International Trade, Skill Premium and Endogenous Firm Organization

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Abstract

In this paper, I propose a new theory to address a critical question in international trade: ‘What factors explain the worldwide increase in skill premiums following international trade integration’? I show how differences in the labor specialization of skilled and unskilled workers can explain the increase in the skill premium following trade liberalizations, independent from a country to be south or north. In this model, high-skilled workers are imperfect substitutes of each other, in contrast to the perfectly substitute low-skilled ones. A firm decides about its optimal degree of specialization for its high and low skilled workers by paying a fixed cost for each division of labor. A more productive and larger firm takes advantage of this economy of scale and increase the high-skilled workers labor division, organizing itself to a more skill-intensive firm. In a general equilibrium setting with heterogeneous firms, I show that trade cost reduction

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directly increases aggregate skill intensity of the firms and the skill premium. Moreover I find that more openness induces higher aggregate productivity for the economy above and beyond what Melitz-type models predict. Lastly, I calibrate this model to the US data and show that a 20% rise in trade costs can lower the skill-premium by 6%.

**Keywords:** Skill Premium, International Trade, Labor Specialization, Endogenous Skill Intensity, Firm Organization

**JEL:** F12, L22, J3

### 1 Introduction

The rise in income inequality following the trade integration in the last three decades is a striking and robust empirical fact. Both developed and developing countries have experienced increases in skill premiums\(^1\) after their liberalization periods. This is in contrast to the predictions of standard trade models, such as Heckscher-Ohlin, which states that the skill premium should fall in developing countries after a reduction in trade costs.

On the other hand, empirical researches show that trade openness induces reallocations in the labor market between firms in an industry, especially the high skilled workers towards more productive firms and exporters, increasing the skill intensity and organizational complexity of these firms. Given these facts and one natural question here could be that whether there exist any firm-level decisions that may lead to the rise in the skill premiums by trade liberalization.

To address this question, this paper proposes, and quantifies a tractable model of international trade with endogenous firm-organization and labor-specialization. In this new framework, consistent with these empirical findings\(^2\), I show how international trade openness, whether in a north or a south country, induces firms to take advantage of the imperfect substitutability of the high skilled workers and increase their own skill intensity endoge-

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\(^1\)Skill premiums refer to the relative wages of high skilled to low skilled workers.

\(^2\)See Goldberg and Pavnick (2007) as a survey on the literature about international trade and inequality.
nously\textsuperscript{3}. I show how firms’ decisions on their division of labor and horizontal organizational expansion result in a rise in the skill premium after a trade liberalization.

In this model, a firm hires perfectly substitutable low-skilled workers and imperfectly substitutable high skilled workers and to produce the output. High skilled workers work in several imperfectly substitutable groups or divisions: high skilled workers in each group, have specific specialties and are assigned tasks which can hardly be assigned to others, making them imperfect substitutes. For example, it is not easy to substitute a surgeon with a nurse, or an MBA graduate with an engineer. A firm needs to assign tasks which needs specialties to groups of high skilled workers which are not perfectly substitutable with each other. Therefore, a firm benefits more from the specialization of the high skilled workers, due to their imperfect substitutability.

On the other hand, a firm enjoys the higher gains from assigning special tasks to more divisions of high skilled workers by paying a fixed cost to set up each division. These fixed costs could be in the form of training, capital purchases, coordination or monitoring costs. The firm can take advantage of these specialization groups and its more labor divisions from high skilled workers and increase the firm’s labor productivity. The trade-off between paying these costs and the gains resulted from specialization of high-skilled workers determines the optimal number of divisions in this framework - which I name it the level of "horizontal expansion" of the firm; and it endogenously determines the firm’s skill intensity. This trade-off generates an economy of scale: a firm’s higher production demand or higher productivity results in more specialization groups in the firm, making the firm more skill intensive. Thus, an increase in a firm’s production demand or an increase in its productivity are causes that lead to a firm’s decision for expanding its horizontal organization. This biased expansion consequently increases a firm’s relative labor demand for high skilled workers.

The market structure in this model is similar to the Melitz (2003) model of trade. There are potential heterogeneous firms to enter: they pay a sunk cost and their productivity shock

\textsuperscript{3}See Rossi-Hansberg, Caliendo and Monte (2011 & 2012) for the effect of international trade on the organization of firms.
is then realized from a cumulative distribution function. If production is profitable, the firm pays a fixed cost of production, organizes itself as explained above and then produces its good to sell in a monopolistic competition market. The monopolistic rent should pay for all the fixed costs; this would endogenously determine the threshold for the entry. Free entry determines the size of the market. If the economy is open to trade, a firm should pay a fixed cost of exporting and export. This would entail that only more productive firms can export. Again, the rent from exporting determines the productivity threshold for exportation.

International trade introduces a higher demand for the industry, inducing more productive firms to reorganize and increase their division of high skilled workers, be more specialized and become more skill-intensive and more productive and then export\(^4\). Therefore, the model predicts that the exporters are more productive, have higher level of specialization and are more skill intensive. The prediction which is consistent with the findings in the data, like the one in Bustus (2011a).

I show that more openness in trade induces a rise in the industry aggregate skill intensity; it results in a higher labor demand for high skilled workers relative to low-skilled ones, generating a reallocation of high-skilled labor towards the exporting firms. Starting from Autarky, a reduction in trade costs would increase aggregate relative labor demand towards high-skilled workers.

In general equilibrium with two symmetric open countries, I show analytically that bilateral reduction in trade costs results in higher aggregate specialization, higher aggregate productivity, and higher aggregate skill intensity. Moreover, trade openness raises relative labor demand for the high-skilled workers and as the relative supply of high-skilled workers are fixed, trade openness would inevitably result in a rise in the skill premium, whether in a south or a north country: the fact that is consistent with decades of observations from developed and developing countries, as surveyed in Pavnick and Goldberg (2007).

\(^4\)Using French data, Caliendo, Monte and Rossi-Hansberg (2012) show that exporting firms have higher layers of hierarchy in their organisations. Also Bustus (2011a) uses Argentinian data and show after trade costs reductions, new firm adopt higher and more skill intensive technologies.
A by-product of the described model is the introduction of a new channel for the gains from international trade, too. As previously described, trade integration endogenously increases the old and the new exporters’ degree of specialization, raising their overall productivity; hence, it leads to a rise in the aggregate productivity. The increase in aggregate productivity would translate to reduction in aggregate prices and increases in real wages; thus making this mechanism a new source for the gains from trade.

Finally, to show how the model behaves quantitatively, I calibrate the model to US data by matching the identifying moments from the model to the ones in the US data. I match the model’s prediction for the skill premium, fraction of exporters and the firms’ death rate with the US data to calibrate the specialization fixed costs, exportation fixed costs and the entry sunk costs. I calibrate other parameters from other related literature. Then I analyze counterfactual scenarios to find out how much of the changes in trade costs or the specialization costs affect the skill premium. I find that with a 20% rise in US trade costs, the skill premium decreases by 6% and the aggregate welfare drops by 5%.

**Related Literature:** The goal of the paper is to propose a framework that is consistent with salient features of data and show how trade liberalization can raise the global skill premium through firm level decisions. Empirical findings show that skill premium has been increased in the developed and developing countries post trade liberalizations. Also, it has been shown that most of the resource reallocations due to the trade costs reduction have been occurred within industries and there is an increase in demand for skilled workers in sectors with larger tariff cuts. Moreover, the data shows that more skill intensive and productive

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5 See Chaney and Ossa (2012) as a closely related work.


firms are involved in export activities and have a higher complexity of organization. In this paper, my goal is to propose a new theoretical framework explaining and connecting these facts through firms’ endogenous decisions.

The impact of international trade on inequality is a classical question in economics. Traditional theories like Stolper-Samuelson Theorem predicts a decline in the skill premium in unskilled labor abundant countries through an inter-industry reallocation of labor. These predictions are in sharp contrast to the recent empirical researches surveyed in Goldberg and Pavnick (2007). To explain this puzzle, as surveyed in Goldberg and Pavnick (2007) and Acemoglu and Autor (2011), there are recent new theories examining the problem through different channels. The current paper proposes a new channel that complements the literature showing that the rise in the skill premium after liberalization is a phenomenon due to the larger market size, no matter in a north or a south country.

Feenstra and Hanson (1996, 1997, 1999, 2003) show that following trade openness, the developed countries out-source intermediate production into countries with less expensive labor. Production of these intermediate goods raises the demand for high-skilled workers from developing countries and thus raises their skill premium. Burstein and Vogel (2010), Parro (2010), Krusell et al. (2000), Stokey (1996) and Cragg and Epelbaum (1996) uses the complementarity of capital with skilled-labor and the growth in the global capital flows to explain the rise in the skill premium after trade openness. Verhoogen (2008 and 2009), in his influential work, show that Quality upgrading of exporters in developing countries increases the demand for better skilled workers and addresses the same question. This paper introduces another mechanism along these theories to explain the same fact, using the notion

8Bernard (1997) shows that the increases in employment at exporting plants contribute heavily to the observed increase in relative demand for skilled labor in manufacturing. Also, as notes in Verhoogen 2008, 2009, Bustos 2011, Harrigan 2012, Helpman, Elhanan, Itskhoki, and Redding 2010, exporters account for almost all of the increase in the wage gap between high- and low-skilled workers.

9Rossi-Hansberg & Caliendo 2012

10For the evidence on intra-industry reallocation, see: Mexico: Revenga (1997), Hanson and Harrison (1999), and Feliciano (2001); Colombia: Attanasio, Goldberg, and Pavcnik (2004); Morocco: Currie and Harrison (1997); India: Topalova (2004a); cross-country: Wacziarg and Seddon (2004); Latin American: Haltiwanger, Kugler, Kugler, Micco and Pages (2004)
of imperfect substitutability of high skilled workers.

On the other hand, many empirical findings have shown that due to "skill-biased technology" phenomenon, in both developing and developed countries, there is a quantifiable increase in the share of skilled workers and their relative wages within a narrowly defined category of industries. Attanasio, Goldberg, and Pavcnik's (2004) findings show that in Mexico, even low-skill intensity industries have been skill biased in technological advancements. Trade openness and skill-biased technologies affect the relative skill demand, thus increasing the skill premium. Burstein, Cravino and Vogel (2012) is a novel work using this notion. Wood (1995) and Thoenig and Verdier (2003) show that trade induces more R&D in exporters. Acemoglu (2003) introduce a model of endogenous technological change to explain the increase in wage premium. Matsuyama (2007) argues that export sectors are inherently more skill intensive and the rise in trade would raise the demand for high skilled workers. Helpman and Itskhoki (2010) show the more productive firms and exporters are better in screening their workers which results in a bias for high skilled workers in exporters, inducing a reallocation of labor toward exporters after trade opening within an industry.

Another examples are Bustus (2011a), and Harrigan (2011) which use the notion that exportation is a skill-biased activity and explain the higher skill intensity of exporters and the rise in the skill premium. This paper is very close to these ones making this bias an endogenous decisions of the firms, aggregating to a macro phenomenon in the general equilibrium. Bustus (2011a) shows that more productive firms and exporters bear an inclination toward upgrading their skill-biased technologies and hence endogenously evolve to become more skill intensive. In contrast, this paper constructs a micro-founded model that takes the increase in the skill intensity to be due to the firms' organizational decisions and their workers' specialization levels.

On the other hand, Burstein, Cravino and Vogel (2012) and Harrigan (2011) employ a skill-biased production technology where productivity is inherently correlated with skill intensity. They make the case that an increase in trade will ensure that only more productive
firms, which are supposedly more skill intensive, will remain in the market. Therefore, the overall skill demand within the industry picks up, resulting in an increase in the skill premium. In contrast, my model has a micro-foundation for this correlation with a firm level decision. Burstein, Cravino and Vogel (2012) elegantly implement a general equilibrium quantitative analysis to discover the extent in which their model can explain the rise in the skill premium across countries. Since they define firms to be inherently more skill intensive through being more productive, they show that any policy in favor of more productivity will induce higher skill intensity. In contrast to this paper, in their framework, firm’s skill intensity does not respond to changes in the scale and any demand shock to the firm has no impact on the firm’s structure and skill intensity. As such, new exporters choose to be more skill intensive through changing their organizations as empirically shown in Caliendo, Monte and Rossi-Hansberg (2012) and Bustus (2011b).

On the other hand, an abundant number of papers in economics are concerned with showing the productivity gains of labor specialization and assigning narrow measure of tasks to workers. Based on the classical concepts in the works of Adam Smith (1776), Hayek (1945), Rosen (1983), Becker and Murphy (1992)\(^{11}\) this paper examines the distributional effects of international trade, through the lens of labor division. In this model, it is the size of the market (the aggregate demand of home and foreign) that induces a firm to choose the level of its labor specialization, making it more productive and more skill-intensive. These are the decisions that in equilibrium lead to higher inequality between low and high skilled workers.

Moreover, the imperfect substitutability of the high skilled workers are similar to the ideas in Card and Lemieux (2001) where different age groups within an education group are imperfect substitutes. Card and Lemieux model the imperfectly substitutability hypothesis in a nested, two-level CES model. Versions of the multi-level CES are applied in a number of researches like Krusell (2000), Borjas (2003) and Acemoglu, Autor and Lyle (2004).\(^{11}\)

\(^{11}\)For related literature see Krishna et al (2009), Bombardini et al (2012) and Noblet (2010)
Similarly Card (2009) and Peri and Ottaviano (2011) consider a model where the skilled immigrants and natives are imperfect substitutes. Jäger (2016) shows that coworkers in the same occupation are substitutes, while high-skilled workers and managers appear to be complements to coworkers in other occupations. These findings support a key assumption of models featuring that skilled workers raise the productivity of other workers at the same firm (see, e.g., Lucas (1978); Murphy, Shleifer, and Vishny (1991)).

From another aspect, this paper is connected to the literature on firms’ organizations: Caliendo and Rossi-Hansberg (2011), Caliendo, Monte and Rossi-Hansberg (2012) employ the concept of firm organization in firms’ heterogeneity frameworks and their responses to trade integration. They show, both theoretically and empirically, that increasing trade openness induces firms to augment the number of layers in their hierarchy and become more productive. They also show that exporters have more vertically layered organizations. In contrast to trade-impact on the productivity gains and real wages as their main focus, my paper analyzes the distributional effects of trade through the lens of firm organization. Moreover, organizational expansions in this paper is horizontal, in contrast to the vertical expansions in their work.

Finally, starting from Bernard (1997) and following with huge empirical and theoretical works, most importantly Bernard, Eaton, Jensen and Kortum (2003) and Melitz (2003), it is shown that firm heterogeneity plays a crucial role in explaining the international trade patterns and reallocation of resources after trade liberalization. This paper builds upon these studies and connect the literature on division of labor and firms’ organizations to find the distributional effects of international trade.

2 Model

The model is a static two-symmetric countries model of international trade with two types of labor: High skilled and low-skilled workers. As in a Melitz model, production of varieties of
goods takes place in a continuum of heterogenous firms which now should decide on their optimal horizontal organizational expansion. The general setup is similar to the Krugman-Melitz type models with heterogeneous firms and monopolistic competition framework. There are two symmetric countries, \( j = h, f \) (home and foreign) with \( L \) low-skilled and \( H \) high-skilled workers.

### 2.1 Preferences

In country \( j \), the representative household supplies both types of labor and has constant elasticity of substitution (CES) preferences (as in Spence-Dixit-Stiglitz) over the consumption \( c_{ij}(A_i) \) of differentiated varieties, which are produced by a measure \( M_{ij} \) continuum of producers in \( i \), selling in \( j \); Its producer has productivity \( A_i \) which is randomly drawn from a cumulative distribution function \( F(A) \), such that

\[
U_j = \left( \sum_{i=h,f} \int_{A_i} c_{ij}(A_i)^{\frac{\sigma-1}{\sigma}} M_{ij} dF(A_i) \right)^{\frac{\sigma}{\sigma-1}}.
\]

Parameter \( \sigma \) is the elasticity of substitution between the differentiated varieties. Trade is balanced; therefore the representative household has the following budget constraint

\[
\sum_{i=h,f} \int_{A_i} d_{ij} p_i(A_i) c_{ij}(A_i) M_{ij} dF(A_i) = X_j = w_{Lj} L_j + w_{Hj} H_j + \Pi_j
\]

where \( X_j \) is the total expenditure of country \( j \), \( \Pi_j \) is the total profit of the firms in country \( j \), and \( w_{kj} \) is the wage for labor of type \( k \) (= \( H \) or \( L \)) in country \( j \).

Therefore, the demand for variety \( A \) is \( c_{ij}(A) = (d_{ij} p_i(A))^{-\frac{1}{\sigma}} \left( P_j^{\sigma-1} X_j \right) \) where \( p_i(A) \) is the price of the variety produced by the a firm with productivity \( A \) in country \( i \). Parameter \( d_{ij} \) is the variable iceberg trade cost of exporting goods from \( i \) to \( j \). Lastly, \( P_j = (M_{ij} \int_A p_i(A)^{1-\sigma} dF(A))^{\frac{1}{1-\sigma}} \) is the aggregate price index.

### 2.2 Market Structure

The Market structure is the same as Krugman (1980) so that each firm sells its differentiated good monopolistically in the market. Because of SDS preferences, the demand elasticity is
constant $\sigma$. Thus for a producer in country $i$ with productivity $A$, its total production demand is

$$y_i(A) = p_i^{-\sigma}(A) D_i$$

(1)

where $D_i$ is a "demand indicator" for a producer in $i$, such that $D_i = D_{ii}$ for a domestic producer and $D_i = \sum_j D_{ij}$ for an exporter, where $D_{ij} = d_i^{1-\sigma} P_j^{\sigma-1} X_j$ with $d_{ii} = 1$. Since the firm sells its unique variety as a monopoly in the market, it sets its price a constant markup $m = \frac{\sigma}{\sigma-1}$ over marginal cost.

**2.3 Production**

As in a Melitz-type framework, firms are heterogenous in productivity in this model. They pay a sunk entry cost to draw a random productivity $A$ from cumulative distribution function $F(A)$, and after observing the productivity level, decide to pay a fixed entry cost to enter a market if it is profitable. To enter an international market they need to pay an extra fixed exporting cost. For notational simplicity, I drop the country and firm subscripts.

A firm hires $l$ numbers of low-skilled and $h$ numbers of high-skilled workers. Low skilled workers are perfect substitute with other while high-skilled workers are imperfect substitutes. The firm produces output $Y$ such that:

$$Y = A \left( l^{\frac{\sigma-1}{\sigma}} + \int_0^S h_s^{\frac{\sigma-1}{\rho}} ds \right)^{\frac{\rho}{\rho-1}}$$

(2)

where $h_s$ is the number of high skilled workers in group $s$ from a measure $S$ of groups and

$$h = \int_0^S h_s ds$$

(3)

is the total number of employed high-skilled workers$^{12}$. The elasticity of substitution between

$^{12}$We can use discrete $S$ number of groups as well, but for more tractability we take it as continuous.
high skilled workers is \( \rho > 1 \) which is equal to the elasticity of substitution between the low-skilled and high-skilled group\(^\text{13}\).

For a given \( S \), a firm decides how to allocate its high skilled workers in different groups which are not perfect substitutes. A firm can pay a fixed cost of specialization \( w_h f S \) and generate measure \( S \) groups. These costs can be interpreted as coordination, training or monitoring costs. Here, the firm faces a trade-off between paying more fixed costs and having more groups of high skilled workers to enjoy the benefit of more specialization and higher productivity.

Therefore, a firm’s cost minimization problem is

\[
\min_{S,l,h,\{h_s\}^S} w_l l + w_h h + w_h f S \\
\text{s.t. (2) & (3)}
\]

for a given \( Y \) where \( w_l \) and \( w_h \) are wages for lows-skilled high skilled-workers respectively.

Due to symmetry, the firm chooses all the \( h_s \)'s equal. Therefore the firm’s production function is simplified to \( Y = A \left( \frac{l^{\frac{1}{1-\rho}}}{1-\rho} + S^\frac{1}{\sigma} h_s^{\frac{1}{\rho-1}} \right)^{\frac{\rho}{\rho-1}} \). Thus for a given \( S \), optimal relative labor demand is \( \frac{h}{l} = S \Omega^{-\rho} \) (with \( \Omega = \frac{w_h}{w_l} \)) which is increasing in the degree of specialization; leaving us with \( C(S; Y) = w_l l + w_h h = Y A \left( w_l^{\frac{1}{1-\rho}} + S w_h^{\frac{1}{\rho-1}} \right)^{\frac{1}{\rho-1}} \) as the total labor cost. As it is clear, this total cost is decreasing in \( S \), the degree of of specialization(DoS); meaning that the firm can lower its costs by increasing the number of specialized groups and becoming more skill intensive.

2.3.1 Firm’s Organizational Problem

As it was shown, a firm should pay fixed cost of specialization and choose \( S \) groups of specialization for its high skilled workers. Choosing \( S \) optimally can be interpreted as choosing the optimal horizontal degree of expansion in the firm organization; i.e. how a firm decides

\(^\text{13}\)The more general case with \( \rho_H > \rho \) is also true without loss of generality, where \( \rho_H \) is the elasticity of substitution between the high skilled workers.
to increase the number of groups of high skilled workers and enjoy the gains from specializing them and having lower cost of production \( C(S) \).

Solving the firm’s organizational problem means setting \( w_h \bar{f} = -\frac{\partial C(S; Y)}{\partial S} \), giving us the optimal degree of specialization as

\[
S^*(Y) = \left( \frac{Y/A}{(\rho - 1) \bar{f}} \right)^{\frac{\rho}{\rho - 1}} - \Omega^{\rho - 1}
\] (4)

and the conditional marginal cost as

\[
mC(Y) = \left( \frac{(\rho - 1) \bar{f}}{Y A^{\rho - 1}} \right)^{\frac{1}{\rho}} w_h
\]

which are both functions of total production; the former is an increasing one, while the later is a decreasing one.

For a given \( Y \), the firm chooses the optimal level of specialization and its labor demand such that

**Lemma 1** (a) The optimum degree of specialization \( S^*(Y) \) is increasing in \( Y \) and decreasing in \( \bar{f} \).

(b) The relative labor demand of high-skilled vs. low-skilled workers (skill intensity), \( \frac{N_H(Y)}{N_L(Y)} \), is increasing in \( Y \).

(c) The marginal cost of producing \( Y \), \( mc(Y) \), is decreasing in \( Y \).

**Proof.** See Appendix.

As the lemma states, the conditional firm organizational problem results in a decreasing marginal cost function with respect to total production; meaning that the firm can enjoy an increasing returns due to this endogenous specialization of its workers. Basically, a firm with higher production demand has more incentive to invest on its organizational expansion, increase its division of labor and be more specialized in its high skilled workers, thud
2.3.2 Profit Maximization

As in Krugman type framework, a firm pays a fixed production cost and sells its production in a monopolistic market. Since the demand elasticity is constant, the firm has a constant markup $m = \frac{\sigma}{\sigma - 1}$ over marginal costs, which is itself a function of $Y$. To solve for the optimum level of production $Y$ and price $p$, I use the firm demand equation (1) which results in solving the following fixed-point problem:

$$Y = \left( \frac{\sigma}{\sigma - 1} mc(Y) \right)^{-\sigma} D \quad (5)$$

Therefore, given the demand indicator $D$, a firm with productivity $A$ chooses the optimum level of production $Y(A, D)$ and price $p(A, D)$. Because marginal cost is decreasing in $Y$ (as shown in the previous lemma), the firm’s price is also decreasing in $Y$. Figure 1 shows this feature. Any increase in the firm’s production demand shifts the marginal revenue curve to the right, inducing a reduction in the firm’s price. Next lemma describes the optimal firm’s decision about its output and price.

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14 In a more general case it has been shown that without loss of generality, we can have specialization of workers for both types of workers. But since high skilled workers’ substitutability is less than the low skilled workers, for sake of tractibility, I use this extreme limit of perfect substitutability for low skilled workers and imperfect substitutability for high skilled ones.

15 $\sigma$ is the elasticity of substitution between different varieties.
Lemma 2 If the $\rho > \sigma > 1$, then

(a) The firm’s optimum action exists and output and prices are positive and finite.

(b) The firm’s optimum level of production $Y^* (A, D)$ is increasing in $A$ and $D$, and decreasing in $\bar{f}$.

(c) The firm’s price $p^* (A, D)$ is decreasing in $A$ and $D$, and increasing in $\bar{f}$.

(d) The optimum degree of specialization (DoS), $S^* (A, D)$, is increasing in $A$ and $D$ and decreasing in $\bar{f}$.

(e) The relative labor demand \( \frac{N_H^* (A, D)}{N_L^* (A, D)} \) is increasing in $A$ and $D$, decreasing in $\bar{f}$.

(f) The firm’s optimum revenue $R (A, D)$ is

\[
R (A, D) = \bar{g}D^{\frac{\rho-1}{\rho-\sigma}} A^{\frac{\sigma-1}{\rho-\sigma}(\sigma-1)} \tag{6}
\]

(g) The cost of hiring high and low skilled workers are:

\[
C_H (A, D) = w_H N_H = \frac{\sigma - 1}{\sigma} R (A, D) - (\rho - 1) \bar{f} \Omega^{\rho-1} \tag{7}
\]

\[
C_L (A, D) = w_L N_L = (\rho - 1) \bar{f} \Omega^{\rho-1} \tag{8}
\]

where $\Omega = \frac{w_H}{w_L}$ is the skill premium and \( \bar{g} = \left( \frac{\sigma}{\sigma-1} \right)^\rho (\rho - 1) \bar{f} w_H^{\rho-1} \right)^{-\frac{\sigma-1}{\rho-\sigma}}.$

Proof. See Appendix. ■

Note that output $Y (A, D)$ and $p (A, D)$ can be easily calculated accordingly. Variable profit function $\Pi (A, D)$ can be calculated as $\Pi (A, D) = \frac{\rho-\sigma}{\sigma (\rho-1)} R (A, D) + \bar{f} \Omega^{\rho-1}$. Also, specialization of high-skilled workers would be $S (A, D) = \frac{C_H (A, D)}{(\rho-1) \bar{f}}$ which is increasing and convex in $A$ and $D$.

As expected from a Krugman-Melitz type model, more productive firms have lower prices (quality adjusted) and higher productions, revenues and profits and the relationship of productivity and price is one to one. In contrast, what is new here is that 1) the relationship of productivity and prices are more than one to one; i.e. one percent higher productivity results
in more than one percent lower prices, 2) between two firms with the same productivity $A$, the one with higher demand $D$ has a lower price; it shows a scale effect which it lacks in conventional trade models. This efficiency gain is the result of the economy of scale that exists in the firm’s organizational expansion. Firms with higher demands are more horizontally expanded in their organization and have higher degrees of specialization for skilled workers, decreasing their marginal costs; thus their prices. This analysis shows another margin of gain from economy of scale; I call it the "within-firm margin".

Also it can easily be shown that the firm’s revenue and output increases more than one to one with respect to demand $D$ which is again due to the productivity gain from specialization and horizontal expansion; i.e. since $\frac{e-1}{p-\sigma} = 1 + \frac{e-1}{p-\sigma} > 1$, showing larger effects of $D$ on these variables in comparison to the conventional models.

Also, the elasticity of revenue with respect to productivity is $\eta = (1 + \frac{e-1}{p-\sigma})(\sigma - 1) > \sigma - 1$. This inequality shows that this model generates a more significant effect of productivity and aggregate demand compared to the typical Krugman-Melitz type models.

The most important result is that the high-skilled labor demand is increasing with productivity $A$ and production demand $D$ and the labor demand for low-skilled workers is constant with respect to these two variables. Therefore, the relative labor demand increases with productivity and production demand, making skill intensity positively correlated with these two variables. In other words, since specialization of the high-skilled workers brings gains for the firm, an increase in the firm’s productivity has a biased effect in labor demand toward high-skilled workers, too. Thus, the skill intensity of the firm is endogenously determined by the firm’s decisions; this feature does not exist in conventional models. This biased effect productivity is consistent with data where we observe that the skill intensity of a firm has positive correlation with the firm’s productivity as in Harrigan (2012) or Bustus (2011a). This biased effect arises from the notion that a firm can make a decision on its horizontal organizational expansion and its labor intensity. This choice gives a more-productive firm the opportunity to raise its skill intensity. Therefore this model generates an endogenous
process for biased technological change.

2.4 Market Entry, Aggregation and Partial Equilibrium

2.4.1 Entry

The countries are similar, so they have the same allocation and prices; thus I do not use any country specific subscripts. There is a measure $M_e$ of potential firms that pay a sunk entry cost $f_e$ to draw a productivity level $A$ with cumulative distribution function $F(A) = \Pr(A \geq \bar{A})$. Again, following Melitz (2003) and Chaney (2008), I assume a Pareto distribution with parameter $\theta$ and minimum productivity level $\bar{A}$ such that $F(A) = \left(\frac{\bar{A}}{A}\right)^{-\theta}$.

To guarantee the convergence in the aggregation, the following condition should hold:

**Assumption:** $\theta > \eta = \frac{(\rho - 1)(\sigma - 1)}{\bar{A} - \sigma}$.

After observing the productivity $A$, a firm pays an operational fixed cost $f_o$ to enter the domestic market, if it’s profitable. Also the firm can pay a fixed exporting cost $f_x$ to export, if it can earn more profit from exporting. This means that a firm operates domestically if $\Pi(A, D) \geq f_o$ and it exports if $\Pi(A, D + D_f) - \Pi(A, D) \geq f_x$, where $D$ and $D_f$ are the demand indicators for home and the foreign market, where $D_f = d^{1-\sigma}D$.

**Assumption:** "Home market" is softer than "foreign market." It means that exporting fixed costs are high enough so that if a firm operates in the foreign market, it would surely operate also in the domestic market.

These entry conditions result that more-productive firms can only enter and most productive ones enter the export market, too, defining entry and export productivity thresholds $\bar{A}_o$ and $\bar{A}_x$:

\[
\bar{A}_o = Z_1 D^{\frac{\sigma - 1}{\sigma - \eta}} \tag{9}
\]

\[
\bar{A}_x = Z_1 D^{\frac{\sigma - 1}{\sigma - \eta}} \left(\frac{f_x}{f_o}\right)^{\frac{\eta}{\sigma - 1}} \left(d^{-\eta} - 1\right)^{-\frac{1}{\eta}} \tag{10}
\]
where $\tilde{d} = (1 + d^{1-\sigma})^{\frac{1}{1-\sigma}}$, $Z_1 = m^m \eta^{\frac{1}{\rho}} (\rho - 1)^{\frac{1}{\rho - 1}} w_H f_a^{\frac{1}{\rho - 1}} f_a^{-\frac{1}{\rho}}$, $f_a = f_o - \bar{f} \Omega^{\rho - 1}$ is an adjusted fixed cost variable.

Equations (9) and (10) show the negative effect of demand $D$ and the positive effects of trade barriers ($d$ and $f_x$) on the entry and export thresholds. $\bar{A}_o$ and $\bar{A}_x$ are decreasing in $D$, since, profits are increasing in productivity $A$ and production demand $D$; therefore, as in the Melitz model, an increase in demand induces more firms to pay fixed costs to operate or to export; thus it lowers these thresholds.

Figure 2 schematically shows how firms with different productivities decide about their prices, outputs, skill intensities, entry and export activities. As shown in the previous section, the relative labor demand is increasing in productivity $A$ and demand indicator $D$. Thus more-productive firms choose to be more skill-intensive and demand more high-skilled workers relative to low-skilled ones. And since very productive firms decide to enter the foreign market and face a larger demand, they decide to be more specialized and also become more skill-intensive because of higher production demand. Therefore they have more organizationally expanded firms, charging lower prices and choosing to be much more skill-intensive than non-exporters.
2.4.2 Partial Equilibrium, Aggregate Price and Aggregate Production Demand

In partial equilibrium, wages $w_H$ and $w_L$, aggregate expenditure $X$ and the measure of entrants $M_e$ are given. Using the polynomial form of the revenue function, Pareto distribution assumption for the productivities, and the entry conditions, I can solve and simplify the aggregate revenue as

$$
\tilde{R}(D) = (\rho - 1) m \zeta M_e (\mu_o f_a + \mu_x f_x)^{16}
$$

where $\zeta = \frac{\eta (\sigma - 1)}{(\theta - \eta) (\rho - \sigma)}$, $\mu_o = \left( \frac{\bar{A}_o}{\bar{A}} \right)^{-\theta}$ and $\mu_x = \left( \frac{\bar{A}_x}{\bar{A}} \right)^{-\theta}$ are the fractions of producers and exporters. Thus aggregate revenue $\tilde{R}(D)$ is a function of aggregate demand $D = P^{\sigma - 1} X$ because the thresholds $\bar{A}_o$ and $\bar{A}_x$ depend on $D$ as in (9) and (10).

Market clearing and trade balance imply that the aggregate revenue is equal to the aggregate expenditure. Hence $X = \tilde{R}(P^{\sigma - 1} X)$. This fixed-point problem can pin down the equilibrium aggregate price index $P$ as a function of aggregate expenditure $X$:

$$
P = \frac{Z_2}{X^{\frac{1}{\sigma - 1} - \frac{1}{\beta}} (1 + O)^{1/\theta}}
$$

where $Z_2 = \frac{Z_1}{\bar{A}((\rho - 1) m \zeta f_a M_e)^{1/\theta}}$ and

$$
O = \left( \frac{f_x}{f_a} \right)^{1 - \frac{\theta}{\eta}} \left( \tilde{d}^{-\eta} - 1 \right)^{\frac{\theta}{\eta}}
$$

is an openness parameter which is decreasing in the trade costs.

Equation (12) shows that trade openness reduces aggregate prices through three sources. First is the typical Krugman type channel which is the availability of cheaper foreign varieties. This is inherited in the $\tilde{d} = (1 + d^{1-\sigma})^{1/\tau}$ term inside the openness parameter. As trade costs $d$ goes down, foreign producers face lower marginal costs; thus they lower their prices. This would push down the aggregate price. The second source is the extensive margin-of-trade channel as in the Melitz (2003) and Chaney (2008), where the number of foreign exporters change and home country’s households have access to increasingly more varieties 

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of foreign goods. This mechanism shows up with the terms \( f_x \) and \( \theta \), in the price equation. The third source, which is the newly introduced source in this paper is the within-firm margin of adjustment where the old and new exporters re-organize to a more horizontally expanded and more specialized firm. This reorganization increases their labor productivity, reducing marginal costs, hence reducing their prices, resulting in declines in the aggregate price index. This notion shows up through the parameter \( \eta = \left( 1 + \frac{\sigma-1}{\rho-\sigma} \right) (\sigma - 1) > \sigma - 1 \) in the openness parameter and the fixed specialization cost \( \bar{f} \) inside the variable \( Z_2 \). Lowering \( \bar{f} \) would lower the specialization costs, inducing more incentive for labor specialization and having increasingly more productive firms which will reduce the aggregate price index.

In contrast to this model, in the conventional model, where there is no within-firm margin of adjustment, the extensive margin of trade appears with \( \frac{\theta}{\sigma-1} \) instead of \( \frac{\theta}{\eta} \), as noted in Chaney (2008). Also, the intensive margin effect shows up with \( d^{-\theta} \) instead of \( d^{-\eta} \).

Also, having \( P \) solved analytically and using the definition of \( D (=P^{\sigma-1}X) \), the fraction of exporters and non-exporters can be solved as:

\[
\mu_0 = \left( \frac{1}{(\rho - 1) m \zeta} \right) \frac{X}{M_e (1 + O)} \frac{1}{f_a} \quad (14)
\]
\[
\mu_x = \left( \frac{1}{(\rho - 1) m \zeta} \right) \frac{X}{M_e (1 + O)} \frac{O}{f_x} \quad (15)
\]

As expected from the Melitz-type model, the fraction of exporters increases with openness \( O \) but the fraction of domestic producers decreases with it; making the market less crowded. This analysis shows that reducing trade costs increases foreign production demand, inducing more firms to become exporters. The production demands of older exporters also change, inducing them to re-organize, too.

To see how old and new exporters re-organize, I should look into the demand that they face. There are two sources that affect the exporters’ demands. Since \( D_f = (P^\sigma) \left( \frac{X}{P} (1 + d^{1-\sigma}) \right) \), one source is the aggregate industry demand that shows up as \( \frac{X}{P} (1 + d^{1-\sigma}) \). In other words, 

\[17\]Details are explained in the seminal work of Chaney (2008) which elaborates on the Melitz type model and distinguishes between different mechanisms affecting the prices.
when aggregate real expenditure on the industry goes up or trade costs goes down, total production demand for the industry goes up, increasing the production demand for each firm, specifically for the exporters; let’s call it the Direct Channel. The second source is substitutability of the firms within the industry. A decrease in industry’s aggregate price index, $P$, reduces a firm’s production demand because of the substitution effect; since the competitors’ prices are lower and consumers substitute away from this firm. This mechanism shows up with the term $P^\sigma$ in the demand indicator $D_f$; let’s call this second channel the Indirect Channel. As it was shown above, reducing $d$ reduces $P$; thus reduces the aggregate demand. Therefore, reducing $d$ would increase $D_f$ through the Direct Channel and decrease it through the Indirect Channel. To find out which force dominates, I calculate $D_f$ explicitly and I get

$$D_f (d) \propto \left( \tilde{d}^\eta + \left( \frac{f_x}{f_a} \right)^{1-\frac{\theta}{\eta}} \left( 1 - \tilde{d}^\eta \right)^{\frac{\theta}{\eta}} \right)^{-\frac{\sigma-1}{\sigma}}$$

where $\tilde{d} = (1 + d^{1-\sigma})^{\frac{1}{\sigma-1}} \in \left( \frac{1}{\sigma-1} \right)$. Function $D_f (d)$ has been illustrated in Figure 3. It is easy to show that reducing trade cost $d$ would initially raise foreign demand $D_f$ initially through the Direct Channel, but bring it down later because of the drop in the aggregate price$^{18}$ through the Indirect Channel. Thus reducing variable trade cost may raise or drop the demand for old exporters in the equilibrium. Therefore, old exporters may increase or decrease their level of labor specialization depending on $D_f$.

New exporters are different. With a marginal change in $d$, some old non-exporters become new exporters. The demand they were facing has been multiplied by $1 + d^{1-\sigma} > 1$, which is a large increase compared to a marginal change through the Indirect Channel. Consequently, new exporters always expand their specialization level. Figure 4 shows the skill intensity of the firms with respect to their productivities before and after a change in variable trade costs.

---

$^{18}$This maximum occurs when $\tilde{d} = \left( 1 + \frac{f_x}{f_a} \right)^{-1/\eta}$ and hence, maximum $D_x$ is $(\tilde{d})^{\sigma-1} \left( \frac{x}{f_x} \right)^{\frac{\eta - 1}{\eta}} \left( 1 + \frac{f_x}{f_a} \right)^{(\tilde{d}^{-1})^{\frac{\eta - 1}{\eta}}}$. 

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On the other hand, it is clear from (16) that reducing the fixed export costs $f_x$ can only reduce $D_f$ for the old exporters, because it can only reduce the price index from the Indirect Channel and has no positive Direct Channel. This means that reducing fixed export costs results in a contraction in old exporters. New exporters with the same reason as above will face expansion of labor division.

### 2.4.3 Aggregate Relative Labor Demand

Due to the polynomial form, the aggregate labor demand for both types of workers can be solved analytically. According to (8), labor demand of the low-skilled workers is independent of productivity $A$ and demand $D$; hence $\tilde{C}_L$, the total cost of low-skilled workers of all the firms is $\tilde{C}_L = M_o \mu_x (\rho - 1) \tilde{f} \Omega^{\rho - 1}$. On the other hand, from (8) and (7), total labor demand $C_H + C_L$ equals $\frac{R}{m}$ for each firm. Therefore, from (11), the aggregate cost of labor is $\tilde{C}_L + \tilde{C}_H = (\rho - 1) \zeta M_o (\mu_o f_o + \mu_x f_x)^{19}$. Proposition 3 presents the aggregate relative labor

\[ ^{19}\tilde{C}_H \text{ and } \tilde{C}_L \text{ are the aggregate cost of high skilled and low workers, respectively.} \]
demand equation.

Proposition 3  If \( \Omega < \left( \frac{\sigma-1}{\rho+1} \right)^{\frac{1}{\sigma+1}} \), the aggregate relative labor demand equals

\[
\frac{\bar{N}_H}{\bar{N}_L} = \Omega^{-1} \left( \zeta \left( \frac{f_o}{f} \Omega^{1-\rho} - 1 \right) (1 + O) - 1 \right) \quad (17)
\]

where \( \zeta = \frac{\theta(\sigma-1)}{(\theta-n)(\rho-\sigma)} \), \( \bar{N}_k \) is the aggregate total labor demand of type \( k = H, L \) for all firms in the industry, \( \Omega = \frac{w_H}{w_L} \) is the skill premium and \( O \) is the openness parameter in (13) and in terms of the exogenous variables \( w_H, w_L, X \) and \( M_e \), the aggregate relative labor demand is only a function of the skill premium \( \Omega = \frac{w_H}{w_L} \).

Proof. See Appendix. ■

With higher trade barriers, the openness parameter decreases and therefore aggregate relative labor demand decreases. In the extreme case where trade barriers are infinity, openness becomes zero \( (O = 0) \); hence equation (17) presents the model under autarky (closed economy)\(^{20}\). Increase in the aggregate relative labor demand following a reduction in trade barriers is the result of labor reallocation in three types of firms: New exporters, old exporters and non-exporters. First of all, new exporters are those highly productive previous non-exporters who found it optimal to expand their organization and hire more skilled workers due to the large international demand; thus they become more skill intensive. Second, old exporting firms, depending on \( d \), may face more or less demand because of the drop in the aggregate price, as discussed earlier; therefore, they will restructure and become more or less skill-intensive; increasing or decreasing their demands for high-skilled workers. Third, less productive non-exporters, who do not find it optimal to export, face lower demand \( (D) \) because of the presence of much more productive firms in the industry;

\(^{20}\)Note that (17) is the analytical equation for relative labor demand in the Upper Boundary Case, and it is valid when \( \Omega < \left( \frac{f_o}{f_H} \right)^{\frac{1}{\sigma+1}} \). This condition ensures that the specialization costs should be low enough relative to the entry costs \( f_o \) so that all the firms have incentive to hire high-skilled workers. As long as this condition holds, lowering trade barriers would increase the aggregate relative labor demand.
Table 1: Effect of trade cost reduction in partial equilibrium

<table>
<thead>
<tr>
<th></th>
<th>Production Demand</th>
<th>Relative Labor Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic producers</td>
<td>Less</td>
<td>Less</td>
</tr>
<tr>
<td>New exporters</td>
<td>More</td>
<td>More</td>
</tr>
<tr>
<td>Old exporters (high trade costs)</td>
<td>More</td>
<td>More</td>
</tr>
<tr>
<td>Old exporters (low trade costs)</td>
<td>Less</td>
<td>Less</td>
</tr>
</tbody>
</table>

thus, relative labor demand for these firms would decline since they shrink their organization and become less skill intensive. Table 1 summarizes the above discussion.

These organizational changes result in a reallocation of high-skilled workers from domestic producers toward new and old exporters in an environment with high trade costs. In a low trade cost environment, the reallocation of high-skilled workers is from domestic producers and old exporters toward new exporters. Nevertheless, from (17) it turns out that the reallocation towards the new exporters dominates and the aggregate relative demand increases with lowering trade costs.

Finally, note that this framework is a new source of gain from international trade. Lowering trade costs induces an exporter to re-organize to a more specialized one and become more productive. Therefore, a reduction in trade costs affects aggregate productivity through a new margin, other than the intensive and extensive margin of trade; I call it "within-firm margin".

2.5 General Equilibrium Results

In general equilibrium, all the goods and labor markets clear. Trade is balanced between the two countries; hence, aggregate income equals aggregate expenditure. Entry is free; therefore aggregate profit equals zero. Thus, the measure of potential firms $M_e$, aggregate expenditure $X$ and wages become endogenous.

Without loss of generality, I assume that all the fixed specialization costs, operational
costs, and export and entry costs are paid in terms of high-skilled labor; therefore \( f_e = w_h \bar{f}_e \), \( f_o = w_h \bar{f}_o \) and \( f_x = w_h \bar{f}_x \). Relaxing these assumptions does not change the results qualitatively, as is shown in the Appendix. I take low-skilled labor to be the numéraire; hence \( w_L = 1 \).

**Assumption:** Specialization costs are low enough so that \( \frac{e-1}{\sigma-1} \bar{f} \Omega^{\sigma-1} \leq \tilde{f}_o \).

If the above assumption holds, the equilibrium conditions can be simplified as:

\[
\frac{H}{L} = \Omega^{-1} \left( m \zeta \left( \frac{\bar{f}_o}{\bar{f}} \bar{\Omega}^{1-\rho} - 1 \right) (1 + O) - 1 \right) \quad \text{(relative labor demand and trade balance)} \tag{18}
\]

\[
\bar{f}_e M_e = \frac{L \Omega^{-1} + H}{m \theta} \quad \text{(aggregate zero profit)} \tag{19}
\]

\[
L = (\rho - 1) M_e \bar{f}_e \Omega^\rho \quad \text{(low-skilled labor demand market clearing)} \tag{20}
\]

The skill premium \( \Omega \) can be solved by using the only equation (18). Measure of entrants would be solved from (19). Fraction of active producers \( (\mu_o) \) can be solved from (20)\(^{21}\) and therefore aggregate expenditure would be easily calculated from (11). Proposition 4 presents conditions for the unique equilibrium and its results.

**Proposition 4** If \( \frac{H}{L} > v^* \), then

(a) There is a unique equilibrium.

(b) The skill premium increases as trade costs (fixed or variable) decreases.

(c) Openness \( O \) increases as trade costs (fixed or variable) decreases.

The parameter \( v^* \) is defined as: \( v^* = \frac{\sigma \theta (d-\eta-1) \hat{\theta} \left( \frac{e-\theta}{\rho+1} \bar{f}_x \right)^{\hat{\eta}-1} + \theta + (\sigma-1) \eta}{(\sigma-1)(\theta-\eta) \left( \frac{e-\theta}{\rho+1} \bar{f}_x \right)^{\hat{\eta}-1}}. \)

**Proof.** (a) The skill premium \( \Omega \) can be determined using equation (18). The right-hand side of this equation is decreasing in trade costs since the openness parameter \( O \) is decreasing

\(^{21}\)Fraction of exporters can also be solved as \( \mu_x = \mu_o \bar{O} \frac{L_e}{f_e} \).
in both of the variable and fixed trade costs. This ensures that reducing trade costs shifts the relative labor demand curve up resulting in an increase in the skill premium. Finally, the condition above insures that $\frac{\theta-1}{\theta-1} \bar{f} \Omega^{\theta-1} \leq \bar{f} o$.

(b) The right-hand side of (18) is decreasing in the trade costs. Therefore reducing these costs shifts up the demand curve; increasing the skill premium.

(c) Multiplying both sides of (18) by $\Omega$ and rewriting $\Omega$ in terms of $O$ leads to a downward sloping function of $O$ on the left-hand side and an increasing function in terms of $O$ and trade costs on the right-hand side. Therefore reducing trade costs, decreases the right-hand side shifting down the upward sloping function of the right-hand side, and $O$ increases. This completes the proof. ■

Reductions in variable or fixed exporting costs would increase the aggregate relative labor demand; but since the relative labor supply is fixed, the skill premium rises, as illustrated in figure 5. It is important to note that the argument above is true when the condition in the proposition holds. This condition is violated when trade costs are very low.

Equation (19) will pin down the measure of a potential firm such that $M_e = \frac{\Omega^{\theta-1} + H}{n \theta \bar{f}_e}$. Because a reduction in trade costs will raise $\Omega$, $M_e$ declines. Moreover, equation (20) will pin down the measure of actual producers as $M_o \mu_o = \frac{\Omega^{\theta-1}}{(\rho-1)\bar{f}_o}$ which will also reduce along with a reduction in trade costs, as expected to happen as a result of any Melitz-type model, stating that the market becomes less crowded following the trade liberalization. Moreover, equation (15) reduces to $\mu_x = \frac{O^{\theta \bar{f}_e}}{1+O^{\theta-1}}$. Since openness $O$ increases with a reduction in trade costs
in the equilibrium, the fraction of exporters would increase. Finally, export intensity can be calculated as \( \frac{X_{ij}}{X} = \frac{1-{d}^{\rho}}{1-{d}^{\rho-1}} \frac{O}{O+1} \) which can be easily shown to be decreasing in trade costs, as expected.

As discussed above, this within-firm margin is a new source for generating an endogenous skilled bias technological change and a new source of gains in aggregate productivity and welfare. Also along with the intensive and extensive margins of trade, it is a new margin in the gravity equation where it allows firms to expand their organization and become more productive. In figure 6, responses of the model to trade costs for different values of \( \rho \) have been shown. What this model predicts is that the parameter \( \rho \), which is a notion of gains from labor specialization (lower \( \rho \), higher gain from specialization) also affects the trade elasticity as \( \theta \) and \( \sigma \) do. Therefore, this parameter also becomes a crucial determinant of gains from trade, just like \( \theta \) and \( \sigma \).

The predictions of the model regarding the reallocations of labor within industries are also consistent with the empirical works. Many empirical works cited above have shown that more-productive firms and exporters are more skill-intensive and they have increased their skill-intensity after trade openness. Also, it is a robust feature of the data that the skill premium has increased after trade liberalizations\(^{22}\). In the next section, I quantify the model by calibrating the model to the US data and show that the model can generate a large

\(^{22}\)See Goldberg and Pavnick (2007) for a survey on the literature.
increase in the skill premium with small reductions in trade costs.

3 Quantitative Analysis: Calibration to U.S. Economy

In this section, I show numerically how the model behaves in the general equilibrium by calibrating the model to US economy. I run some comparative statics and counterfactual analysis to show how the economy would change in different scenarios.

To calibrate the model, first I normalize the productivity parameter $\bar{A}$ and the fixed cost of operation $\bar{f}_o$ to one. These two variables can only change the definition of number of produced goods and the measure of firms which could be normalized to anything. Then I calibrate the other parameters using the existing related literature. I use Acemoglu (2010) and set $\frac{H}{L}$ to 1.28 from the 2003 data. In our analysis, only the ratio of high skilled to low skilled labor matters and thus I use the ratio $\frac{H}{L}$. I use Chaney 2005 and set $\sigma = 3.9$ which is close to 3.79 as in BEJK (2003). I also take the relative skill intensity equal to 1 which is in the range that Burstein & Vogel (2011) have shown. Then, I calibrate $d_{ij} = 1.3$ as in Ghironi & Melitz (2006). Chaney (2005) estimates the distribution of firms’ sales which is Pareto by parameter $\sigma - 1$ and show that $\frac{\theta}{\sigma - 1} = 1.89$. In our model, distribution of firms’ sales is Pareto with parameter $\eta$ and therefore we use his estimate to take $\frac{\theta}{\eta} = 1.89$. I take $\rho = 5.5$ which results in $\theta = 4.69$ very close to that of most of heterogenous firm models of trade estimations.

For the remaining parameters $\bar{f}_x, \bar{f}_e$ I need to match the parameters with the model. I take $\Omega = 1.91$ from Acemoglu (2010) for the year 2003. Also I take the fraction of exporters $\frac{M_x}{M} = 21\%$ as in BEJK (2003). Finally I take the firms’ death rate $\delta = 10\%$. Then I match these three variables using the model as below:
Table 2: Calibration to US data summary

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum productivity</td>
<td>$A$</td>
<td>1 Normalized</td>
</tr>
<tr>
<td>Fixed Operational Cost</td>
<td>$f_x$</td>
<td>1 Normalized</td>
</tr>
<tr>
<td>College/High school</td>
<td>$H/L$</td>
<td>1.28 In 2003 from Acemoglu 2010</td>
</tr>
<tr>
<td>Variable Trade cost</td>
<td>$d_x$</td>
<td>1.3 Ghironi &amp; Melitz 2005</td>
</tr>
<tr>
<td>Ex-post Firm Sale Het.</td>
<td>$\frac{e}{\eta}$</td>
<td>1.89 Chaney 2005 (EKK = 2.46)</td>
</tr>
<tr>
<td>Goods Subs. Elasticity</td>
<td>$\sigma$</td>
<td>3.9 Chaney 2005 (BEJK= 3.79)</td>
</tr>
<tr>
<td>Skill Intensity</td>
<td>$\beta_h$</td>
<td>0.5 Burstein Vogel 2010 (0.1 to 0.6)</td>
</tr>
<tr>
<td>Labor Subs. Elasticity</td>
<td>$\rho$</td>
<td>5.5 ( \Rightarrow \theta = 4.69 )</td>
</tr>
<tr>
<td>Skill Premium</td>
<td>$\Omega$</td>
<td>1.91 In 2003 from Acemoglu 2010</td>
</tr>
<tr>
<td>Fraction of Exporters</td>
<td>$\frac{M_x}{M_o}$</td>
<td>21% BEJK 2003</td>
</tr>
<tr>
<td>Firms Death rate</td>
<td>$1 - \frac{M_x}{M_e}$</td>
<td>10% Ghironi Melitz 2005</td>
</tr>
</tbody>
</table>

Table 3: Results of matching model moments to US data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Matched Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Exporting Cost</td>
<td>$f_x$ 1.0659</td>
</tr>
<tr>
<td>Fixed Specialization Cost</td>
<td>$\bar{f}$ 0.0435</td>
</tr>
<tr>
<td>Fixed Entry Cost</td>
<td>$\bar{f}_e$ 0.4945</td>
</tr>
</tbody>
</table>

Estimated Parameters from matching moments to US data

\[
1 + \Omega H/L = \frac{\sigma \theta}{\rho - \sigma \theta - \eta} \frac{\bar{f}_a}{\bar{f}_o - \bar{f}_a} \left( 1 + \left( \bar{d}^{\eta - 1} - 1 \right) \frac{\bar{f}_a}{\bar{f}_x} \right) \left( \frac{\bar{f}_a}{\bar{f}_x} \right)^{\frac{\theta}{\eta} - 1} \\
\frac{M_x}{M} = \left( \bar{d}^{\eta - 1} - 1 \right) \frac{\bar{f}_a}{\bar{f}_x} \left( \frac{\bar{f}_a}{\bar{f}_x} \right)^{\frac{\theta}{\eta}} \\
\delta = 1 - \frac{M_o}{M_e} = 1 - \frac{\theta - \eta}{\eta} \frac{\bar{f}_e}{\bar{f}_a + \frac{M_x}{M} \bar{f}_x} 
\]

Table 2 shows the summary of the calibration and estimations from matching the parameters:
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic Entry cutoff</td>
<td>1.007</td>
</tr>
<tr>
<td>High-skill Employer Entry cutoff</td>
<td>1.085</td>
</tr>
<tr>
<td>Foreign Entry cutoff</td>
<td>1.114</td>
</tr>
<tr>
<td>High-skilled Real Wages</td>
<td>1.096</td>
</tr>
<tr>
<td>Low-skilled Real Wages</td>
<td>0.574</td>
</tr>
<tr>
<td>Fraction of only Low-skilled Firms</td>
<td>0.682</td>
</tr>
</tbody>
</table>

Figure 7: Impact of reduction in trade cost in US on the skill premium, real wages and welfare

3.1 Bilateral Variable Trade Cost Reduction

In this sub-section, I quantify the model under the upper boundary case with two symmetric economies, by using the calibrated model and measure the effects of changes in important variables of interests. I vary the variable trade cost and show that how much the reduction in trade costs can decrease the skill premium. I find that by reducing the variable trade costs, both type of workers’ real wages increases but this change is so meaningful for the high-skilled workers. Also, as it is shown in Figure 7, by 20% rise in trade cost, the skill premium decreases by 6% and the aggregate welfare drops by %5.

Figure 8 illustrates the equilibrium distribution of high-skilled workers in the industry and degree of specialization of firms from 10% reduction in trade costs from \(d = 1.4\) to \(d = 1.3\). As it was expected, reducing trade costs shrinks the measure of entrants and increases the entry threshold. Also, the new exporter would expand their organization and increase their
Figure 8: Impact of reduction in trade cost in US on the distribution of specialization and allocation of high skilled workers

degree of specialization. Finally, aggregate demand will also decline since aggregate prices drop in the equilibrium. Therefore, reduction in trade costs results in lower demand for domestic producers which is also called import competition. Finally, due to the drop in the demand, old exporters also lower their specialization a little bit since now they have more competitors and less demand. The conclusion is that the reduction in trade costs induces a reallocation of labor from domestic producers and old exporters toward the new exporters which are now highly specialized.

3.2 Specialization Fixed Cost Reduction

In this sub-section, I analyze numerically the effect of reducing the specialization costs of high-skilled workers using the calibrated model as before. Obviously, decreasing the fixed cost of specialization induces firms to specialize high-skilled workers more and therefore the demand for the high-skilled workers goes up. As we can see from the Figure 9, a 20% decrease in this cost can raises the wage premium by 5%.
Figure 9: Impact of reduction in specialization cost in US

4 Conclusion

In this paper, I introduce a new model of skill specialization that can explain several stylized facts about distributional effects and the labor market effects of international trade. The most important one is that it proposes a new mechanism in explaining the increase in the skill premium in developing and developed countries after trade liberalization. By modeling the internal firm organization, I introduce a channel through which trade affects the skill premium through firms’ organizational decisions about their labor divisions and degrees of specialization of their skilled workers.

By introducing a model where a firm can specialize its workers into different divisions of labor and then optimize the degree of specialization, I found that the more-productive firms choose to specialize more and to demand relatively more skilled labor. Also, I show that for exporters, there’s a jump in the degree of specialization, relative labor demand, and level of production and sales. An increase in the product demand will also result in more specialization and will induce a reduction in the marginal cost of production.

After a productivity or demand shock, more skilled workers reallocate to more productive firms. Therefore opening up to trade will initially induce more productive firms to enter the foreign markets and expand their degrees of specializations and their demands for high-skilled labor. This would generate an increase in the relative demand for high-skilled workers, which
will result in an increase in the skill premium.

I could also find that an unbiased change in a firm’s productivity results in changes in the average degree of specialization, and therefore biased changes in relative demand for skilled workers and consequently biased labor productivity changes. This skill-biased technological change will induce also an increase in the skill premium.

Finally, I calibrate the model to US data and numerically analyze the model’s performance in explaining the changes in the skill premiums. I show that a 20% rise in the variable trade costs can reduce the skill premium by 6% and a 5% welfare losses in the US economy.

References


Appendix A - "For Online Publication" Canonical Model

Proof of lemma 2:

For revenue $R$:

$$R(A, D) = m^{\frac{\rho(\sigma-1)}{\rho-\sigma}} \left( D^{\frac{1}{\rho-1}} A \right)^{\gamma} Q(A, D) \frac{1-\sigma}{\rho-\sigma}$$

For the cost of low-skilled labor, we had $C_L(A, D) = (\rho - 1) \bar{f} \Omega^{\rho-1}$ which is independent of $A$ and $D$. For the high-skilled workers, I use the following simple relationship which can also be found from definition of $Q$ as well:

$$R(A, D) = m (C_H(A, D) + C_L(A, D))$$
thus

\[ C_H (A, D) = \frac{R (A, D)}{m} - C_L (A, D) \]
\[ = \frac{\bar{g}}{m} D^{\frac{\rho-1}{\rho-\sigma}} A^{\frac{\rho-1}{\rho-\sigma}} (\sigma-1) - (\rho - 1) f_H \Omega^{\rho-1} \]

also, as I showed in the previous lemma, we have \( \bar{f} S = \frac{w_N N_H}{\rho-1} \). Hence

\[ S (A, D) = \frac{\bar{g}}{m (\rho - 1)} D^{\frac{\rho-1}{\rho-\sigma}} A^{\frac{\rho-1}{\rho-\sigma}} (\sigma-1) - \Omega^{\rho-1} \]

Finally

\[ \Pi (A, D) = R (A, D) - (C_H (A, D) + C_L (A, D) + f_H S_H (A, D)) \]
\[ = \frac{R (A, D)}{\sigma} - f_H S_H (A, D) \]
\[ = \frac{R (A, D)}{\sigma} - C_H (A, D) \]
\[ = \frac{R (A, D)}{\sigma} - \frac{R(A,D)}{m} - C_L (A, D) \]
\[ = \frac{\rho - \sigma R (A)}{\rho - 1} + \bar{f} \Omega^{\rho-1} \]

This completes the proof. Note that this is the variable profit. The fixed operational fixed costs, fixed export costs and fixed sunk costs would be subtracted in the next steps.

**Lemma 5** (a) the firm in \( i \) produces domestically if:

\[ A \geq \bar{A}_o = Z_1 D^{-\frac{1}{\sigma-1}} \quad (21) \]

and the firm in \( i \) exports to \( j \) if

\[ A_i \geq \bar{A}_x = Z_1 D^{-\frac{1}{\sigma-1}} \left( \frac{f_x}{f_a} \right)^{\frac{1}{\eta}} \left( \left( 1 + \frac{D_f}{D} \right)^{\frac{\rho-1}{\rho-\sigma}} - 1 \right)^{-\frac{1}{\eta}} \]

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where \( f_a = f_{oi} - \frac{\beta_i}{\mu_i} w_{hi} \tilde{f}_i \Omega_i^{\rho - 1} \), \( Z_1 = Z_1 = \mu^\eta \eta^\frac{1}{\eta} w_H \tilde{f}_i \Omega_i^{\rho - 1} f_a^\frac{1}{\eta} \).

(b) In the two symmetric countries model, \( \tilde{A}_x \) simplifies to

\[
A_i \geq \tilde{A}_x = Z_1 D^{-\frac{1}{\sigma - 1}} \left( \frac{\tilde{f}_x}{f_a} \right)^\frac{1}{\eta} \left( d^{\eta} - 1 \right)^{-\frac{1}{\sigma - 1}}
\]

**Proof.** (a) A firm operates in the domestic market if \( \frac{\rho - \sigma}{\rho - 1} \frac{\rho - 1}{\sigma} A_1 + w_H \tilde{f} \Omega^{\rho - 1} - f_a \geq 0 \); thus

\[
A \geq \left( \frac{\rho - 1}{\rho - \sigma} \frac{f_a - w_H \tilde{f} \Omega^{\rho - 1}}{D^{\rho - 1} D^{\rho - 1}} \right)^{\frac{1}{\eta}}
\]

\[
= \left( \frac{\rho - 1}{\rho - \sigma} \frac{f_a}{D} \right)^{\frac{1}{\eta}} D^{-\frac{1}{\sigma - 1}} \left( \left( 1 + \frac{D}{f_a} \right)^{\frac{\rho - 1}{\rho - \sigma}} - 1 \right)^{-\frac{1}{\sigma - 1}}
\]

and a firm export if \( \frac{\rho - \sigma}{\rho - 1} \frac{\rho - 1}{\sigma} A_1 + w_H \tilde{f} \Omega^{\rho - 1} - f_a \geq 0 \); thus

\[
A \geq \left( \frac{\rho - 1}{\rho - \sigma} \frac{f_a}{D} \right)^{\frac{1}{\eta}} D^{-\frac{1}{\sigma - 1}} \left( \left( 1 + \frac{D}{f_a} \right)^{\frac{\rho - 1}{\rho - \sigma}} - 1 \right)^{-\frac{1}{\sigma - 1}}
\]

(b) \( \tilde{A}_x = \left( \frac{\rho - 1}{\rho - \sigma} \frac{f_a}{D} \right)^{\frac{1}{\eta}} D^{-\frac{1}{\sigma - 1}} \left( \left( 1 + \frac{D}{f_a} \right)^{\frac{\rho - 1}{\rho - \sigma}} - 1 \right)^{-\frac{1}{\sigma - 1}}
\]

\[
\left( \left( 1 + \frac{D}{f_a} \right)^{\frac{\rho - 1}{\rho - \sigma}} - 1 \right)^{-\frac{1}{\sigma - 1}} = \left( 1 + d^{\frac{1 - \sigma}{\rho - 1}} \right)^{\frac{\rho - 1}{\rho - \sigma}} - 1)^{-\frac{1}{\sigma - 1}}
\]

\[
= \left( 1 + d^{\frac{1 - \sigma}{\rho - 1}} - \frac{\rho - 1}{\rho - \sigma} \right)^{-\frac{1}{\sigma - 1}}
\]

\[
= \left( \tilde{d}^{\eta} - 1 \right)^{-\frac{1}{\eta}}
\]

This completes the prove. \(\blacksquare\)
Proof of Proposition 3:

In the Upper Boundary Case, the fraction of producers out of total potential producers is the fraction of firms which can pay the operational fixed cost and enter the domestic market. Also, productivities are Pareto distributed, thus \( \mu_o = \left( \frac{A_o}{A} \right)^{-\theta} \) and the fraction of exporters are \( \mu_x = \left( \frac{A_x}{A} \right)^{-\theta} \).

Revenue for a domestic producer is \( R_d (A) = \bar{g} D^{\frac{\rho - 1}{\rho - \sigma}} A^\eta \) and revenue for an exporter is

\[
R_x (A) = \bar{g} (D + D_f)^{\frac{\rho - 1}{\rho - \sigma}} A^\eta \\
= R_d (A) + \bar{g} \left( (D + D_f)^{\frac{\rho - 1}{\rho - \sigma}} - D^{\frac{\rho - 1}{\rho - \sigma}} \right) A^\eta
\]

thus the aggregate revenue is

\[
R = M_e \left( \int_{\tilde{A}_o}^{\hat{A}_x} R_d (A) \, dF (A) + \int_{\tilde{A}_o}^{\hat{A}_x} R_x (A) \, dF (A) \right)
= M_e \left( \int_{\tilde{A}_o}^{\hat{A}_x} R_d (A) \, dF (A) + \int_{\tilde{A}_o}^{\hat{A}_x} \left( \bar{g} \left( (D + D_f)^{\frac{\rho - 1}{\rho - \sigma}} - D^{\frac{\rho - 1}{\rho - \sigma}} \right) A^\eta \right) \, dF (A) \right)
= \frac{n \theta \eta}{\theta - \eta} (\mu_o f_a + \mu_x f_x) M_e
\]

On the other hand since the low-skilled labor demand is independent of \( A \) and \( D \) and is equal to \( ((\rho - 1) w_H \bar{f}) \Omega^{\rho - 1} \), the aggregate Demand for labor of type \( L \) is

\[
C_L = ((\rho - 1) w_H \bar{f}) \Omega^{\rho - 1} \mu_o M_e
\]

Therefore I can solve for the aggregate demand for the high-skilled using the notion that

\[
C_H = \frac{R}{m} - C_L
\]
So I find that

\[ \frac{C_H}{C_L} = \frac{R}{mC_L} - 1 \]

\[ = \frac{\theta \eta}{\theta - \eta} \left( \mu_o f_a + \mu_x f_x \right) M_e \frac{1}{((\rho - 1) f_H) \Omega^{\rho - 1} \mu_o M_e} - 1 \]

\[ = \frac{\theta \eta}{\theta - \eta} \frac{f_a}{(\rho - 1) w_H f_H \Omega^{\rho - 1}} \left( 1 + \frac{\mu_x f_x}{\mu_o f_a} \right) - 1 \]

\[ = \frac{\theta \eta}{\theta - \eta} \frac{f_a}{(\rho - 1) f_o - f_a} \left( 1 + \left( \frac{f_x}{f_a} \right)^{\theta \eta} \left( \tilde{d}^{-\eta} - 1 \right)^{\eta} \right) - 1 \]

\[ = \xi \frac{f_a}{f_o - f_a} (1 + O) - 1 \]

where \( \xi = \frac{\theta \eta}{(\theta - \eta)(\rho - 1)} \) and \( O = \left( \frac{f_x}{f_a} \right)^{\theta \eta} \left( \tilde{d}^{-\eta} - 1 \right)^{\eta} \). This completes the proof.

**Proof of Proposition 4:**

(a) In the general equilibrium with balanced trade, aggregate income equals aggregate demand. Also free entry ensures that aggregate profit is zero. Thus aggregate income equals \( w_L L + w_H H \), and also aggregate revenue equals aggregate income. Thus,

\[ w_L L + w_H H = R = \frac{m \theta \eta}{\theta - \eta} \left( \mu_o f_a + \mu_x f_x \right) M_e \quad (22) \]

In equilibrium all the markets clear. Since all the fixed costs are being paid in terms of high-skilled workers, aggregate low-skilled labor demand equals aggregate labor supply; thus,

\[ w_L L = C_L = ((\rho - 1) w_H f_H) \Omega^{\rho - 1} \mu_o M_e \quad (23) \]

Dividing (22) and (23) results in

\[ 1 + \Omega \frac{H}{L} = m \xi \frac{f_a}{f_o - f_a} (1 + O) \quad (24) \]

LHS is an increasing function of the skill premium \( \Omega \) which changes from 1 to infinity.
RHS is a decreasing function of $\Omega$ since both $f_a$ and $O$ are decreasing in $\Omega$. If the assumption in the proposition hold, then RHS goes from infinity to zero. Hence, there exists an equilibrium intersection which solves for $\Omega$. In order to see how this assumption hold, we can rewrite the above equation in terms of $f_a$ and thus RHS would be a function of $f_a$ such that $f_a \in (0, f_o)$. LHS would be a decreasing function of $f_a$. In order that we have all the firms employing high-skilled workers we should have $C_H (\tilde{A}_o) > 0$. By imposing this condition, then the inequality $\frac{1+\nu_1 \tilde{d}_0 f_a}{1+\nu_1 \tilde{d}_0} \left( \tilde{f}_0 f_a \right)^{\frac{\theta}{\sigma}} \left( \tilde{f}_0 \right)^{\frac{1}{\sigma}} < \frac{(\sigma-1)(\theta-\eta)}{\sigma \theta}$ ensures a solution to the equation.

(b) RHS of (24) is decreasing in both of the trade costs $d$ and $\tilde{f}_x$. Thus the equilibrium $\Omega$ is decreasing in the trade costs.

(c) Rewriting (24) in terms of $O$ ensures that reducing trade costs results would results in an increase in the equilibrium $O$.

Proposition 6 In general equilibrium, by reducing trade costs, the skill premium initially rises and then it falls.

Proof. It has been shown numerically using the algorithm mentioned in the paper.

A.1 Relaxing Assumptions

Suppose the fixed costs are in terms of Final good: In this case we have: $f_i = P_i \tilde{f}_i$, $f_{oii} = P_i \tilde{f}_{oii}$, $f_{oij} = P_i \tilde{f}_{ij}$ (or $P_j \tilde{f}_{ij}$), $f_{ei} = P_i \tilde{f}_{ei}$. Equilibrium condition for low-skilled labor doesn’t change. For the revenue equation, we have

\[ \text{Total Revenue} - \text{Fixed costs} = \text{Total Household Income} \]

Thus we have the following Equilibrium equations ($f_H = w_H \tilde{f}$):

\[ w_L L = M_o (\rho - 1) f_H \Omega^{\rho-1} \]
\[ w_L L + w_H H = \frac{\theta \eta}{\theta - \eta} (M_o f_o + M_x f_x) \]

\[
P = \left( \frac{(\rho - 1) \sigma}{\rho - \sigma} \right) \left( \frac{M_o f_o}{D} + \frac{M_x f_{oij}}{D + D_x} \frac{1 - (1 - z_x)_{\rho-1}}{z_x} + \frac{M_a d^{1-\sigma} f_x}{D + D_x} \right)^{\frac{1}{1-\sigma}}
\]

with definitions:

\[ f_H = P f_H, f_o = P f_o, f_x = P f_x \text{ (or } P f_x), f_e = P f_e. \] \hspace{1cm} (25)

\[ X = w_L L + w_H H + \Pi \]

\[
\begin{cases} 
\text{No Free Entry} & \quad \Pi = \frac{w_L L + w_H H}{\theta} - M_e f_e \\
\text{Free Entry} & \quad M_e = \frac{1}{\theta} X \\
\end{cases}
\]

\[ D_{ik} = a^{1-\sigma} P_{k}^{\sigma-1} X_{k} \]

\[ M_o = a^{\theta} \left( \frac{\rho - \sigma}{\sigma (\rho - 1)} \frac{P^{\frac{1-\sigma}{\rho-\sigma}} D^{\frac{\rho-1}{\rho-\sigma}}}{f_o - f_H \Omega^{\rho-1}} \right) \left( \frac{\theta}{\eta} \right) M_e \]

\[ M_x = a^{\theta} \left( \frac{\rho - \sigma}{\sigma (\rho - 1)} \frac{F^{\frac{1-\sigma}{\rho-\sigma}} (D + D_x)^{\frac{\rho-1}{\rho-\sigma}} - D_x^{\frac{\rho-1}{\rho-\sigma}}}{f_x} \right) \left( \frac{\theta}{\eta} \right) M_e \]

Here the equations are of the same form as the original assumption except with a change in parameter in the revenue equation. Therefore by division of the first two, we get the following relative labor demand:
$1 + \Omega H/L = \bar{r} \frac{\bar{f}_a}{f_o - f_a} \left( 1 + O \right)$

where

$O = \left( \frac{z_x}{1 - z_x} \right)^{\frac{\theta}{\eta}} \left( \frac{\bar{f}_a}{f_x} \right)^{\frac{\theta}{\eta} - 1}$

$\bar{r} = \frac{\theta \eta}{(\theta - \eta)(\rho - 1)}$