1 Readings

1. Read "Economics and Reality" by Uhlig"

2. Read DLS ch 6,9, Barro ch 6, Wickens ch 2,4

2 Value Function Calculation

1. Consider the following Bellman equation:

\[ V(k) = \max_{k'} \{ \ln (Ak^\alpha - k') + \beta V(k') \} \]

We guess that the solution is of the following form:

\[ V(k) = c \ln k + d \]
\[ k' = eAk^\alpha \]

Verify this guess and solve for \(c, d, e\). Explain the intuitions.

2. Consider the following Bellman equation:

\[ V(k) = \max_{k'} \{ (Ak^\alpha - k') + \beta V(k') \} \]
We guess that the solution is of the following form:

\[ V(k) = cAk^\alpha + d \]

\[ k' = k^* \]

Verify this guess and solve for \( c, d, k^* \). Explain the intuitions.

3 Representative Agent model Analysis

Consider the standard representative agent model with no labor supply decision. There is a representative household who solves an infinite-period consumption and investment choices such that

\[
\max_{\{c_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1 - \sigma}
\]

subject to

\[
c_t + i_t = w_t + v_t k_t + \Pi_t
\]

\[
k_{t+1} = (1 - \delta) k_t + i_t
\]

for all \( t = 0, 1, 2, \ldots \) and \( \beta < 1 \).

There is a representative firm which rents capital and employs workers to maximize its profit:

\[
\max_{k_t, l_t} \Pi_t = Ak_t^\alpha l_t^{1-\alpha} - w_t l_t - v_t k_t
\]

Markets clear such that \( l_t^d = l_t^s \) and \( k_t^d = k_t^s \). The final good is the numéraire good with price one. (If needed, you can take \( \sigma = 1 \) (i.e. log utility))

1. Write down the FOCs and the Euler equation. Solve for the steady state equilibrium.

2. Explain how does your S.S. results (all important macro variables like \( y, c, k, i \)) depend on \( A, \beta, \delta \) and \( k_0 \).
Suppose the economy is in the steady state at $t = 0$. Analyze the following situations using Dynare. Specifically explain (as we did in class) what would happen to each variable over time using supply-demand analysis in different markets. You can take $\beta = 0.95, \delta = 0.1$. For $\sigma$ try $\sigma = 0.5, 1, 2$

3. An unexpected negative productivity shock happens at time $t = 2$.

4. An expected positive productivity shock happens at time $t = 2$.

5. Vary $\sigma$ from 0 to 10 and see how does your responses to $c_t$ vary. Explain intuitively.

4 **Intertemporal Consumption Choice: Uncertainty**

Consider a two-period Intertemporal consumption decision model, where the household’s utility function is given by:

$$U(c_1; c_2) = E[\log (c_1) + \beta \log (c_2)]$$

where $c_1$ and $c_2$ denote period one and two consumptions, respectively, and $\beta < 1$ is the discount factor. The income in period 1 is $y$. But in period 2 the income is $y(1 + \varepsilon)$ with probability $\frac{1}{2}$ and it is $y(1 - \varepsilon)$ with probability $\frac{1}{2}$ where $\varepsilon > 0$ and it is small relative to 1. Household saves (or borrows) $b$ in period 1 and receives $b(1 + r)$ in period 2.

1. Setup the household maximization problem and write down the FOCs accurately.

2. Solve the optimum household choice for $c_1,c_2$ and saving $b$ for a given $r$. How does your answer vary with $\varepsilon, r$ and $y$. Explain the economic intuition.

3. Solve the general equilibrium problem and find the market rate of return $r$. How does your answer vary with $\varepsilon$ and $y$. Explain the economic intuition.

4. (optional) Now repeat the above exercises for the utility function: $U(c_1; c_2) = E \left[ \frac{c_1^{1-\eta}}{1-\eta} + \beta \frac{c_2^{1-\eta}}{1-\eta} \right]$. 
