Principles of Economics
Problem Set 6

1. **Readings:**

   (a) Mankiw’s book chapters 5, 6, 7, 14
   
   (b) Dr Nili’s book chapter 3
   
   (c) Murphy’s book pages 50-69
   
   (d) Becker’s book chapter 4 (Optional)

2. **Reading:** Read the article in the link below. In one paragraph elaborate what does the writer want to deliver and in another paragraph, write your position against or pro the writer. Bring sufficient arguments to support your claims. Be critical.


3. Suppose a profit-maximizer firm produces good $Y$ using capital and labor according to a production function $Y = F(K, L) = AK^aL^b$. The firm hire labor at wage $w$ and rent the capital at rate $r$.

   (a) Write down the cost minimization problem.
   
   (b) Write the FOCs (First Order Conditions)
   
   (c) Solve for the optimum level of $K$ and $L$ given $Y$ and the market prices $w, r$. We call these $K(Y; w, r)$ and $L(Y; w, r)$ the "conditional factor demand" functions.
   
   (d) Explain mathematically and intuitively, how do the conditional factor demands vary with $A, w, r$ and $Y$.
   
   (e) Find the cost function.
   
   (f) Find the marginal cost function. Is it constant? Why?
   
   (g) Explain mathematically and intuitively, how the does the marginal cost vary with $A, w, r$ and $Y$. 

(h) Now find the firm’s supply function. (Find \( Y(p) \))

(i) Now replace \( Y \) with \( Y(p) \) in the conditional demand function and solve for \( K(p; w, r) \) and \( L(p; w, r) \). We call them the unconditional demand functions.

(j) Explain mathematically and intuitively, how do the unconditional factor demands vary with \( A, w, r \) and \( p \).

(k) Find the elasticity of unconditional factor demands with respect to \( A, w, r \) and \( p \).

4. (optional) Consider the above problem and suppose we are in the short run so that the firm cannot adjust capital; i.e. the firm cannot change its capital level and can only decide about the labor. How do your answers change to the previous questions.

5. (Optional) Redo the above original problem using the production function

\[
Y = F(K, L) = AK^\alpha L^{1-\alpha}
\]

and explain the differences.

6. Suppose a firm that produces good \( Y \) uses labor only according to a production function \( f(L) = AL^\alpha \). Suppose the firm maximizes profit where \( \Pi(L) = pf(L) - wL \). Assume that both the output market and factor market (labor market) are competitive and the firm takes prices \( p \) and \( w \) as given. To have a decreasing return to scale production function which satisfies the law of diminishing marginal returns, assume that \( 0 < \alpha < 1 \). Finally assume that the fixed cost of production equals 7 million tomans.

(a) Find the level of \( L \) that maximizes profits.

(b) Find the firm’s cost function.

(c) Find the firm’s marginal and average costs.

(d) Find the firm’s supply function.

(e) What is the optimal labor demand?

(f) What is the market supply of good \( y \) if there are 1000 producers in the market.
(g) If there are only \( L = 10000 \) people available in the market, what is the equilibrium wage in the market.

From now on assume that \( A = 2, \alpha = \frac{1}{3} \) and wage \( w \) is 5 thousands tomans per hour.
Also assume that the demand for good \( Y \) is \( D(p) = 10000 - \frac{1}{2}p \).

(h) What is the market entry price?

(i) Find the market price of good \( Y \). Do firms enter at all?

(j) Find a firm’s total profit.

(k) What happens to market price and total profit if there are 1100 producers in the market?

In the long run, more firms can enter the market as long as their total profit is positive. Therefore, in the long run, so many firms have entered the market and the price have been lowered such that the total profit gets equal to zero. Thus, there is no incentive for more firms to enter the market. This price is called the "long run equilibrium price".

(l) Find the long run equilibrium price.

(m) How many firms are there in the market in the long run?

Now suppose that there is an income effect on demand due to the government subsidies so that the new demand for good \( y \) is \( D(p) = 20000 - \frac{1}{2}p \).

(n) What happens to the equilibrium price, quantity and profit in the short run where the number of firms does not change. Plot a graph that shows this change.

(o) What happens to the equilibrium price, quantity and profit in the long run where the number of firms changes. Plot a graph that shows this change.

Suppose that there is a shock in the labor market and therefore wages drop to \( w = 4 \) thousands tomans per hour.

(p) Using the same new demand curve, find the new equilibrium price, quantity and profit in the short run where the number of firms does not change. Plot a graph that shows this change.
(q) Find the new equilibrium price, quantity and profit in the long run where the number of firms changes. Plot a graph that shows this change.

(r) Now suppose due to the sanctions, the firms’ productivity drops to $A = 1$. How many firms exit the market right away? (use the new demand curve and the new wage).

7. (Optional) Similar to the previous problem, suppose a firm produces output $Y$ using labor only according to a production function $f(L) = AL^a$. But in contrast to the previous problem assume that the firm gets some utility from employing more people, for a fixed level of profits. Therefore, the firm chooses a level of employment to maximize utility, $U = U(L, \Pi(L))$ where $\Pi(L) = pf(L) - wL$. Assume that both the output market and factor market are competitive and the firm takes prices $p$ and $w$ as given. To be more concrete, assume $U$ is a Cobb-Douglas utility function. So $U = L^\beta \Pi(L)^{1-\beta}$, where $0 < \beta < 1$

(a) Find the level of $L$ that maximizes profits. What is the supply function of the firm?

(b) Find the level of $L$ that maximizes the firm’s utility. What is the supply function of the firm?

(c) In which case is the optimal $L$ larger? Why?

(d) Using the same demand function $D(p) = 10000 - \frac{1}{2}p$ and same parameters as in problem 1, what is the market price of good $Y$ in each case if there are 1000 firms in the market?

(e) Explain your findings.

(f) Now suppose $\beta = 1$ and the firms operates as long as $\Pi(L) \geq 0$. What is the interpretation of this utility function for the firm? Plot a graph that shows firm’s revenue and cost vs. labor and show the optimal decision point of this firm and compare it to a firm that maximizes profit only ($\beta = 0$).

8. True/False/Ambiguous: Argue.

(a) The more elastic the supply of a good, the smaller will be the deadweight loss from imposing a marginal tax on sales of that good.
(b) If corn and soybeans can be produced on the same land and are both used as animal feed, then a tax on soybeans will reduce the price of corn and increase corn production.

(c) A tax on imported oil will increase U.S. oil prices, increase U.S. oil production, and reduce U.S. oil consumption.

(d) An increase in the cost of refining oil should lead to greater production of coal and higher coal prices.

(e) An increase in the cost of refining oil should lead to higher prices for oil products (like gasoline) but lower prices for crude oil.

9. Show that there does not exist a Cobb-Douglas production function that exhibits increasing marginal product and decreasing return to scale?