1. Consider the single output \((q)\) single input \((z)\) production sets in Figures 1-(a) and 1-(b).

(a) Explain whether each of the production sets exhibits CRS, IRS, or DRS.
(b) Which of these production sets features fixed production costs. Is this a sunk cost?
(c) In each case draw total cost, average cost, marginal cost, and supply curves as accurately as you can. Clearly explain how you have derived the curves from the production set. Assume output and input prices are \(p\) and \(w\), respectively.

![Production Sets](attachment:production_sets.png)

(a) Production set 1  
(b) Production set 2

Figure 1: Production sets for question 1

2. Assume the production set \(Y\) is closed and satisfies monotonicity (free disposal). Prove the following statements.
(a) $\pi(p)$ is convex function in $p$.
(b) $\pi(p)$ is nondecreasing in output prices.
(c) If the profit function is differentiable and net supply is a function then $y(p) = D_p\pi(p)$.

3. Consider the cost minimization problem for the case of single output technology

$$\min_{z \geq 0} \quad w \cdot z$$
$$\text{s.t.} \quad f(z) \geq q$$

Assume $f(z)$ is differentiable. Using the perturbation argument show that at an interior solution we must have

$$\frac{w_l}{w_k} = \frac{\partial f(z^*)/\partial z_l}{\partial f(z^*)/\partial z_k} = TRS_{l,k}$$

4. Derive cost function, conditional factor demand functions, output supply function, and profit function for the following production functions. Assume price of input $z_i$ is $w_i$ and price of output is $p$.

(a) $q = f(z) = (z_1^\rho + z_2^\rho)^{1/\rho}$
(b) $q = f(z) = \min\{z_1, z_2\}$
(c) $q = f(z) = \begin{cases} 0 & \text{if } \min\{z_1, z_2\} < a \\ z_1 + z_2 & \text{if } \min\{z_1, z_2\} \geq a \end{cases}$ where $a > 0$ is a parameter.
(d) [optional] $q = f(z) = (\min\{z_1, z_2\})^\alpha$

5. Consider a Cobb-Douglas production function $q = f(l, k) = l^\alpha k^\beta$ with $\alpha, \beta \in (0, 1)$ and $\alpha + \beta < 1$. Output price is $p$. $l$ is labor and the wage rate is $w$ and $k$ is capital with a gross rental rate of $r$. We want to compare two types of taxes. A case of input taxation and a case of profit taxation.

(a) No taxes: Find the cost function, conditional factor demands, profit function, and supply function when no taxes are imposed on the firm.
(b) Profit taxation: the government takes away $\tau$ percent of the profits of the firm. Derive the functions in part a) and interpret. Does profit taxation change the choice of inputs? Does it affect supply?
(c) Input taxation: the government imposes ad valorem taxes on labor and capital. This means the wage rate and the rental rate are now $w(1 + \tau_w)$ and $r(1 + \tau_r)$, where $\tau_w$ and $\tau_r$ are tax rates for labor and capital. Derive the functions in part a) and interpret. Does input taxation change the choice of inputs? Does it affect supply?
(d) Assume the government wants to raise a total revenue of $T$. Based on the results in the previous parts, what type of taxes do you recommend?
6. A firm has two plants that could be used to produce the same output $y$. The cost functions for these plants are given by $c_1(y_1) = \frac{y_1^2}{2}$ and $c_2(y_2) = ay_2$, where the indices show the plant number, i.e. $y_1$ is the level of production in plant 1. Also $a > 0$ is a parameter.

(a) Think about a situation where the firm wants to produce a fixed amount of output equal to $q$. What is the optimal distribution of production across the two plants? Could you derive the overall cost function of this firm?

(b) Write down the profit maximization of this firm and derive the profit function and output supply. Give a graphical presentation of your answer as well and discuss the intuition for your result.

7. Consider a profit-maximizing firm that produces a good which is sold in a competitive market. It is observed that when the price of the output good rises, the firm hires more skilled workers but fewer unskilled workers. Now the unskilled workers unionize and succeed in getting their wage increased. Assume that all other prices remain constant.

(a) What will happen to the firm’s demand for unskilled workers?

(b) What will happen to the firm’s supply of output?

8. [Optional] Consider a firm using a single output production technology with $f(l, k) = \sqrt{l + ak}$, with $a > 0$. The price of output and inputs are $p$, $w$, and $r$ (all positive). The firm also faces a credit constraint in the sense that it has only access to $M$ dollars before production starts.

(a) Write down the profit maximization problem (PMP) for this firm. Carefully explain the constraints.

(b) Solve the PMP and derive profit function and net supply.

(c) Interpret your results. What is the role of credit constraints in this problem? How does it limit output?

(d) Could you still claim that the profit maximizing production vector is efficient?