Representative Agent Model

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Dynamic Macro problem

- Setup the dynamic problem
  - Determine the exogenous parameters and the endogenous allocations and prices.
- Write First Order Conditions (FOCs) and simplify them
- Solve for the Steady State solution in terms of parameters
- Comparative Statics on the exogenous parameters
- Solve for the transitional dynamics
- Comparative statics for the speed of convergence
A Simple Social Planner Problem

\[
\max_{\{c_t, k_{t+1}, i_t\}} \sum_{t=0}^{\infty} \beta^t U(c_t)
\]

s.t. \(c_t + i_t = f(k_t)\)

\(k_{t+1} = (1 - \delta) k_t + i_t\)
Dynamic Programming version of the Social Planner Problem

\[ V(k) = \max_{\{c,k',i\}} \{ U(c) + \beta V(k') \} \]

s.t. \( c + i = f(k) \)
\[ k' = (1 - \delta) k + i \]

Simplified:

\[ V(k) = \max_{k'} \{ U(f(k) + (1 - \delta) k - k') + \beta V(k') \} \]
A simple Representative Agent model

- $N_t$ Households

\[
\max_{\{c_t, k_{t+1}, i_t\}} \sum_{t=0}^{\infty} \beta^t U(c_t)
\]

s.t. \[c_t + i_t = w_t \ast 1 + v_t k_t + \pi_t\]

\[k_{t+1} = (1 - \delta) k_t + i_t\]

- Representative Firm:

\[\max \pi_t = F(K_t, L_t) - v_t K_t - w_t L_t\]

- Market Clearing

\[K_{d}^t = N_t k_{t}^s\]

\[L_{d}^t = N_t \ast 1\]
A Representative Agent model with labor-leisure decision

- $N_t$ Households

$$\max_{\{c_t, k_{t+1}, i_t, h_t\}} \sum_{t=0}^{\infty} \beta^t U (c_t, l_t)$$

s.t. $c_t + i_t = w_t * l_t + v_t k_t + \pi_t$

$k_{t+1} = (1 - \delta) k_t + i_t$

- Representative Firm:

$$\max \pi_t = F (K_t, L_t) - v_t K_t - w_t L_t$$

- Market Clearing

$$K_t^d = N_t k_t^s$$

$$L_t^d = N_t * l_t^s$$
A neoclassical growth model

- $N_t$ Households

  \[
  \max_{\{c_t, k_{t+1}, i_t, h_t, b_{t+1}\}} \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)
  \]

  \[\text{s.t. } c_t + i_t + b_{t+1} = w_t * l_t + \nu_t k_t + \pi_t + R_t b_t\]

  \[k_{t+1} = (1 - \delta) k_t + i_t\]

- Representative Firm:

  \[\max A_t F(K_t, L_t) - \nu_t K_t - w_t L_t\]

  \[A_{t+1} = A_t (1 + g)\]

- Market Clearing

  \[K^d_t = N_t K^s_t\]

  \[L^d_t = N_t * l^s_t\]

  \[N_t b_{t+1} = 0\]
What if the firms own the capital?
Lessons

- No Free Lunch
- No arbitrage
- Effect of productivity
- Consumption smoothing
- Capital accumulation process
- Investment rate
- Capital inflow
- Infinite elastic capital supply
- Leisure effect
- Calibration
- Role of price flexibility
Intertemporal Budget Constraint (IBC)

- How to setup the IBC.
- Interpretation! (People only care about their lifetime income)
Continuous version

- Problem set up
- Hamiltonian