1. Prove that if the preference relation $\succeq$ is strictly convex, then Walrasian demand $x(p, w)$ is single-valued. What does strict convexity imply about the shape of the utility function? Give an intuition about why you would expect this type of utility function to have a unique maximum over a compact set like $B(p, w)$.

2. Consider the choice of transportation for people living in a city, who choose between taking a bus or driving their own car to get to work. Choice of transportation depends on its costs and speed. Buses are less expensive but they are more time-consuming. Assume that, at first, fraction $\alpha$ of the population are using buses and the rest are driving to their workplace. Now, a subsidy is placed on the price of gas, which only reduces the cost of driving to work. Ignore how this subsidization is financed and answer the following questions.

   (a) What is the effect of gas price change on the choice of transportation for each group?

   (b) How the well-being of people who take buses after the subsidization has changed from the past? Does the well-being of the group who chose to drive to work before the subsidization happened, necessarily increases after lowering the price of fuel? (Hint: Think about how the price of fuel is related to transport duration for each group).

3. Suppose that in a two commodity world, the consumer’s utility function takes the form $u(x) = [\alpha_1 x_1^{\rho} + \alpha_2 x_2^{\rho}]^{1/\rho}$. This utility function is known as the constant elasticity of substitution (or CES) utility function

   (a) Show that when $\rho = 1$, indifference curves become linear.

   (b) Show that as $\rho \to 0$, this utility function comes to represent the same preferences as the (generalized) Cobb-Douglas utility function $u(x) = x_1^{\alpha_1} x_2^{\alpha_2}$.

   (c) Show that as $\rho \to \infty$, indifference curves become “right angles”; that is, this utility function has in the limit the indifference map of the Leontief utility function $u(x_1, x_2) = \text{Min}\{x_1, x_2\}$.

4. A consumer is deciding about her work hours ($h$) and consumption ($c$). Her preference over bundles of work and consumption are as follows

$$u(c, h) = c + \sqrt{24 - h}$$

The consumer would get an hourly wage of $w$ and the price of consumption is normalized to 1. In addition to her wage income, the consumer also receives a non-labor income equal to $y$.

   (a) Draw the indifference curves in the $(c, h)$ space.
5. Consider an individual deciding on the choice between consumption and leisure with the following utility function \( u(c, l) = c + \phi(l) \), where \( c \geq 0 \) is consumption, \( l \in [0, 24] \) is leisure, and \( \phi \) is an increasing twice differentiable function. Assume price of consumption is 1 and the wage rate is \( w \).

(a) Write down the utility maximization problem (UMP) for this individual. Draw the constraint set and the indifference curves in the space of \((c, l)\). Could you argue that the KT necessary conditions are sufficient?

(b) Assume \( \phi(l) = \sqrt{l} \) and that \( w(1 - t) > 1 \). The government levies an income tax according to the following schedule

\[
T(y) = \begin{cases} 
0 & \text{if } y \leq z \\
(ty - z) & \text{if } y > z.
\end{cases}
\]

where \( T \) is tax liability, \( y \) is before tax labor income (i.e. \( w(24 - l) \)), \( z \) is the exemption threshold, and \( t \in (0, 1) \) is the marginal tax rate.

i. Write the new UMP and draw the constraint set. Is the constraint set convex?

\[ \bullet \] Show that the budget constraint is equivalent to the following two inequalities

\[
c + w(1 - t)l \leq 24w(1 - t) + zt
\]

\[
c + wl \leq 24w.
\]

i. Find consumption demand, labor supply, and indirect utility when \( t = \frac{1}{4} \) and \( z = 4w \).

6. State your opinion on the following statement in less than 250 words.

The rational choice theory which you all should be familiar with by now is not only applicable to model human behavior in choosing between drinking orange juice and smoking cigarettes, but also it could be used to model many once in a lifetime and pivotal decisions made by humans, such as marriage and education. For now, keep your focus on the choice of education. After completing high school, you should first choose between going to university or joining the labor market directly, and if the former is chosen, then you should decide on which major you want to pursue. At first, describe, what you consider as a proper and concise model on the choice of education. Now, given that we have assumed education as a rational choice, what explains the considerable proportion of students who are unhappy with the choice they have made regarding their education.