1. Consider a two good world with $x_1$ and $x_2$. Prove the following statements.

(a) If LNS holds then at least one good needs to be normal. What can you say for the general $L$ commodity case?
(b) If LNS holds and one good is inferior then the other good needs to be a luxury.
(c) If LNS holds and $\frac{\partial x_2}{\partial x_1} \geq \frac{x_2}{x_1}$ for a given point on the wealth expansion path, then $x_2$ is a luxury and $x_1$ is a necessity. What is the graphical interpretation of the inequality condition?

2. [Hard] Consider the following utility function

$$u(x_1, x_2) = \min \left\{ \sqrt{x_1 x_2}, \left(\sqrt{x_1 x_2}\right)^{1/3} \right\}$$

(a) Derive Walrasian demand functions when prices are $p_1$ and $p_2$ and the consumer has wealth $w$.
(b) How does $x_1(p, w)$ and $x_2(p, w)$ vary with prices? Are any of these goods Giffen?
(c) Are these goods normal or inferior? Does Walras Law hold here?

3. Consider a consumer with wealth $w$ and Cobb-Douglas utility, $u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$, where $\alpha \in (0, 1)$.

(a) Write down the UMP and derive Walrasian demand functions.
(b) Find the wealth expansion path when $w \in [0, \infty)$.
(c) Could you show whether the two commodities here are inferior, necessity, or luxury?
(d) Find the offer curve when $p_1 \in (0, \infty)$.

4. Consider the indirect utility function given by

$$v(p_1, p_2, w) = \frac{w}{p_1 + p_2}$$
(a) What are the demand functions?
(b) Derive the Hicksian demand and expenditure functions?
(c) Show that the Hicksian demand function derived in (b) satisfy the compensated law of demand?
(d) Could you draw the indifference curves for the preferences that gave the above indirect utility function?

5. Prove that a solution to the EMP exists if \( p \gg 0 \) and there is some \( x \in \mathbb{R}_+^l \) satisfying \( u(x) \geq u \).

6. Starting from \( h(p, u) = x(p, e(p, u)) \) derive the Slutsky equation
   \[
   \frac{\partial x_l(p, w)}{\partial p_k} = \frac{\partial h_l(p, u)}{\partial p_k} - \frac{\partial x_l(p, w)}{\partial w} x_k(p, w)
   \]
   for all \( w = e(p, u) \) and for all \( l, k = 1, \ldots, L \).

7. Prove that when commodity \( l \) is inferior then
   \[
   \frac{\partial h_l(p, u)}{\partial p_l} < \frac{\partial x_l(p, w)}{\partial p_l}
   \]
   for all \( w = e(p, u) \). What is the graphical representation of this inequality? Intuitively explain your result.

8. Consider a consumer with constant elasticity of substitution (CES) utility over two goods \( u(x_1, x_2) = (x_1^\rho + x_2^\rho)^{1/\rho} \) with \( \rho \in [0, \infty) \).
   (a) Is the underlying preference relation represented by the CES utility function homothetic? Explain.
   (b) Draw the indifference curves (ICs) for \( \rho \to 0, \rho \in (0, 1), \rho = 1, \rho > 1 \), and \( \rho \to \infty \). Explain the shape of the ICs.
   (c) Define the elasticity of substitution as
   \[
   \epsilon_{21} = \frac{\partial \left( \frac{x_2}{x_1} \right)}{\partial \text{MRS}} \times \frac{\text{MRS}}{\left( \frac{x_2}{x_1} \right)}
   \]
   where \( \text{MRS} \) is the marginal rate of substitution between \( x_2 \) and \( x_1 \). Find \( \epsilon_{21} \) for the CES utility function. Discuss.
   (d) Write down the UMP and solve for the Walrasian demand function when \( \rho \in (0, 1) \).
   (e) Write down the EMP and solve for the Hicksian demand function when \( \rho \in (0, 1) \).
   (f) Verify that the slutsky equation, written above, holds (assume \( \rho \in (0, 1) \)).

9. Discuss the following in less than 200 words.
   Consider a country with a low population growth rate. In order to address this problem, policy makers are assessing two feasible child bearing incentive packages. The first is
to give a fixed stipend to each newborn’s mother, until the child reaches a certain level of age. The second policy is to increase funding on public schools and reduce cost of health insurance and medical therapies for children. Which policy do you think is most effective in accelerating population growth? Discuss how characteristics of respondents (mothers who are willing to bear a child after the policy implementation) might differ in each of these policies. Which policy is the best, from society’s perspective?