Microeconomics I

Assignment 1: Consumer theory
Properties of preference relations
Due: 2 Mehr 1396

44715
Fall 1396

1. Utility function: Let \( \preceq \) a preference relation on the set \( X \). Suppose there exists a real function \( u \) on \( X \) such that \( u(x) \leq u(y) \) iff \( x \preceq y \). Show that \( \preceq \) is rational.

2. Prove that strong monotonicity \( \Rightarrow \) monotonicity \( \Rightarrow \) local nonsatiation.

3. Suppose there are 16 colors (all shades of gray) to choose for your room paint. You are offered pairwise choices between consecutive shades from the darkest to the lightest. You feel indifferent between each pair, yet when offered the pair of darkest shade with the lightest you prefer the lightest. Does your preference relation satisfy rationality?

4. Plot the indifference curves for the preference relation represented by \( u(x_1, x_2) = x_1^2 + \sqrt{x_2} \). Derive a formula for the slope of the indifference curves.

5. An individual consumes cigarettes and orange juice. The more he consumes cigarettes the more he desires them. He is willing to exchange 1 pack with 1 liter of oranges when he consumes 1 pack per week. But when his consumption is at 2 packs per week he is willing to exchange 2 packs with 1 liter of oranges. What is the likely shape of the indifference curves?

6. A farmer has a deep well that has enough water to irrigate its farm for 1 year. The reservoir does not recharge. The farmer lives for 2 years. Farmer’s preference relation is defined over consumption in each year \( (c_1, c_2) \in R^2_+ \). He is willing to exchange 1 dollar of consumption in year \( t \) with 1.2 dollars of consumption in year \( t + 1 \). The farmer prefers more consumption to less.

   (a) Draw a set of indifference curves that match the above description.

   (b) The farmer could earn 1.3 dollars in \( t + 1 \) if he invests 1 dollar in \( t \). His income from farming in a given year would be 10 dollars. Discuss how this farmer would decide on farm production and how he splits his consumption.