Microeconomics 1

Assignment 5: Consumer theory
UMP and EMP
Due: 29 Mehr 1397

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Fall 1397

1. Prove that if the preference relation $\succeq$ is convex, then the Hicksian demand $h(p, u)$ is a convex set.

2. Suppose $u$ is continuous and that the underlying preferences satisfy LNS.
   (a) Prove that the expenditure function is strictly increasing in $u$ and non-decreasing in $p$.
   What happens to these properties if we drop the LNS assumption?
   (b) Give an example of a situation where the expenditure function is constant as one price increases.

3. A consumer of two goods has utility function $u(x, y) = \max\{ax, ay\} + \min\{x, y\}$, with $0 < a < 1$.
   (a) Draw the indifference curves for these preferences.
   (b) Derive the Marshallian and Hicksian demands.

4. Consider the Leontief utility function $u(x_1, x_2) = \min\{x_1, x_2\}$.
   (a) Write down the UMP. Derive Walrasian demand functions and the indirect utility function.
   (b) Check whether Roy’s identity holds.
   (c) Write down the EMP. Derive Hicksian demand functions and the expenditure function.
   (d) Check whether Shephard’s lemma holds.

5. A consumer has a Cobb-Douglas utility function $u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$, where $\alpha > 0$ and $x_1, x_2 \in \mathbb{R}^+$. Assume that the prices vector satisfies $p \equiv (p_1, p_2) >> 0$, and wealth $\omega > 0$.
   (a) Write the consumers utility maximization problem. Find Walrasian demands, and the indirect utility function $v(p_1, p_2, w)$.
   (b) Write the consumers expenditure minimization problem. Find Hicksian demands, and the expenditure function $e(p_1, p_2, u)$.
   (c) Evaluate the Hicksian demands $h(p_1, p_2, u)$ at $u = v(p_1, p_2, w)$, and show that Hicksian and Walrasian demands coincide, that is, $h(p_1, p_2, v(p_1, p_2, w)) = x(p_1, p_2, u)$.

6. A consumer is deciding about the proportion of day she works ($h$) and consumption ($c$). Her preference over bundles of work and consumption are as follows

$$u(c, h) = c + \sqrt{1-h}$$

The consumer would get a daily wage of $w$ (only source of income) and the price of consumption is normalized to 1.
(a) Write down the UMP and solve it.

(b) Government decides to impose an income tax, so effectively the wage rate becomes \( w(1 - \tau) \), where \( \tau \in [0, 1] \) is the tax rate. How does this policy affect consumer utility and work decision? Assume \( w(1 - \tau) > 0.5 \).

(c) Now the government decides to give back the collected tax as a lump sum subsidy. How does this new policy change utility and work decision.

(d) Redo parts a, b and c for \( u(c, h) = \ln(c) + \gamma \ln(1 - h) \).

7. Preferences are represented by \( u = \phi(x) \) and expenditure function, indirect utility function and demands are calculated. If the same preferences are now represented by \( u^* = \psi(\phi(x)) \) for a strictly increasing function \( \psi(.) \), show that \( e(P, u) \) is replaced by \( e(P, \psi^{-1}(u^*)) \), and \( v(P, w) \) by \( \psi(v(P, w)) \), and \( h(P, u) \) by \( h(P, \psi^{-1}(u^*)) \). Also, check that Marshallian demand \( x(P, w) \) is unaffected.

8. Show that if \( u(.) \) is homogeneous of degree one, then \( h(P, u) \) and \( e(P, u) \) are homogeneous of degree one in \( u \) [i.e., they can be written as \( h(P, u) = \bar{h}(P)u \) and \( e(P, u) = \bar{e}(P)u \)].