1 Readings

1. Read Barro Chapters 5, 6.
2. Read DLS Chapters 2, 3, 5, 6.

2 Crusoe’s Preferences and Optimal Choice

Suppose that Crusoe has a utility function of the following form:

\[ u(c, l) = (1 - \gamma) \log c + \gamma \log (1 - h) \]

\(c\) and \(h\) are defined as in class (Consumption and hours of work). The production function of producing coconut is \(y = Ah^\alpha\), where \(0 < \alpha < 1\).

1. Solve for the optimal allocation using Lagrangian.
2. How does a change in \(A\) affect hours of work and total production. Explain in terms of income and substitution effects.
3. How does your result change if \(\alpha = 1\).
   Now consider the decentralized economy.
4. Setup the competitive equilibrium problem.
5. Find for the Robinson’s labor supply function.

6. How does a change in \( w \) affect Robinson’s labor supply and consumption. Explain your finding in terms of income and substitution effects.

7. How does a change in the profit of the firm to Robinson (\( \pi \)) affect Robinson’s labor supply and consumption. Explain your finding in terms of income and substitution effects.

8. What if there is a tax on wages? How does it affect labor supply and consumption. Explain intuitively.

9. Find the representative firm’s labor demand.

10. In a competitive equilibrium, find the real wage \( w/p \).

11. Solve for the equilibrium hour of work and total production.

12. Intuitively, discuss about the effect of aggregate productivity on labor demand, labor supply, equilibrium hours of work and equilibrium total production.

13. How does your results change if \( \alpha = 1 \)

14. (Optional) Now redo parts 1-6 with the following preferences:

   \[ u(c, l) = \frac{c^{1-\eta}}{1-\eta} - \frac{l^{1+\phi}}{1+\phi} \]

15. (Optional) In the class example, income and substitution effect canceled out each other when productivity term \( A \) changes. Now discuss the income and substitution effect in this case. What happens when \( A \) goes up? Which effect dominates? Be clear about the general equilibrium (GE) effect. Specifically, determine which effect comes from the GE?

16. (Optional) How does \( \phi \) matter?

3 Crusue in Two Periods

Now suppose there are two periods, \( \alpha = 1 \) and the Crusoe (Households) can borrow at rate \( r \). By writing down the model and FOCs using \( A_1 \) and \( A_2 \) as the productivity of the two periods.
1. Show what happens to $h_1$ if $r$ goes up, taking everything else constant? In other words, how does labor supply respond to a shock in the rate of return $r$.

2. Solve the previous two-period household problem and show how $h_1$ responds to $r$. Explain the economic intuition of your result.

3. Solve for the general equilibrium where the aggregate borrowing is zero.

4. Suppose $A_1$ goes up.
   
   (a) In the partial equilibrium where $R$ is given, how does it affect the labor supply in periods 1 and 2? How about the consumptions? and the borrowings.
   
   (b) Now answer the same problem in the general equilibrium where aggregate $B = 0$.

5. Now suppose $A_2$ goes up instead. Answer the previous problem in this case. Explain the differences and the economic intuitions.

4 Intertemporal Consumption Choice: Identical Agents

Consider the two-period model of Intertemporal consumption decision that we discussed in class, where the household’s utility function is given by:

$$U(c_1; c_2) = \frac{c_1^{1-\eta} - 1}{1 - \eta} + \beta \frac{c_2^{1-\eta} - 1}{1 - \eta}$$

where $c_1$ and $c_2$ denote the consumption in periods one and two, respectively, and $\beta < 1$ is the discount factor and $\eta < 1$. Suppose $y_1 = y_2 = 1$. Here we study the case where $\eta \neq 1$

1. Solve for the allocations analytically for $\eta = 1$.
   
   From now, consider a general $\eta$:

2. Write down the Euler Equation.

3. Solve for $s, c_1$ and $c_2$.

4. Plot $s, c_1$ and $c_2$ versus $r$ for $r \in (0, 1)$ and $\eta = 0.5$
5. On the same graph but with a different color, plot $s, c_1$ and $c_2$ versus $r$ for $r \in (0, 1)$ and 
$\eta = 1$.

6. On the same graph but with a different color, plot $s, c_1$ and $c_2$ versus $r$ for $r \in (0, 1)$ and 
$\eta = 2$.

7. Explain how does $\eta$ affect your results. Can you explain it intuitively?

8. Now for $r = 0.1$ plot $s, c_1$ and $c_2$ versus $\eta \in (0.1, 10)$

9. Now consider the general equilibrium and assume that all the agents are all the same and 
there are $N$ agents in the economy. Numerically solve for $r$ for $\eta = (0.1, 10)$. ploy $r$ versus 
$\eta$. Explain how does $\eta$ affect $r$. Intuition?

5 Intertemporal Consumption Choice: Heterogeneous Agents

Consider the two-period Intertemporal consumption decision model that we discussed in class, 
where the household’s utility function is given by:

$$U(c_1, c_2) = \frac{c_1^{1-\eta} - 1}{1-\eta} + \beta \frac{c_2^{1-\eta} - 1}{1-\eta}$$

where $c_1$ and $c_2$ denote period one and two consumption, respectively, and $\beta < 1$ is the 
discount factor and $\eta < 1$.

Here we study a case where all the household are NOT identical; and there are two types of 
households. Type $a$ is poor in the beginning and wealthy later but the other type $b$ is wealthy 
today and poor tomorrow. More specifically, suppose that there are $N_a = 60$ millions of 
households of type $a$ whose income are $y_{1a} = $2000 and $y_{2a} = $4000 and there are $N_b = 20$ 
millions of households of type $b$ whose income are $y_{1b} = $5000 and $y_{2b} = $1000 . You can take 
$\eta = 1$.

1. What is the aggregate borrowing of type $a$? (Write it as a negative number.)

2. What is the aggregate saving of type $b$? (As a positive number)
3. What is the net saving of the economy?

4. In equilibrium where the net saving is zero, what is the market value for the rate of return $R$?

5. (Optional) Solve for $R$ in the general case where we don’t have the assumption of $\eta = 1$.

6. What is the equilibrium $b^*_i$ for each type of household? Do we still have zero lending and borrowing?

7. Now suppose $N_a$ increases to $N_a = 400$ millions.
   
   (a) What is the new equilibrium $R$? Explain and give an economic intuition for this change using a supply-demand analysis.
   
   (b) How does the consumption of each type would change? Then use your findings of the previous problem regarding the income and substitution effects to justify your answer.
   
   (c) How does the Saving/Borrowing of each type would change? Explain it using the supply and demand curves.

8. Now suppose $y_{1a}$ increases to $y_{1a} = $20000.
   
   (a) What is the new equilibrium $R$? Explain and give an economic intuition for this change using a supply-demand analysis.
   
   (b) How does the consumption of each type would change? Then use your findings of the previous problem regarding the income and substitution effect to justify your answer. (Hint: be careful! There are couple of effects)
   
   (c) How does the Saving/Borrowing of each type would change? Explain it using the supply and demand curves.

9. Now suppose the government issues $\bar{B}$ amount of bonds, meaning that they borrow $\bar{B}$. Show how does it affect $r$. Explain the economic intuition.
6 Intertemporal Consumption Choice: Uncertainty (Optional)

Consider the two-period Intertemporal consumption decision model that we discussed in class with a minor change, where the household’s utility function is given by:

\[ U(c_1; c_2) = \log(c_1) + \beta E[\log(c_2)] \]

where \( c_1 \) and \( c_2 \) denote period one and two consumption, respectively, and \( \beta < 1 \) is the discount factor. The income in period 1 is \( y \). But in period 2 the income is \( y(1 + \varepsilon) \) with probability \( \frac{1}{2} \) and it is \( y(1 - \varepsilon) \) with probability \( \frac{1}{2} \) where \( \varepsilon > 0 \) and it is small relative to 1. Use linear approximations, if needed anywhere.

1. Setup the household maximization problem and write down the FOCs accurately.

2. Solve the optimum household choice for \( c_1, c_2 \) and saving \( b \). How does your answer vary with \( \varepsilon \) and \( y \)? Explain the economic intuition.

3. Solve the general equilibrium problem and find the market rate of return \( r \). How does your answer vary with \( \varepsilon \) and \( y \)? Explain the economic intuition.