1 Readings

1. Read Dr Nili’s book chapters 9,11 (11_1 to 11_4)

2. Read Intermediate Macroeconomics by Pablo Kurlat (on dropbox) chapters: 2 & 3 (This is a perfect substitute note for DLS)

2 Problem: Solow with Government

Let us introduce government spending in the basic Solow model. Consider the basic model without technological change and suppose that

\[ Y(t) = C(t) + I(t) + G(t) \]

with \( G(t) \) denoting government spending at time \( t \). Imagine that government spending is given by \( G(t) = \sigma Y(t) \).

1. Discuss how the relationship between income and consumption should be changed. Is it reasonable to assume that \( C(t) = sY(t) \)?

2. Suppose that government spending partly comes out of private consumption, so that \( C(t) = (s - \gamma \sigma)Y(t) \), where \( \lambda \in [0; 1] \). What is the effect of higher government spending (in the form of higher \( \sigma \)) on the equilibrium of the Solow model?
3. Now suppose that a fraction of $G(t)$ is invested in the capital stock, so that total investment at time $t$ is given by $I(t) = (1 - s - (1 - \lambda)\sigma + \phi\sigma)Y(t)$: Show that if $\phi$ is sufficiently high, the steady-state level of capital-labor ratio will increase as a result of higher government spending (corresponding to higher $\sigma$). Is this reasonable? How would you alternatively introduce public investments in this model?

3 Problem: Solow example (Optional)

Suppose that the world has two types of economies: industrial and agricultural. Both have a production function of the following form:

$$Y = K^\alpha L^{1-\alpha}$$

Suppose, agricultural economies have $\alpha = \frac{1}{3}$, industrial economies have $\alpha = \frac{2}{3}$. All countries have the same population and there is no population growth. Assume that all countries have the same savings rate $s = \frac{1}{2}$ and depreciations rate of capital $\delta = \frac{1}{8}$. An economy can only become industrial if it can finance a lot of research and development. Assume that, in this world, this happens suddenly. Specifically, assume that an economy becomes industrial (its changes) once it has a level of 27 units of capital per capita.

1. What is convergence? Would we observe convergence in this world? Explain and show graphically.

2. Suppose that the World Bank wants to help poor economies develop. It decides to give a gift to any economy with an output per capita of less than 3. The gift consists of 2 units of capital per capita. Will this make any help to the convergence? What if the gift is 19 units of capital per capita?

3. Suppose that there are no World Bank subsidies and that Agricultural countries have a government that can set the savings rate of the economy. Is there a savings rate that will allow Agricultural economies to become industrialized? What if the world bank subsidizes the economy by donating a dollar for every dollar that the country saves?
4 Problem: Neoclassical growth model

Consider the continuous time neoclassical growth model as in the class where the social planner maximizes

\[ W = \int_0^\infty e^{-\rho t} U (c_t) \, dt \]

where \( U (C) = \frac{C^{1-\sigma} - 1}{1-\sigma} \), subject to the resource constraint

\[ c_t + \dot{k}_t - \delta k_t = A_t k_t^\alpha \]

1. Write the two differential equations in terms of \( c \) and \( k \) that determines the equilibrium.

2. Suppose \( A_t = \bar{A} \). Solve for the steady state values of \( c \) and \( k \).

3. Find \( \frac{b_t}{m} \) in the steady state. Calibrate your model to US data using typical numbers \((\rho = 0.04, \delta = 0.1, \alpha = \frac{1}{3})\) and to Iran data \((\alpha = \frac{2}{3})\).

4. Go to Iran’s data from cbi.ir and calculate \( \frac{b_t}{m} \) for the years 1370-1389. What is the average number? How should you change \( \rho \) in the model so that \( \frac{\bar{k}}{\bar{y}} \) matches the data in the steady state?

5. (Optional) Find \( \frac{\dot{k}_t}{k_t - \bar{k}} \) in the steady state. This shows how fast we are converging to the steady state. Use typical numbers to calculate the number of years to get as half path to the steady state.

6. Now suppose \( A_t = \bar{A} e^{gt} \). Calculate the growth rate of consumption and capital \((\bar{g} = g_k = g_c)\).

7. Suppose we are on the Balanced Growth Path such that all the variables grow with constant rate. What is the Euler Equation? Solve for the marginal product of capital.

8. Define \( \tilde{k}_t = k_t e^{-\bar{g}t} \) and \( \tilde{c}_t = c_t e^{-\bar{g}t} \) and setup the problem using these variables. Do we have steady state for this problem? What are the two differential equations that solve for them? What are the steady state values if exist? This will show the transition to the
balanced growth path. Draw a diagram that shows how capital or consumption converges to the BGP.

5 Problem: Neoclassical growth model: Population Growth

Consider the discrete neoclassical model as in the class. Now suppose the population grows with rate \( g \) such that \( N_t = N(1 + g t) \).

1. Setup the problem, Write the First order conditions and find the Euler equation.

2. Explain how your results differ for the aggregate variables and per capita variables with the one we discussed in class.

6 Matlab: Neoclassical growth model

Consider the discrete neoclassical model as in the class. We have the infinite-period model of consumption-investment choices, where the social planner solves:

\[
\max_{\{c_t,k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \log c_t
\]

subject to

\[
c_t + i_t = Ak_t^\alpha
\]
\[
k_{t+1} = (1 - \delta) k_t + i_t
\]

for all \( t = 0, 1, 2, \ldots \). Again \( \beta < 1 \).

1. Write a code in Matlab to solve for the steady state values of the variables given the parameter values.

2. Write a code in Matlab to solve for the transition path to the steady state given the parameter values. To do so, your code receives \( k_0 \). It makes a guess for \( c_0 \), then it goes forward using the resource constraints and the Euler equation and solves for \( c_t \) and \( k_t \) for
\[ t = 1 \ldots T \] (large enough). Then you should check whether you are violating any boundary conditions or assumptions. If so, you should update your guess in the right direction (for example change \( c_0 \) to \( c_0 + \varepsilon \)) and try again. Iterate this process until you converge to a specific value of \( c_0 \). (Note that You can also make your guesses for \( k_1 \) instead of \( c_0 \)).

3. Now use your code and make a plot of \( k_1 \) versus \( k_0 \) for different values (at least 100 data points) of \( k_0 \) between \([0, \bar{k}]\). What does this graph show? What does it mean?

4. Now we use this code to find the transition paths. Suppose the initial capital stock drops by 10% below the steady state value. Show how do capital, output, consumption and investment respond and how do they converge to the steady state values (plot the transition paths). You can use the following numbers for your parameters: \( \beta = 0.96, \delta = 0.10, \alpha = 0.33. \)

5. Now suppose we are initially at the steady state, but the productivity rises by 10%. How do maro variables evolve over time?