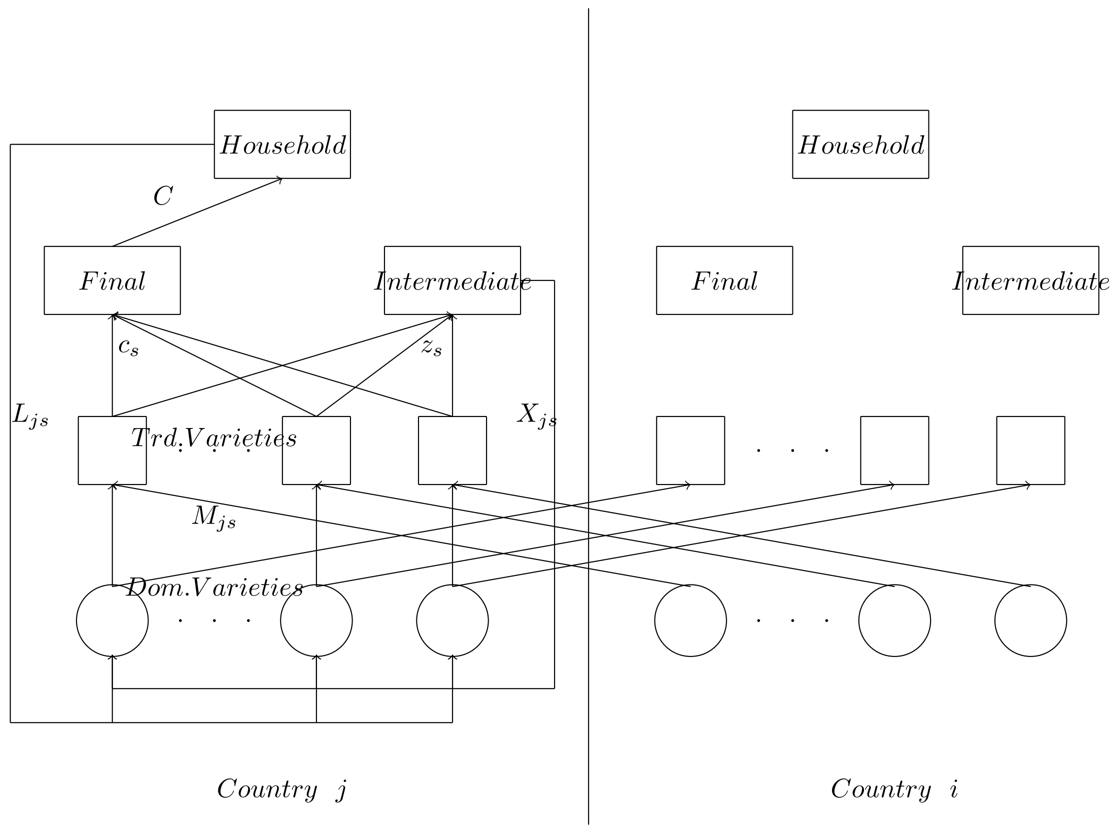

Figure 2: Schematic of the Open Economy Model

PROPOSITION 2: (The Competitive Equilibrium in an Open Economy): GDP of the country j in the competitive equilibrium for the open economy is:

$$Y_j = \left(T_j\left(\sigma,\lambda\right)^{\gamma} T_j\left(\eta,\lambda\right)^{1-\gamma}\right)^{\frac{1}{\gamma}} \bar{L}_j \tag{12}$$

where

$$T_j\left(\eta,\lambda\right) = \left(\sum_{s=1}^{S} A_{js}^{\eta-1} \lambda_{js}^{\frac{1-\eta}{\mu_s-1}}\right)^{\frac{1}{(\eta-1)}} \tag{13}$$

and $T_j\left(\sigma,\lambda\right)$ is defined in a way analogous to $T_j\left(\eta,\lambda\right)$. $\lambda_{js} = \frac{V_{jjs}}{E_{js}}$ is the own trade share in industry s of country j, where E_{js} is the total expenditure of the country j on the productions of different countries including itself in the industry s and V_{jjs} is the total expenditure of country j on its productions in the industry s and it is a particular case of a more general Gravity equation for the value of industry s trade flowing from country i to country j as:

$$V_{ijs} = p_{ijs}^{1-\mu_s} p_{js}^{\mu_s-1} E_{js} \tag{14}$$

Proof: The proof of this proposition can be found in the appendix.

In order to measure the welfare gains from international trade, one should compare the GDP calculated from the open economy model with the GDP in the autarky. In the autarky, in all industries, domestic goods absorb all of the expenditures, where for all of the s's, $\lambda_{js} = 1$. Therefore, the welfare gains from international trade would be:

$$welfare - gain = \frac{Y_j}{Y_j^{aut}} = \left[\left(\frac{T_j\left(\sigma,\lambda\right)}{B_j\left(\sigma\right)}\right)^{\gamma}\left(\frac{T_j\left(\eta,\lambda\right)}{B_j\left(\eta\right)}\right)^{1-\gamma}\right]^{\frac{1}{\gamma}} \tag{15}$$

$B_j\left(\sigma\right)$ and $B_j\left(\eta\right)$ are defined in the Proposition 1. Equation 15 is a useful one to discuss the role of the complementarity of the factors of production in the welfare gains from international trade and the next two sections we will discuss different aspects of this equation numerically and we will show that international trade helps by decreasing domestic goods' share in total expenditures, especially in the weaker industries which are acting as weak links in the production's chain; in the autarky these weak links of the economy absorb much of the resources in the market allocation due to the complementarity of intermediate goods in producing varieties in the economy.

Calculating welfare gains from international trade using equation 15 requires the values of the productivity in various sectors which is hard to measure. In order to calculate the welfare gains from international trade with the observable data, we modify equation 15 in a way that the welfare gains can be calculated by an industries' share in the total expenditures of the country on final and intermediate goods. Thus, the welfare gains from trade can be obtained as in the proposition 3:

PROPOSITION 3: (Welfare Gains from Trade): The welfare gains from international trade concerning various industries' share of the total expenditures on final and intermediate goods is:

$$welfare - gain = \left[G_j\left(\sigma, \lambda\right)^{\gamma} G_j\left(\eta, \lambda\right)^{1-\gamma}\right]^{\frac{1}{\gamma}} \tag{16}$$

where

$$G_j\left(\sigma, \lambda\right) = \left(\sum_{s=1}^{S} e_{js}^{F} \lambda_{js}^{\frac{\sigma-1}{\mu_s-1}}\right)^{\frac{1}{(\sigma-1)}} \tag{17}$$

and

$$G_j\left(\eta, \lambda\right) = \left(\sum_{s=1}^{S} e_{js}^{X} \lambda_{js}^{\frac{\eta-1}{\mu_s-1}}\right)^{\frac{1}{(\eta-1)}} \tag{18}$$

e_{js}^{F} and e_{js}^{X} are industry s's share in the total expenditures on final and intermediate usages respectively.

Proof: The proof of this proposition can be found in the appendix.

Finally, we assume that the productivity and domestic goods' share are jointly normally distributed. In this case, the welfare gains from trade obtain as the proposition 4:

PROPOSITION 4: (Competitive Equilibrium, Open Economy, Random Productivity and Domestic Goods Share in Total Expenditure): Let $a_s = log(A_s)$ and $\omega_s = log(\lambda_s)$ be jointly normally distributed so that $a_s \sim N(\mu_a, \nu_a^2)$, $\omega_s \sim N(\mu_\omega, \nu_\omega^2)$, and $cov(\omega_s, a_s) = \nu_{a\omega}$. Then the welfare gains from trade would be:

$$\begin{aligned} welfare - gain = \gamma \left(\frac{\mu_w}{1-\mu} + \frac{1}{2}\left(\sigma - 1\right)\left(\left(\frac{1}{1-\mu}\right)^2 \nu_\omega^2 + 2\nu_{a\omega}\right)\right) + \\ (1-\gamma)\left(\frac{\mu_w}{1-\mu} + \frac{1}{2}\left(\eta - 1\right)\left(\left(\frac{1}{1-\mu}\right)^2 \nu_\omega^2 + 2\nu_{a\omega}\right)\right) \end{aligned} \tag{19}$$

In the welfare gain expression for the random productivity and random domestic goods share expression, for simplicity, we assume that the elasticity of substitution among different countries' productions is identical among all of the industries, which is denoted by μ.

Proof: The proof of proposition 4 is in the appendix.

According to Proposition 4, in order to have a larger welfare gains from trade with lower elasticity of substitution among intermediate goods, the covariance of logarithm of productivity and logarithm of domestic goods' share in total expenditures should be negative enough; this means that the domestic goods' share in total expenditures should be larger for weak industries compare to stronger industries, which could be a result of higher trade barriers in weak links of the production chain, due to the protection and import substitution policies.

4. Numerical Solutions

In order to calculate the welfare gains from trade, the following parameters should be calibrated: the elasticity of substitution of final goods and intermediate goods, σ and η respectively, the elasticity of substitution among various countries' production for different industries, μ_s, labor share from output, γ, and the productivity in producing different industries, A_s.

For the elasticity of substitution among final goods, following Jones (2011) and Hsieh and Klenow (2009), we take $\sigma = 3$ as the benchmark value. For the elasticity of substitution among intermediate goods, we use $\eta = 0.5$ as in Jones (2011), which is smaller than similar studies trying to calculate the welfare gains from trade. We use other values of η for the sensitivity analysis. For instance, we investigate the results when $\eta \to 0$ which is equivalent to the Leontief production function and in which different goods cannot be substituted with the other intermediate goods. In this case, the industry with the lowest productivity is the weakest link in the production chain. The other case to study is $\eta \to 1$, the Cobb-Douglas production function, where TFP in the intermediate goods section is the geometric mean of different industries productivity. Finally, we assume that elasticity of substitution of intermediate goods section is no different from the elasticity of substitution of the final goods section and define $\eta = 3$; this helps our results to be comparable to other studies in calculating the welfare gains from trade, which do not separate the intermediate goods and final goods in the degree of substitutability. For the labor share in the output we use $\gamma = 0.5$ as in Jones (2011) and Alvarez and Lucas (2007) which has been calibrated using World Bank data, United Nations databank and U. S. Bureau of Economic Analysis data. Again, we use other values for this parameter, to investigate how the welfare gains from trade are affected by the value of the labor share in the output.

5. Results

The main idea that makes this paper distinguished from similar studies in the welfare gains from trade is the assumption of the complementarities among intermediate goods which shows itself in TFP and trade elasticity. As discussed in the previous sections, the core idea on the role of the complementarity of factors of productions is that, an industry with low productivity takes the role of the weak link in the production chain and affects the other sectors through the TPF and the sectors link via the aggregate intermediate good which is used in production of all of the industries and leads to lower GDP and losses in the welfare of the economy. Therefore, if the weak links in the economy are fortified, due to the complementarity of the intermediate goods, larger multiplier factor for the TFP would be acquired. Opening to international trade and reducing trade barriers in the weak links of the economy are of the beneficial proposed policies to increase TFP and consequently GDP and welfare in the economy.

First, to show the effect of the complementarity of intermediate goods on the welfare gains of trade via a simple mechanism, assume that there are just two industries in the economy where the productivity in one of the industries is lower than the other, which takes the weak link role in our model. Assume that the weak industry's productivity is equal to 0.2, $A_1 = 0.2$, and for the other industry, we have $A_2 = 1$. Assume that initially, the economy is in autarky, $\lambda_1 = \lambda_2 = 1$. Now suppose that the stronger industry still remains closed and the weaker industry, as the weak link of the economy moves toward the freer trade, say λ_1 moves from1 to 0.1. We assume $\mu_1 = \mu_2 = 4$, $\sigma = 3$, $\gamma = 0.5$ and calculate and show the welfare gains from trade for four different values for the elasticity of substitution among intermediate goods: $\eta \to 0$, $\eta \to 1$, $\eta = 0.5$ and $\eta = 3$. The results have been shown in the figure 3.

Figure 3 shows that if the elasticity of substitution among intermediate goods decreases, i.e. if the degree of their complementarity with each other increases, the welfare gains from international trade increase significantly, in that the industry which was the Achilles Heel of the economy and used to absorb more substantial amounts of economy's endowments, due to its lower productivity and higher degree of the complementarity of intermediate goods, now gets smaller by moving toward the free trade, owing to the possibility of importing from foreign countries which is now available in the open economy analysis. Therefore, labor and intermediate goods would move toward the industries with higher productivity and would lead to higher TFP of the economy.

Intermediate goods' share in the output is another parameter which affects the welfare gains from trade. As discussed in the previous section, there is a multiplier effect in the presence of intermediate goods which provide links between sectors which is similar to the multiplier effect in the growth and development models. So if the intermediate goods' share in outputs gets larger, the multiplier effect would be larger and the welfare gains from international trade increases, especially in more significant degrees of complementarity.

Figure 4 shows this for three values of elasticity of substitution among intermediate goods, $\eta \to 0$, $\eta = 0.5$ and $\eta \to 1$ for three amounts for labor share, and consequently intermediate goods share, in output, $\gamma = 0.4$, $\gamma = 0.5$, and $\gamma = 0.6$. Figure 4 displays that if the elasticity of substitution among intermediate goods decreases, the welfare gains from trade increase, and also the more intermediate goods share is in the output, which is equivalent to lower values for γ, the more welfare could be gained from trade.

The elasticity of substitution among final goods also affects welfare gains

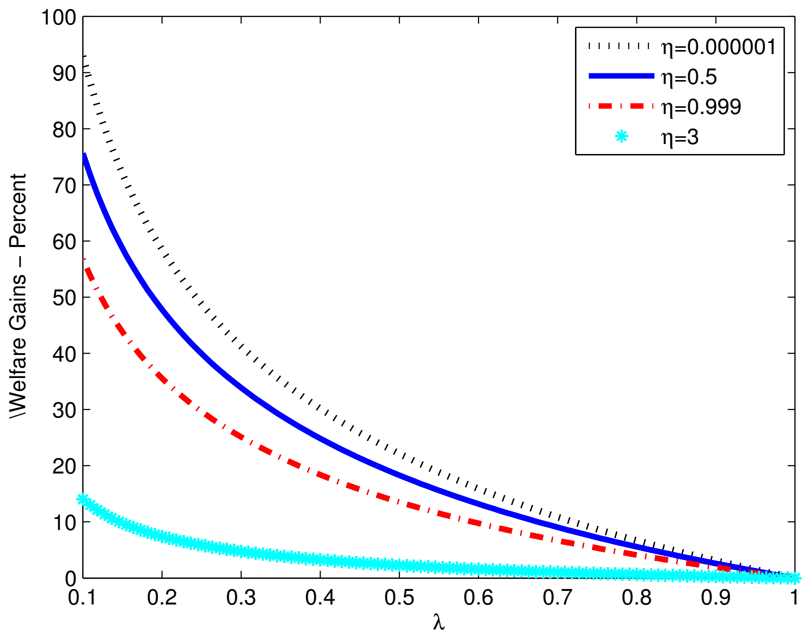

Figure 3: Welfare Gains from Decreasing Trade Barriers of the Weaker Sector for Different Values for Intermediate Goods' Elasticity of Substitution

Note. The horizental axis shows the domestic goods' share in total expenditure and the vertical axis shows the welfare gains from trade in percent in a two industry economy, where one industry's productivity is $A_1 = 0.2$ and the other's is $A_2 = 1$. The welfare gains from trade have been calculated by maintaining the second industry in autarky and reducing trade barriers in weaker industry. trade elasticity for both industries is $\mu_s = 4$, elasticity of substitution among final goods is $\sigma = 3$ and intermediate goods' share in the output is $1 - \gamma = 0.5$. Welfare gains from trade is displayed for four different values of the elasticity of substitution among intermediate goods, μ.

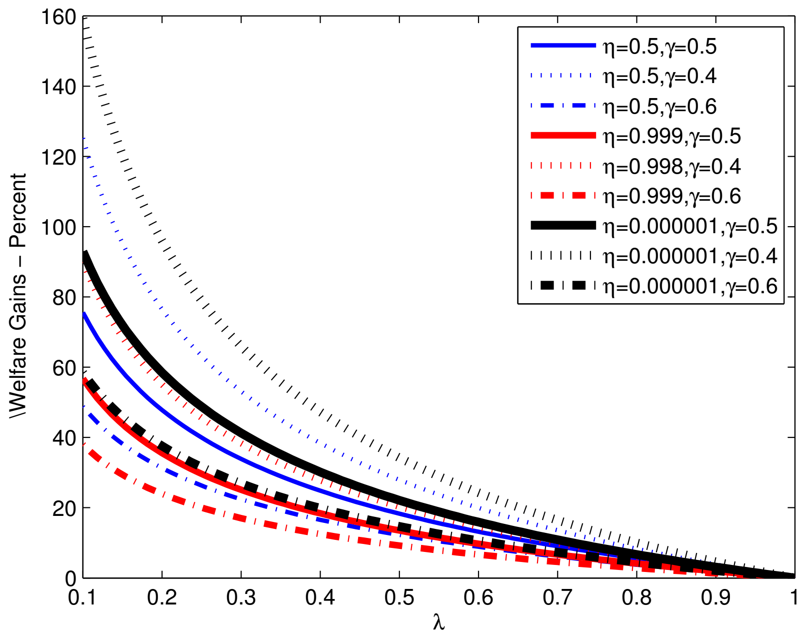

Figure 4: Intermediate Goods' Share in Output Effect on the Welfare Gains from Trade

Note. The horizental axis shows the domestic goods' share in total expenditure and the vertical axis shows the welfare gains from trade in percent in a two industry economy, where one industry's productivity is $A_1 = 0.2$ and the other's is $A_2 = 1$. The welfare gains from trade have been calculated by maintaining the second industry in autarky and reducing trade barriers in weaker industry. trade elasticity for both industries is $\mu_s = 4$,and the elasticity of substitution among final goods is $\sigma = 3$. Welfare gains from trade is displayed for two different values of the elasticity of substitution among intermediate goods, μ and three different values of the intermediate goods' share in the output. Lower values for γ is equivalent to bigger intermediate goods' share in the output.

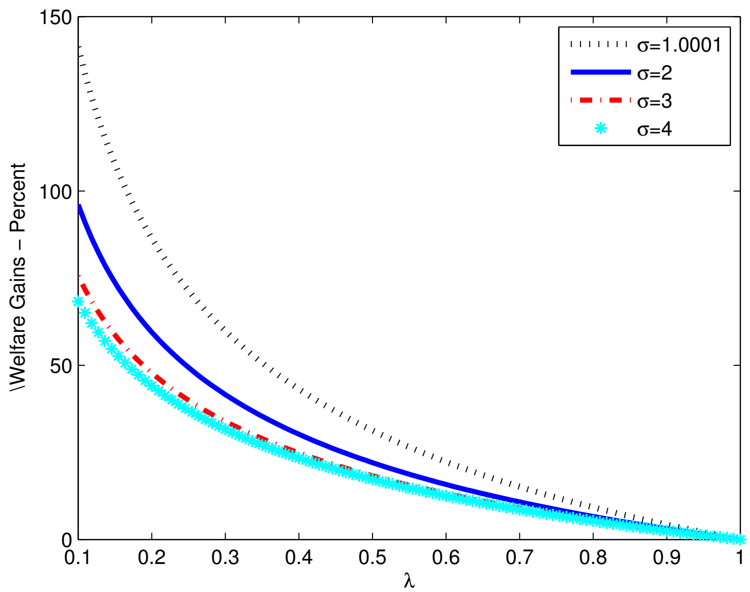

Figure 5: Welfare Gains from Decreasing Trade Barriers of the Weaker Sector for Different Values for Final Goods' Elasticity of Substitution

Note. The horizental axis shows the domestic goods' share in total expenditure and the vertical axis shows the welfare gains from trade in percent in a two industry economy, where one industry's productivity is $A_1 = 0.2$ and the other's is $A_2 = 1$. The welfare gains from trade have been calculated by maintaining the second industry in autarky and reducing trade barriers in weaker industry. trade elasticity for both industries is $\mu_s = 4$, elasticity of substitution among intermediate goods is $\eta = 0.5$ and intermediate goods' share in the output is $1 - \gamma = 0.5$. Welfare gains from trade is displayed for four different values of the elasticity of substitution among final goods, σ.

from trade similar to the elasticity of substitution among intermediate goods, where welfare gains from trade would be larger for lower values of the elasticity of substitution among final goods. This has been shown in figure 5, where the intermediate goods' elasticity of substitution held constant at $\eta = 0.5$ and intermediate goods' share from output and also two sectors' productivity are the same as what assumed in figure 3, and final goods' elasticity of substitution takes four different values, $\sigma = 1, \sigma = 2, \sigma = 3$, and $\sigma = 4$. Figure 5 shows that the welfare gains from trade are higher for lower final goods' elasticity of substitution.

The other parameter which affects the welfare gain from trade is the elasticity of substitution among different countries productions in each of the industries. As Ossa (2015) argues, imports in the average industry do not matter too much, but in some industries are critical to the functioning of the economy. In previous examples, we assumed that the trade elasticity is $\mu = 4$ for both of the sectors. Now assume that for one sector it is $\mu = 3.5$ and for the other is $\mu = 4.5$. We assume benchmark values for other parameters. The results have been shown

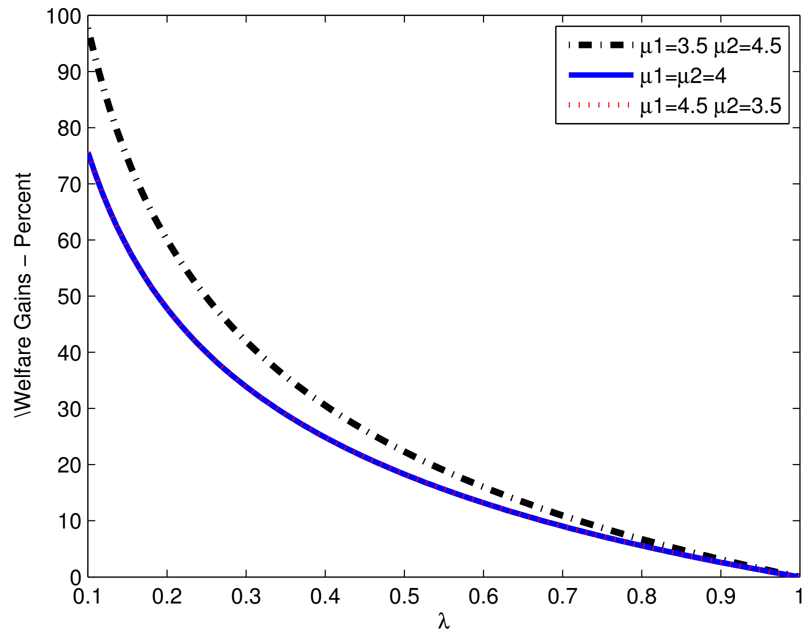

Figure 6: The Effect of the Elasticity of Substitution Among Various Countries' Productions on the Welfare Gains from Trade

Note. The horizental axis shows the domestic goods' share in total expenditure and the vertical axis shows the welfare gains from trade in percent in a two industry economy, where one industry's productivity is $A_1 = 0.2$ and the other's is $A_2 = 1$. The welfare gains from trade have been calculated by maintaining the second industry in autarky and reducing trade barriers in weaker industry. Elasticity of substitution among final goods is $\sigma = 3$ and among intermediate goods is $\eta = 0.5$. Intermediate goods' share in the output is $1 - \gamma = 0.5$. The elasticity of substitution among various countries' productions is different in two industries and the welfare gains from trade is displayed for three cases: $\mu_1 = 3.5$, $\mu_2 = 4.5$; $\mu_1 = \mu_2 = 4$; and $\mu_1 = 4.5$, $\mu_2 = 3.5$.

in figure 6. This figure displays that when trade elasticity decreases, in other words, when the heterogeneity of different countries' production in an industry increases, the welfare gains from reduction of trade barriers in that industry increases which confirms the results of Ossa (2015).

In order to be better compared to Ossa (2015), we assume $\sigma = 1$, where the aggregate final good is the Cobb-Douglas function of the varieties. In this case, trade elasticity could be close to 1. To investigate this issue, first we assume that trade elasticity is homogenous across sectors, $\mu_1 = \mu_2 = 4$, and we compare the welfare gains from the trade in this case with the case that imports in one of the sectors are critical for the functioning of the economy where $\mu = 1.5$ and in the other sector, $\mu = 6.5$. Figure 7 displays this effect on the welfare gains from trade, for the domestic goods share from total expenditure decreasing from 1 down to 0.5. It shows that welfare gains from trade increase dramatically due

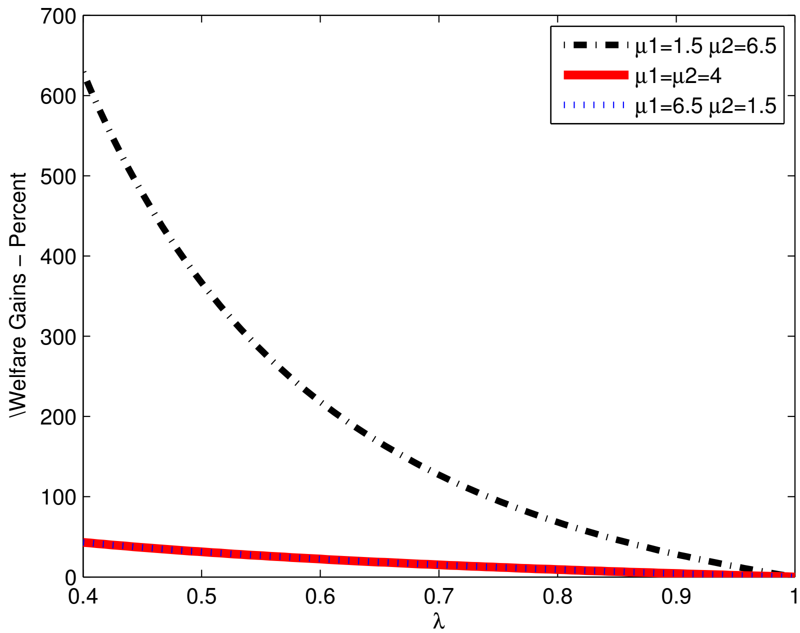

Figure 7: The Effect of the Elasticity of Substitution Among Various Countries' Productions on the Welfare Gains from Trade with One Industry Largely Dependent on Trade

Note. The horizental axis shows the domestic goods' share in total expenditure and the vertical axis shows the welfare gains from trade in percent in a two industry economy, where one industry's productivity is $A_1 = 0.2$ and the other's is $A_2 = 1$. The welfare gains from trade have been calculated by maintaining the second industry in autarky and reducing trade barriers in weaker industry. Elasticity of substitution among final goods is $\sigma = 1$ and among intermediate goods is $\eta = 0.5$. Intermediate goods' share in the output is $1 - \gamma = 0.5$. The elasticity of substitution among various countries' productions is different in two industries and the welfare gains from trade is displayed for three cases: $\mu_1 = 1.5$, $\mu_2 = 6.5$; $\mu_1 = \mu_2 = 4$; and $\mu_1 = 6.5$, $\mu_2 = 1.5$.

to the large dependence of one sector on imports.

In the Figures 3-7, the welfare gains from trade are shown by keeping the stronger industry in autarky and moving the weak link of the economy from autarky toward the free trade. Now, instead of that, assume that the weaker industry remains in autarky and the trade barriers in the stronger industry decrease, so that the domestic goods' share in the industry with higher productivity, which we name it the strong link of the economy, decreases from 1 toward lower values. We calculate the welfare gains from trade for four different values for the elasticity of substitution among intermediate goods, equal to the values assumed in figure 3. The results have been shown in figure 8.

Figure 8 shows that unlike previous results, when the weak industry is kept in autarky and the trade barriers decrease for the stronger industry, the welfare gains from trade would be larger for higher quantities of the elasticity of substi-

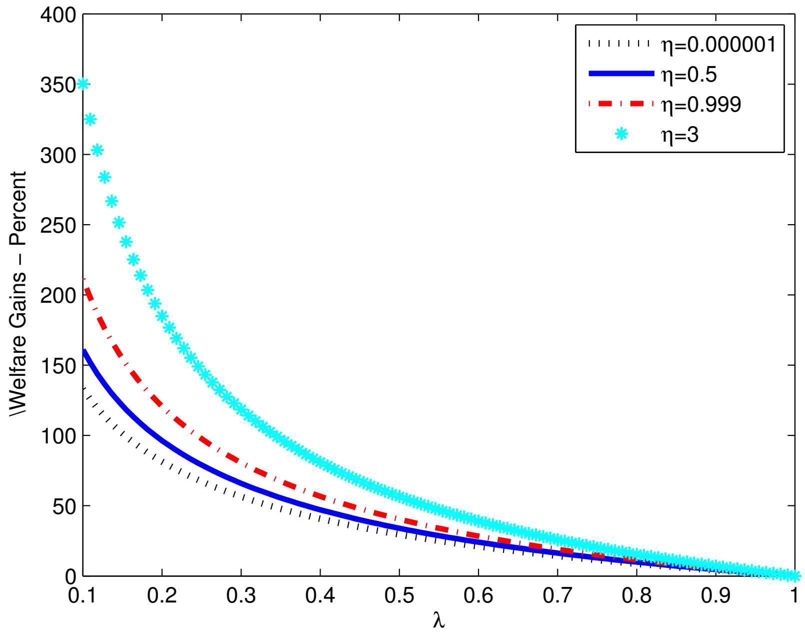

Figure 8: Welfare Gains from Decreasing Trade Barriers of the Stronger Sector for Different Values for Intermediate Goods' Elasticity of Substitution

Note. The horizental axis shows the domestic goods' share in total expenditure and the vertical axis shows the welfare gains from trade in percent in a two industry economy, where one industry's productivity is $A_1 = 0.2$ and the other's is $A_2 = 1$. The welfare gains from trade have been calculated by maintaining the first industry in autarky and reducing trade barriers in stronger industry. trade elasticity for both industries is $\mu_s = 4$, elasticity of substitution among final goods is $\sigma = 3$ and intermediate goods' share in the output is $1 - \gamma = 0.5$. Welfare gains from trade is displayed for four different values of the elasticity of substitution among intermediate goods, μ.

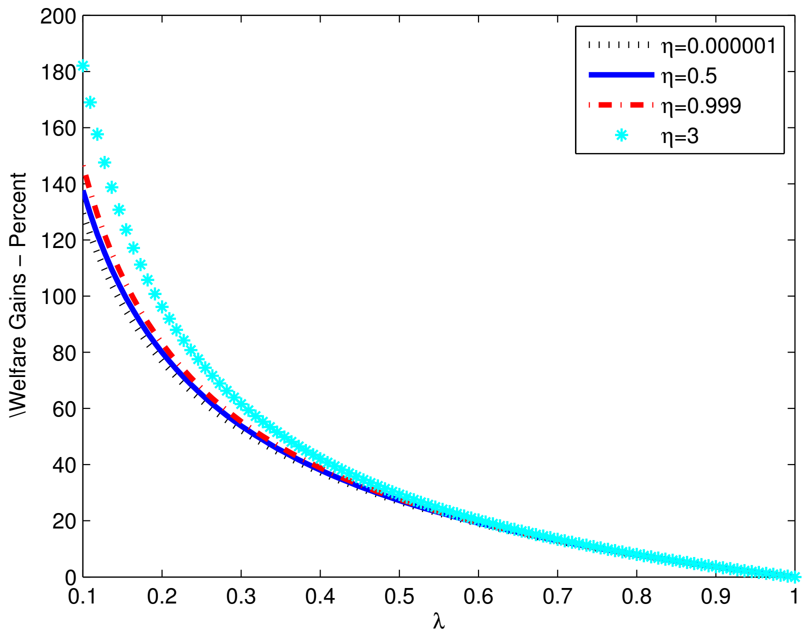

Figure 9: Welfare Gains from Decreasing Trade Barriers of the Stronger Sector for Different Values for Intermediate Goods' Elasticity of Substitution

Note. The horizental axis shows the domestic goods' share in total expenditure and the vertical axis shows the welfare gains from trade in percent in a two industry economy, where one industry's productivity is $A_1 = 0.2$ and the other's is $A_2 = 1$. The welfare gains from trade have been calculated by maintaining the first industry in autarky and reducing trade barriers in stronger industry. trade elasticity for both industries is $\mu_s = 4$, elasticity of substitution among final goods is $\sigma = 3$ and intermediate goods' share in the output is $1 - \gamma = 0.5$. Welfare gains from trade is displayed for four different values of the elasticity of substitution among intermediate goods, μ.

tution among intermediate goods. The reason is that the industry with lower productivity is now kept closed and trade cannot do much for the problem of the weak links in the economy. In this circumstance, if the stronger link of the economy, which now there are fewer trade barriers in, could be substituted for the weak industry with higher intensity in their usages as an intermediate good, which is equivalent to the higher elasticity of substitution between them, the welfare gains from trade would be larger.

The next case to investigate is calculating the welfare gains from trade with homogenous industries, where no industry has the role of the weak link in the economy. In this case, we assume that both of the sectors have the productivity equal to one, $A_1 = A_2 = 1$. The welfare gain from trade has been calculated by keeping one of the sectors in autarky and decreasing the domestic goods' share in the other industry from one to lower quantities down to 0.1. The results have been shown in figure 9.

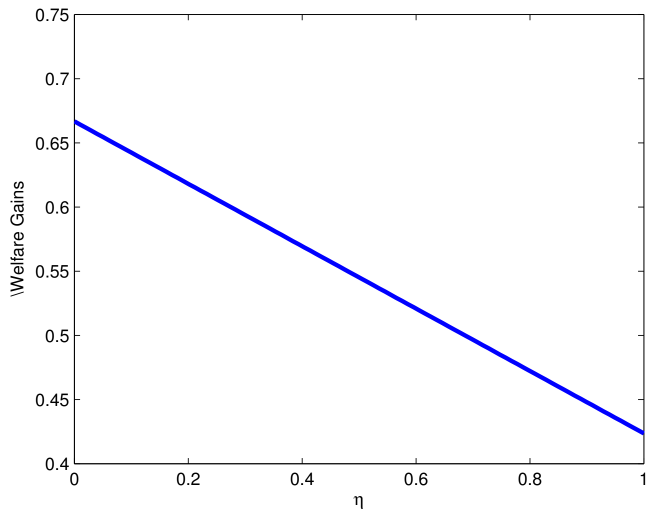

Figure 10: The Welfare Gains from Trade for different Values of the Elasticity of Substitution Among Intermediate Goods with LOG-Normal Distribution of Productivity and Domestic Goods' Share in Total Expenditures

Note. The horizental axis shows the elasticity of substitution among intermediate goods and the vertical axis shows the logarith of the welfare gains from trade.logarithm of various industries' productivity and logarithm of domestic goods' share in total expenditure are jointly normally distributed as $a_s = log(A_s) \sim N(\mu_a, \nu_a^2)$, $\omega_s = log(\lambda_s) \sim N(-0.65, 0.25)$ and $cov(\omega_s, a_s) = \nu_{a\omega}$=-0.5. Elasticity of substitution among final goods is $\sigma = 3$. Intermediate goods' share in the output is $1 - \gamma = 0.5$. The elasticity of substitution among various countries' productions is $\mu = 4$.

Figure 9 displays the welfare gains from international trade for four quantities of the elasticity of substitution among intermediate goods, equal to the quantities assumed in figures 3 and 8. As it is shown in figure 9, the welfare gains from trade would be larger for higher values of elasticity of substitution among intermediate goods. This means that the main result of our model which is larger welfare gains from trade due to the complementarity of the intermediate goods is the consequence of the international trade's role in fortifying the weak links of the economy and is not credible for the international trade models with homogeneous firms.

Figure 10 displays the logarithm of the welfare gains from trade, using the expression obtained from proposition 4, where we assumed that the logarithm of productivity of various industries and domestic goods' share in expenditure are jointly normally distributed. In 10 we assumed values of $\mu_\omega = -0.65$, $\nu_\omega = 0.5$, and $\nu_{a\omega} = -0.5$. This figure shows that if trade barriers are higher for the industries which have lower productivity, then the country would have trouble in providing the required intermediate goods for production, and the more the degree of complementarity of intermediate goods, the more severe this effect would be, therefore the welfare gains from trade would have a direct relation to the degree of complementarity.

6. Conclusion

In this paper we develop a multi-industry, multi-country Armington (1969) type trade model, where the intermediate inputs are complements of each other with the elasticity of substitution less than one. This missed assumption in the literature leads to much larger welfare gains from quantitative trade model. We show that international trade helps to fortify the weak links of the production chain by decreasing domestic goods' share in total expenditure in the industries with lower productivities which would absorb large amounts of the economy's endowments due to the complementarity. We find that introducing this complementarity results in a very large welfare gains from trade, much more than the literature used to find in the standard classical quantitative trade models, as survey in Arkolakis et al. (2012) and Costinot and Rodriguez-Clare (2014). We also show that the heterogeneity of trade elasticities among industries, introduced as in Ossa (2015), greatly increases the welfare gains from trade.

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Appendix

Proof of PROPOSITION 1:

1- The optimization problem of the variety s is as below:

$$\max_{X_s,L_s} p_s A_s \left(\frac{L_s}{\gamma}\right)^{\gamma}\left(\frac{X_s}{1-\gamma}\right)^{1-\gamma} - wL_s - qX_s$$

So the corresponding First Order Conditions for this problem are:

$$[L_s] : p_s A_s \left(\frac{\gamma}{1-\gamma}\right)^{1-\gamma} L_s^{\gamma-1} X_s^{1-\gamma} = w$$

$$[X_s] : p_s A_s \left(\frac{\gamma}{1-\gamma}\right)^{-\gamma} L_s^{\gamma} X_s^{-\gamma} = q$$

By substituting the production function into these equations they can be rewritten as below:

$$[L_s] : p_s \gamma \frac{Q_s}{L_s} = w$$

$$[X_s] : p_s \left(1-\gamma\right) \frac{Q_s}{X_s} = q$$

substituting these first order conditions back into the production function yields an equation for the price of good s:

$$p_s = \frac{w^{\gamma} q^{1-\gamma}}{A_s} \tag{20}$$

2- Now, consider the optimization problems for the Final Good and Intermediate Good.

the optimization problem for the Final Good is:

$$\max_{c_s} \left(\sum_{s=1}^{S} c_s^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} - \sum_{s=1}^{S} p_s c_s$$

And the first order condition for the Final Good aggregator is:

$$[c_s] : c_s = \left(\sum_{s=1}^{S} c_s^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} p_s^{-\sigma} = C p_s^{-\sigma}$$

Substitute this back into the firm's production function gives:

$$Y = C = \left(\sum_{s=1}^{S} c_s^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} = \left(\sum_{s=1}^{S} C^{\frac{\sigma-1}{\sigma}} p_s^{1-\sigma}\right)^{\frac{\sigma}{\sigma-1}}$$

By simplifying this expression we have:

$$1 = \left(\sum_{s=1}^{S} p_s^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{21}$$

And analogously for the Intermediate Good:

$$q = \left(\sum_{s=1}^{S} p_s^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{22}$$

Substituting p_s from equation 20 into equation 21 gives:

$$w^{\gamma} q^{1-\gamma} = B_{\sigma} \tag{23}$$

where

$$B_{\sigma} = \left(\sum_{s=1}^{S} A_s^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \tag{24}$$

Now substitute p_s from equation 20 into equation 22 to get:

$$w^{\gamma} q^{1-\gamma} = q B_{\eta} \tag{25}$$

where B_{η} is defined analogously to B_{σ}.

Combining equation 23 and equation 25 gives:

$$q = \frac{B_{\sigma}}{B_{\eta}} \tag{26}$$

Now substitute q back from equation 26 into equation 23 gives the real wage of the labor force as below:

$$w = \left(B_{\sigma}^{\gamma} B_{\eta}^{1-\gamma}\right)^{\frac{1}{\gamma}} \tag{27}$$

3- Household's budget constraint is:

$$C = Y = w\bar{L} \tag{28}$$

Finally substituting wage from equation 27 into Household's budget constraint yields the main result of the proposition.

$$Y = \left(B_{\sigma}^{\gamma} B_{\eta}^{1-\gamma}\right)^{\frac{1}{\gamma}} \bar{L} \tag{29}$$

Proof of PROPOSITION 2:

Solving the optimization problem for the variety s in the country i yields:

$$p_{ijs} = A_{is}^{-1} w_i^{\gamma} q_i^{1-\gamma} \tau_{ijs} \tag{30}$$

The optimization problem of the aggregator of productions of various countries in the industry s gives:

$$p_{js} = \left(\sum_{n=1}^{N} p_{ijs}^{1-\mu_s} \right)^{\frac{1}{1-\mu_s}} \tag{31}$$

Also the value of industry s trade flowing from country i to country j, V_{ijs} which is known as the Gravity Equation is deducible from the optimization problem for this sector:

$$V_{ijs} = p_{ijs}^{1-\mu_s} p_{js}^{\mu_s - 1} E_{js} \tag{32}$$

where E_{js} is the total expenditure of the country j on the productions of different countries including itself in the industry s. Substituting p_{ijs} from equation 30 in equation 32:

$$V_{ijs} = \left(A_{is}^{-1} w_i^{\gamma} q_i^{1-\gamma} \tau_{ijs} \right)^{1-\mu_s} p_{js}^{\mu_s - 1} E_{js} \tag{33}$$

So the total expenditure of country j on its own productions in the industry s would be:

$$V_{jjs} = \left(A_{js}^{-1} w_j^{\gamma} q_j^{1-\gamma} \right)^{1-\mu_s} p_{js}^{\mu_s - 1} E_{js} \tag{34}$$

Defining $\lambda_{js} = \frac{V_{jjs}}{E_{js}}$ as the own trade share in industry s of country j equation 34 implies:

$$\lambda_{js} = \left(A_{js}^{-1} w_j^{\gamma} q_j^{1-\gamma} \right)^{1-\mu_s} p_{js}^{\mu_s - 1} \tag{35}$$

Therefore, the price index of all industry s variety available in country j is:

$$p_{js} = \lambda_{js}^{\frac{1}{\mu_s - 1}} A_{js}^{-1} w_j^{\gamma} q_j^{1-\gamma} \tag{36}$$

The price index of intermediate goods in country j is implied by solving the Intermediate Sector optimization problem:

$$q_j = \left(\sum_{s=1}^{S} p_{js}^{1-\eta} \right)^{\frac{1}{1-\eta}} \tag{37}$$

By substituting p_{js} from equation 36 into equation 37:

$$q_j = w_j \left(\sum_{s=1}^{S} A_{js}^{\eta-1} \lambda_{js}^{\frac{1-\eta}{\mu_s - 1}} \right)^{\frac{1}{\gamma(1-\eta)}} \tag{38}$$

Now by substituting q_j from equation 38 into 36 we have:

$$p_{js} = \lambda_{js}^{\frac{1}{\mu_s - 1}} A_{js}^{-1} w_j \left(\sum_{s=1}^{S} A_{js}^{\eta-1} \lambda_{js}^{\frac{1-\eta}{\mu_s - 1}} \right)^{\frac{1-\gamma}{\gamma(1-\eta)}} \tag{39}$$

The optimization problem of the Final Sector yields:

$$P_j^F = \left(\sum_{s=1}^{S} p_{js}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{40}$$

Substitute p_{js} from equation 39 into equation 40, yields the price index of Final Goods:

$$P_j^F = w_j \left(\sum_{s=1}^{S} A_{js}^{\eta-1} \lambda_{js}^{\frac{1-\eta}{\mu_s - 1}} \right)^{\frac{1-\gamma}{\gamma(1-\eta)}} \left(\sum_{s=1}^{S} A_{js}^{\sigma-1} \lambda_{js}^{\frac{1-\sigma}{\mu_s - 1}} \right)^{\frac{1}{(1-\sigma)}} \tag{41}$$

By substituting P_j^F from equation 41 into household's budget constraint, GDP, and consequently the consumption of the household, would be:

$$Y_j = C_j = \left(\sum_{s=1}^{S} A_{js}^{\eta-1} \lambda_{js}^{\frac{1-\eta}{\mu_s - 1}} \right)^{\frac{1-\gamma}{\gamma(\eta-1)}} \left(\sum_{s=1}^{S} A_{js}^{\sigma-1} \lambda_{js}^{\frac{1-\sigma}{\mu_s - 1}} \right)^{\frac{1}{(\sigma-1)}} \bar{L}_j \tag{42}$$

Define:

$$T_j(\sigma, \lambda) = \left(\sum_{s=1}^{S} A_{js}^{\sigma-1} \lambda_{js}^{\frac{1-\sigma}{\mu_s - 1}} \right)^{\frac{1}{(\sigma-1)}} \tag{43}$$

and analogously,

$$T_j(\eta, \lambda) = \left(\sum_{s=1}^{S} A_{js}^{\eta-1} \lambda_{js}^{\frac{1-\eta}{\mu_s - 1}} \right)^{\frac{1}{(\eta-1)}} \tag{44}$$

So, GDP and consequently household's consumption could be rewritten as:

$$Y_j = C_j = \left(T_j(\sigma, \lambda)^{\gamma} T_j(\eta, \lambda)^{1-\gamma} \right)^{\frac{1}{\gamma}} \bar{L}_j \tag{45}$$

In order to measure the welfare gains from international trade, one should compare the GDP calculated from the open economy model with the GDP in the autarky. In the autarky, in all industries domestic goods absorb all of the expenditures, where for all of the s's, $\lambda_{js} = 1$. Therefore, equations 43 and 44 are respectively as below:

$$T_j^{aut}(\sigma, \lambda) = \left(\sum_{s=1}^{S} A_{js}^{\sigma-1} \right)^{\frac{1}{(\sigma-1)}} \tag{46}$$

and

$$T_j^{aut}(\eta, \lambda) = \left(\sum_{s=1}^{S} A_{js}^{\eta-1} \right)^{\frac{1}{(\eta-1)}} \tag{47}$$

which are the same as the expressions acquired in the previous section for the closed economy model:

$$T_j^{aut}(\sigma, \lambda) = B_j(\sigma) \tag{48}$$

and

$$T_j^{aut}(\eta, \lambda) = B_j(\eta) \tag{49}$$

Therefore, the welfare gains from international trade would be:

$$welfare - gain = \frac{C_j}{C_j^{aut}} = \left[\left(\frac{T_j(\sigma, \lambda)}{B_j(\sigma)} \right)^{\gamma} \left(\frac{T_j(\eta, \lambda)}{B_j(\eta)} \right)^{1-\gamma} \right]^{\frac{1}{\gamma}} \tag{50}$$

and Proposition 2 is proved.

Proof of PROPOSITION 3:

From the first order condition for the Final Good we have:

$$p_{js} c_{js} = p_{js}^{1-\sigma} C {P_j^F}^{\sigma} \tag{51}$$

so the industry s's share in the expenditures of the Final Goods Aggregator would be:

$$e_{js}^F = \frac{p_{js} c_{js}}{P_j^F C} = p_{js}^{1-\sigma} {P_j^F}^{\sigma-1} \tag{52}$$

By substituting p_{js} from equation 39 and P_j^F from equation 41 we would have:

$$e_{js}^F = \frac{{A_{js}}^{\sigma-1} \lambda_{js}^{\frac{1-\sigma}{\mu_s-1}}}{\sum_{s=1}^{S} A_{js}^{\sigma-1} \lambda_{js}^{\frac{1-\sigma}{\mu_s-1}}} \tag{53}$$

By multiplying two sides of the equation 53 by $\lambda_{js}^{\frac{\sigma-1}{\mu_s-1}}$ and summing up on s we would have:

$$\sum_{s=1}^{S} e_{js}^{F} \lambda_{js}^{\frac{\sigma-1}{\mu_s-1}} = \frac{\sum_{s=1}^{S} A_{js}{}^{\sigma-1}}{\sum_{s=1}^{S} A_{js}^{\sigma-1} \lambda_{js}^{\frac{1-\sigma}{\mu_s-1}}} \tag{54}$$

By combining the equations 54, 43 and 46 we have:

$$\frac{T_j(\sigma,\lambda)}{T_j^{aut}(\sigma,\lambda)} = \left(\sum_{s=1}^{S} e_{js}^{F} \lambda_{js}^{\frac{\sigma-1}{\mu_s-1}}\right)^{\frac{1}{1-\sigma}} \tag{55}$$

Analogously, for the intermediate goods we have:

$$\frac{T_j(\eta,\lambda)}{T_j^{aut}(\eta,\lambda)} = \left(\sum_{s=1}^{S} e_{js}^{X} \lambda_{js}^{\frac{\eta-1}{\mu_s-1}}\right)^{\frac{1}{1-\eta}} \tag{56}$$

By substituting equation 55 and 56 in the equation 50 we would have:

$$welfare - gain = \left[G_j\left(\sigma,\lambda\right)^{\gamma} G_j\left(\eta,\lambda\right)^{1-\gamma}\right]^{\frac{1}{\gamma}} \tag{57}$$

and Proposition 3 is proved

Proof of PROPOSITION 4:

$a_s = logA_s$ and $\omega_s = log(\lambda_s)$ are jointly normally distributed so that: $a_s \sim N(\mu_a, \nu_a^2)$, $\omega_s \sim N(\mu_\omega, \nu_\omega^2)$, and $cov(\omega_s, a_s) = \nu_{a\omega}$.

Let consider the numbers of the active industries in countries as a measure one. So equation 43 and 44 can be rewritten as:

$$T_j\left(\sigma,\lambda\right) = \left(\int_0^1 A_{js}^{\sigma-1} \lambda_{js}^{\frac{1-\sigma}{\mu_s-1}} ds\right)^{\frac{1}{(\sigma-1)}}$$

and

$$T_j\left(\eta,\lambda\right) = \left(\int_0^1 A_{js}^{\eta-1} \lambda_{js}^{\frac{1-\eta}{\mu_s-1}} ds\right)^{\frac{1}{(\eta-1)}}$$

respectively.

Define m_s as:

$$m_s = (\eta - 1)\left(a_s + \frac{1}{1-\mu}\omega_s\right)$$

m_s is normally distributed as:

$$m_s \sim N\left((\eta-1)\left(\mu_a + \frac{\mu_\omega}{1-\mu}\right), (\eta-1)^2 \nu^2\right)$$

where,

$$\nu^2 = \nu_a^2 + \left(\frac{1}{1-\mu}\right)^2 \nu_\omega^2 + 2\nu_{a\omega}$$

Considering the definition of m_s, $T(\eta,\lambda)$ would be:

$$T(\eta,\lambda) = \left(E\left(e^{m_s}\right)\right)^{\frac{1}{(\eta-1)}}$$

By calculating the expected value for $T(\eta,\lambda)$ we have:

$$T(\eta,\lambda) = e^{\mu_a + \frac{\mu_\omega}{1-\mu} + \frac{1}{2}(\eta-1)\nu^2}$$

and analogously for $T(\sigma,\lambda)$:

$$T(\sigma,\lambda) = e^{\mu_a + \frac{\mu_\omega}{1-\mu} + \frac{1}{2}(\sigma-1)\nu^2}$$

and similarly for the autarky the expected values for $B_(\sigma)$ and $B_(\eta)$ would be:

$$B(\eta) = e^{\mu_a + \frac{1}{2}(\eta-1)\nu_a^2}$$

and

$$B(\sigma) = e^{\mu_a + \frac{1}{2}(\sigma-1)\nu_a^2}$$

respectively.

Taking the logarithm of both sides of the wefare gains equation (equation 50), implies:

$$\log(welfare - gain) = \frac{1}{\gamma}\left[\gamma\left(\log T(\sigma,\lambda) - \log B(\sigma)\right) + (1-\gamma)\left(\log T(\eta,\lambda) - \log B(\eta)\right)\right]$$

Substituting $T(\sigma,\lambda)$, $T(\eta,\lambda)$, $B(\eta)$, and $B(\sigma)$ into this equation yields:

$$welfare - gain = \gamma\left(\frac{\mu_w}{1-\mu} + \frac{1}{2}(\sigma-1)\left(\left(\frac{1}{1-\mu}\right)^2 \nu_\omega^2 + 2\nu_{a\omega}\right)\right) + \\ (1-\gamma)\left(\frac{\mu_w}{1-\mu} + \frac{1}{2}(\eta-1)\left(\left(\frac{1}{1-\mu}\right)^2 \nu_\omega^2 + 2\nu_{a\omega}\right)\right)$$

and Proposition 4 is proved.