Evaluating Trade Policies on Intermediate goods

Saber Ahmadi-Renani  Sepehr Ekbatani  Seyed Ali Madanizadeh
Sharif University of Technology  Sharif University of Technology  Sharif University of Technology

July 31, 2015

Abstract

In this paper, we introduce a general equilibrium model of international trade that takes into account the endogenous entry and exit of intermediate goods producers and the industry linkages to quantitatively measure the countries welfare gains and losses from the export and import subsidy policies as in China. Our model extends the inter-industry trade model Eaton and Kortum (2002), intra-industry model of Krugman (1980), and firm heterogeneous model of Melitz (2003). Subsidies on intermediate good imports or export subsidies of specific industries affect welfare of countries through direct effects on wages and cost of input bundles of production (labor and intermediate goods), and indirectly through changes in term of trades, and endogenous entries and exits in many industries due to the general equilibrium adjustment of the input-output linkages which are missed in the Eaton and Kortum (2002) environment. We show that the welfare gain from trade depends on the share of domestic goods in intermediate goods. Moreover, the lower elasticities of substitution of intermediate goods than consumption goods magnifies the welfare gain from trade.

**JEL classification:** F1, F6, F12, F13, F15, F17.

**Keywords:** Trade policy, Gain from trade, Intermediate goods, Input-output linkage, Import subsidies
1 Introduction

We propose a multi-country multi-industry general equilibrium model of international trade that takes into account use of intermediate goods for production and linkage between industries throughout the input-output loop. Our model features inter-industry trade as in Ricardo (1817), intra-industry trade as in Krugman (1980), firm heterogeneity as in Melitz (2003) in the input-output environment.

Our basic framework is similar to Costinot and Rodriguez-Clare (2013). The market structure is monopolistic competition same as in Melitz (2003). Firms in each industry enter as long as the expected profit is higher than the entry sunk cost. After entry, the firms pick their random productivity from a distribution. After realization of productivity only firms that their profit from serving a country is positive, will enter that market. Measure of intermediate good producers are distributed in each country and each sector. Final goods are produced from a CES aggregate production function of the intermediates goods that is available in a country and is called composite (final) good. These final goods is demanded by households for consumption and intermediate goods producers for production.

Although importance of foreign intermediate goods has been well stated theoretically and empirically, there isn’t a general framework that explicitly evaluate the welfare effects of trade policies like import subsidies on intermediate and explains how it affects different industry.

We show how the model can be easily used to evaluate trade policies that specifically target intermediate goods like subsidies to firms for importing intermediate goods. We divide each industry into two sectors: the sector that produce consumption goods that are demanded by consumers, and the sector that produce intermediate goods that are demanded by producers in both sections of an industry.

Our paper stands in the literature of quantitative trade models that estimate the welfare gains from trade. Similar to works of Alvarez and Lucas (2007), Caliendo and Parro (2012), Ossa (2015), Costinot and Rodriguez-Clare (2013), and Halpern et al. (2011) our model takes into account trade of intermediate goods.

Effects of use of foreign intermediate goods on productivity of domestic producers has been extensively documented empirically. Amiti and Konings (2007) show that elasticity of productivity of firms with respect to tariffs on inputs is 1.2 in Indonesia. Goldberg et al. (2010) found that a significant portion of producing new products in India was due to lower inputs tariffs.

In the quantitative trade models surveyed by Arkolakis et al. (2012), it is the share of expenditure in domestic good that determines welfare gain from trade. We show that shares of domestic goods in consumption goods and intermediate goods together determine the welfare gain from trade. However, what is more important in the amount of welfare gain is the share of domestic good in intermediate goods, because its effect magnifies with sectoral linkage.

Similar to Ossa (2015) we also emphasize that variation in trade elasticities between indus-
tries greatly increases the welfare gain from trade. As the elasticities of substitution is lower in intermediate goods than consumption goods, considering different consumption and intermediate goods can magnify the estimated welfare gain from trade.

To decompose the effects of changes in tariffs on welfare, we log-linearized the change in welfare around the factual. We show that change in tariffs affect welfare through four channels: first, it has direct effect on wage of a country, second, it affect the cost of input bundle of production (labor and intermediate goods), third, it changes the term of trade of country, forth, it causes entry and exit in industries.

If a country changes tariffs imposed to exporting intermediate goods to that country, how this affects different industries? Consider a reduction in tariffs of imported intermediate goods. Beside the direct reduction in price of imported intermediate goods, it lowers the cut-off productivity for export to the country. So, more firms start to exporting to the country. It also lower the average productivity of firms that export to the country. With the standard assumptions of trade models, the overall effect of change in tariffs of imported intermediate goods on price index of imported products is positive. Domestic firms in intermediate goods sector will lose some of their sell because of tougher competition, but other sectors that use these intermediate goods for production will gain from lower marginal cost of production. Hence, the cut-off productivity for production and exports to other countries become lower in those industries.

Our paper differs from similar works in different ways. First, we explain how changes in tariffs affect the price indeces and welfare in a country through four different channels. Second, we show how the model can be used to evaluate the welfare effects of trade polices on intermediate goods. Third, we demonstrate how considering the different between consumption and intermediate goods magnifies the welfare gain from trade. Forth, we describe how changes in tariffs on intermediate goods affect producers in different industries.

The remainder of paper is organized as follows. In Section 2, we develop the theoretical framework, characterize the equilibrium, and describe the channels that changes in tariffs affect welfare. Section 3 lays out how trade agreement affect producers in both countries. In section 4, we show how the model can be used to evaluate trade polices on intermediate goods. In section 5, we conclude.

2 Model

2.1 Households

There are N countries and S industries. In each country j there are $L_j$ representative households whom their preferences is defined by:

$$U_j = \prod_{s=1}^{S} C_{j,s}^{\mu_j s}.$$  \hspace{1cm} (1)
The budget constraint is $\sum_{s=1}^{S} P_{js} C_{js} = X_j$, where $X_j$ is country $j$ total expenditure. Because trade is balanced between countries and free entry exist in each industry, $X_j$ is equal to $w_j L_j$.

2.2 Market Structure

The market structure is monopolistic competition same as in Melitz (2003). Firms in each industry enter as long as the expected profit is higher than the entry sunk cost $w_i f_{e,i}$. After entry, the firms pick their random productivity from a distribution $g_i(\phi)$. After realization of $\phi$, firms produce for their domestic if their profit considering fixed cost of production ($f_{ijs}$) is positive. To export to country $j$, firms must pay the fixed cost of $f_{ijs}$ by hiring labor in country $j$ beside the variable cost of export. They decide to export to country $j$ if only their profit from serving that market is positive. We define $\phi_{ijs}$ as the cut-off of productivity of export to country $j$.

We assume that productivities of firms are derived from a Pareto distribution $G_i(\phi) = 1 - (\frac{b_i}{\phi})^{\theta_i}$. Where $b_i$ is the Pareto location parameter (higher $b_i$ means more productive industry) and $\theta_i$ is the Pareto shape parameter (higher $\theta_i$ means less variety in productivity of firms).

The ex-post distribution of firms would be:

$$g_{i}(\phi | \phi > \phi^*_{ijs}) = \frac{g_{i}(\phi)}{1 - G_i(\phi^*_{ijs})} = \frac{\theta_i}{\phi} (\frac{\phi^*_{ijs}}{\phi})^{\theta_i}$$ (2)

In each country and each sector, final good is produced from a CES aggregate production function over all intermediate goods available and is called composite (final) good:

$$Q_{js} = \left(\sum_{i=1}^{N} \int q_{ijs}(\phi) \frac{\phi^{\sigma_s-1}}{\sigma_s} M_{i} d\phi_{i} \right) \frac{\phi^{\sigma_s}}{\sigma_s-1}$$ (3)

where $Q_{js}$ is the final good produced from the intermediates goods $q_{ijs}$ that are available in country $j$, and $M_{i}$ is measure of intermediate good producers in each country and each sector. Part of these final goods are demanded by households for consumption and part of that by intermediate goods producers for production.

2.3 Production

For the production of Intermediate goods, firms use labor and the composite goods from all industries as inputs. Their technology is constant return to scale and their production function is given by:

$$q_{i}(\phi) = \phi \left( l_{i} (\phi) \right)^{\beta_{i}} \left( \prod_{k=1}^{S} (\tilde{Q}_{i,s,k} (\phi))^{\eta_{ik}} \right)^{1-\beta_{i}}$$ (4)

Where $\phi$ shows the productivity of firm, $\beta$ is share of labor and $\eta_{ik}(1 - \beta)$ is share of composite good of $k$ industry in production function.
The final good available in each country and each industry is demanded by consumers \((C_{is})\) and intermediate goods producers. So, markets in the intermediate goods clear as:

\[
Q_{is} = C_{is} + \sum_{r=1}^{s} \left[ M_{ir} \sum_{j=1}^{N} \Pr(\phi > \phi^*_{jir}) E(Q_{jir,s}(\phi) \mid \phi > \phi^*_{jir}) \right]
\]  

(5)

Where \(Q_{jir,s}(\phi)\) is the amount of final good demanded from industry \(s\) by a firm in industry \(r\) of country \(i\) with productivity of \(\phi\) for production of its export to country \(j\). Profit maximization of each producer of intermediate good yields the pricing rule:

\[
p_{ij}(\phi) = \left( \frac{\sigma_s}{\sigma_s - 1} \right) \left( \frac{d_{ijs} \tau_{ijs} \bar{c}_{is}}{\phi} \right)
\]  

(6)

Where \(d_{ijs}\) is iceberg cost defined as number of goods that must shipped from country \(i\) to \(j\) for one unit of intermediate good, \(\tau_{ijs}\) is tariff, and \(\bar{c}_{is}\) defined as

\[
\bar{c}_{is} = \left( \frac{w_{i} \beta_{is}}{\beta_{is}} \right)^{\frac{1}{\gamma}} \left( \prod_{k=1}^{S} \left( \frac{P_{ik} \eta_{sk}}{\eta_{sk}^{1-\beta_{is}}} \right) \right)^{1-\beta_{is}}.
\]  

(7)

Where \(\bar{c}_{is}\) is marginal cost of firm with productivity of \(\phi\). \(P_{ik}\) is aggregate price index of \(k\) industry in country \(i\) and is given by

\[
P_{js} = \left( \sum_{i=1}^{N} \int_{\phi_{ij,s}}^{\infty} M_{is}(d_{ijs} \tau_{ijs} p_{is}(\phi))^{1-\sigma} d\phi_{is} \right)^{\frac{1}{1-\sigma}}.
\]  

(8)

### 2.4 Equilibrium for given tariffs

By FOC of firm, the demand of firm for labor and intermediate goods for serving each market are:

\[
l_{ijs}(\phi) = \frac{\beta_{is} \bar{c}_{is} \bar{Q}_{ijs}}{\omega \phi} \tau_{ijs} d_{ijs} q_{ijs}(\phi)
\]  

(9)

\[
P_{js} \bar{Q}_{jir,s}(\phi) = (1 - \beta_{is}) \eta_{is} \frac{\bar{c}_{is}}{\phi} \tau_{ijs} d_{ijs} q_{ijs}(\phi).
\]  

(10)

The total expenditure of industry \(s\) in country \(j\) and equals to:

\[
R_{js} = P_{js} Q_{js} = P_{js} C_{js} + \sum_{r=1}^{S} \left[ M_{jr} \sum_{j=1}^{N} \Pr(\phi > \phi^*_{jir}) E(P_{js} \bar{Q}_{jir,s}(\phi) \mid \phi > \phi^*_{jir}) \right].
\]

Utility maximization yields that revenue of a firm with productivity of \(\phi\) in industry \(s\) of country \(i\) from serving market of country \(j\) is

\[
q_{ijs}(\phi) p_{ijs}(\phi) = \left( \frac{p_{ijs}(\phi)}{P_{js}} \right)^{1-\sigma} R_{js}.
\]  

(11)

The share of total expenditure spent on product of a firm is determined by its price advantage in the market it serves. The more is the elasticity of substitution in an industry, the more the firm lose its sell because of higher price.
Firms serving market of a country must gain positive profit by entering to that market. The productivity of marginal producer in industry $s$ that export from country $i$ to $j$ is given by:

$$\phi_{ij,s}^* = \left(\frac{\sigma_s \omega_s f_{ij,s}}{R_{js}}\right)^{\frac{1}{\rho_s}} \frac{\sigma_s}{\sigma_s - 1} \frac{d_{ij,s} \tau_{ij,s} c_{is}}{P_{js}}$$ (12)

Free entry causes the expected profit of firm to be equal to the entry sunk cost.

$$E(\pi_{js}(\phi)) = \sum_{j=1}^{N} [pr(\phi > \phi_{ij,s})E(\pi_{ij,s}(\phi) \mid \phi > \phi_{ij,s})] = \omega_i f_{e,js}$$ (13)

It can be shown that under Pareto distribution we have

$$E(r_{ij,s}(\phi) \mid \phi > \phi_{ij,s}) = \frac{\theta_s \sigma_s}{\sigma_s - 1} \omega_i f_{e,js}.$$ (14)

So the free entry condition becomes

$$\sum_{j=1}^{N} \left(\frac{b_{is}}{\phi_{ij,s}}\right)^{\theta_s} \frac{\sigma_s - 1}{1 + \theta_s - \sigma_s} \omega_i f_{ij,s} = \omega_i f_{e,js}.$$ (15)

Substituting the expenditure of consumers ($\mu_{is}\omega_i L_i$) and the expenditure of firms from (10) in (11) and using (14) and (15):

$$R_{is} = \omega_i \left[\mu_{is} L_i + \sum_{r=1}^{S} (1 - \beta_{ir}) \eta_{rs} \theta_r M^e_{is} f_{rj,s}\right]$$ (16)

Potential producers hire labors from their country to derive their productivity (sunked cost of production). Active firms must also hire labors from their country for production and from other countries they serve to pay fixed cost of export. Labor market in each country clears as

$$L_i = \sum_{s=1}^{S} \left[M^e_{is} f_{is}^e + \sum_{j=1}^{N} pr(\phi > \phi_{ij,s})E(l_{ij,s}(\phi) \mid \phi > \phi_{ij,s}) + \sum_{j=1}^{N} M_{js} f_{j,s}\right].$$

Substituting (9) and using (14), (15), and (16), the labor market clearing condition yields

$$\alpha_i L_i = \sum_{s=1}^{S} (1 + \theta_s)(1 + \beta_{is}(\sigma_s - 1)) M^e_{is} f_{is}$$ (17)

Where $\alpha_i = \sum_{s=1}^{S} \frac{(\theta_s + 1)(\sigma_s - 1)}{\sigma_s \rho_s} \mu_{is}$ is a constant. Imposing Pareto distribution in (8), the price index is determined by:

$$P_{is} = \gamma_s \left(\frac{\rho_s w_j}{R_{js}}\right)^{\frac{\rho_s - (\sigma_s - 1)}{\rho_s - 1}} \left[\sum_{s=1}^{N} \frac{b_{ij,s} \tau_{ij,s} c_{is}}{d_{ij,s} \tau_{ij,s} c_{is}} \right]^{\theta_s} \frac{M_{is}}{M^e_{is} f_{is}^{\sigma_s - 1}}$$ (18)

Where $\gamma_s = \left(\frac{\rho_s w_j}{R_{js}}\right)^{\frac{\rho_s - (\sigma_s - 1)}{\rho_s - 1}}$ is a constant. Using (18) and (14), we can derive the final free entry condition:

$$\frac{\sigma_s \theta_s}{\sigma_s - 1} f_{is}^e w_i = \sum_{j=1}^{N} \left(M^e_{m,s} f_{mjs}^{\sigma_s - 1} \frac{d_{mjs} \tau_{mjs} c_{ms}}{d_{mjs} \tau_{mjs} c_{ms}} \right) R_{js}$$ (19)

The equilibrium for given tariffs is $N w_i$ and $NS M^e_{is}$, $R_{is}$, $P_{is}$, and $c_{is}$ that satisfy system of $N$ equations of (17) and $NS$ equations of (19), (16), (18), and (7).
2.5 Equilibrium for given tariffs changes

Estimating sunk costs and fixed costs used in previous equilibrium aren’t easy empirically. However, for examining the effects of changes in tariffs we don’t need to know all the parameters. We solved the model in relative changes with the method inspired by Dekle et al. (2002). Defining $X = \frac{X}{X_r}$ for all variables, equations (17), (19), (16), (18), and (7) become:

\[
1 = \sum_{s=1}^{S} \Psi_{is} \hat{M}_{is}^c
\]

\[
\hat{w}_i = \sum_{j=1}^{N} \frac{a_{ij} \tau_{ij}s \hat{e}_{ij}s}{\sum_{m=1}^{N} a_{mjs} \hat{M}_{ms}^c (\hat{\tau}_{mjs} \hat{e}_{ms})^{-\theta_s} \hat{R}_{js}} R_{js}
\]

\[
\hat{R}_{is} = \hat{w}_i \left( \nu_{ij}s + \sum_{r=1}^{S} \kappa_{ij}s \hat{M}_{is}^c \right)^{-\beta_{is}}
\]

\[
\hat{e}_{ms} = \hat{w}_i^{\beta_{is}} \left( \prod_{k=1}^{S} \hat{P}_{ik}^{\eta_{sk}} \right)^{1-\beta_{is}}
\]

\[
\hat{P}_{ik}^{\eta_{sk}} = \left( \frac{\hat{w}_i}{R_{is}} \right)^{\frac{(\sigma_s-1)(1+\beta_s)}{(1+\beta_s)(\sigma_s-1))} \left( \sum_{j=1}^{N} a_{ij}s \hat{M}_{js}^c (\tau_{mjs} \hat{e}_{ms})^{-\theta_s} \right)^{-\frac{1}{\sigma_s}}
\]

Where $\Psi_{is} = \frac{\sum_{j=1}^{N} a_{ij}s \tau_{ij}s \hat{e}_{ij}s}{\sum_{r=1}^{S} \sum_{j=1}^{N} \kappa_{ij}s \tau_{ij}s \hat{e}_{ij}s (1+\beta_{is})(\sigma_s-1))\hat{R}_{js}}$, $a_{ij}s = \frac{T_{ij}s}{\sum_{j=1}^{N} T_{ij}s}$, $\kappa_{ij}s = \frac{T_{ij}s^{\eta_{sk}}}{\sum_{k=1}^{S} T_{ij}s}$, and $\nu_{ij}s = 1 - \sum_{r} \kappa_{ij}s$. The equilibrium for given tariffs changes is N $\hat{w}_i$ and NS $\hat{M}_{is}^c$, $\hat{R}_{is}$, $\hat{P}_{ik}$, and $\hat{e}_{ms}$ that satisfy system of N equations of (21) and NS equations of (21) to (24).

2.6 Welfare effects of changes in tariffs

We are interested to evaluate welfare change caused by general equilibrium adjustment of changes in tariffs. Change in welfare equals to change in nominal wage deflated by change in the ideal aggregate price index: $\hat{W}_j = \hat{w}_j / \hat{P}_j$. Since preferences of consumers are Cobb-Douglas across sectors, change in welfare can be written as $\hat{W}_j = \hat{w}_j / \prod_{s=1}^{S} (\hat{P}_{is})^{\eta_{sk}}$. To decompose the general equilibrium effects of change in tariffs, we used log-linear approximation around factual.

Change in price index of industry $s$ in country $j$ can be written as:

\[
\frac{\Delta P_{js}}{P_{js}} = \sum_{i=1}^{N} \frac{T_{ij}s}{\sum_{m=1}^{N} T_{mjs}} \left[ \frac{\Delta \tau_{ij}s}{\tau_{ij}s} \frac{\Delta e_{is}}{\hat{e}_{is}} + \frac{1}{\theta_s} \frac{\Delta M_{is}^c}{\hat{M}_{is}^c} (1 - \frac{1}{\theta_s - 1}) \left( \sum_{r=1}^{S} \frac{T_{ij}s}{\sum_{m=1}^{N} T_{mjs}} \frac{T_{ij}s}{T_{ij}s} \frac{\Delta M_{is}^c}{M_{is}^c} \right) \right]
\]

Change in price index of each industry is weighted average of changes in price indexes of imported product from each country proportioned to share of import from that country from total final good available. Change in import price index from each country obtained from changes in tariffs, changes in marginal cost of bundles of inputs of production ($\beta_{js} \Delta \frac{w_j}{w_j} + (1 - \beta_{js}) \sum_{k=1}^{S} \eta_{sk} \frac{\Delta P_{js}}{P_{js}}$), and entries to industries in that country.
To understand how changes in tariffs affect price of an industry, imagine a 1% reduction in tariff of an industry. It changes the marginal cost of production in different industries and induces entries in and out of industries. Taking out changing in marginal cost of production and entries to industries, a 1% reduction in tariff changes the price index of exported products by directly decreasing it 1% and by reducing the cut-off productivity of export 1%. The reduction in cut-off productivity of export also affect the price index of exported products from two different channels: change in entry to export market and change in average productivity of exporting firms. Since the cut-off productivity of export is now 1% lower, the entrants to export market increases \( \theta \) (\( M_{ij} = (\frac{\beta_s}{\sigma_s})M_{ij}^r \)). Also, 1% decrease in the cut-off productivity to export reduces the average productivity of exporting firms 1%.

Any change in entry, changes the price index of exported products by elasticity of \(-\frac{1}{\sigma_s - 1}\) and any change in average productivity changes the price index by elasticity of \(-1\). So the price index is reduced \( 1 + \frac{\theta}{\sigma_s - 1} \)%. But, this reduction in price index make competition in export market easier. So, it further reduces the cut-off productivity to export. The overall elasticity that can be calculated from a geometric series is \( \frac{2\theta}{\sigma_s - 1} \). Since it is assumed that \( \theta > \sigma_s - 1 \), this elasticity is always positive and lower than one. It means that if the cut-off productivity of export decreases, the price index of exporting products will be lower.

Therefore, the overall elasticity of change in tariff (beside the direct effects of changes in marginal cost of production and entries) which is sum the initial effect on price index and the effect on price index by changing the cut-off productivity is \( 1^2 \).

As change in marginal cost of production, changes the price index and cut-off productivity for export both 1%, the effect of induced changes in marginal cost of production is same as change in tariffs.

Now consider the channels that change in entry of each industry and other industries affect price index the industry. Change in entry to an industry changes entry to export market and so changes the price index of target market. Hence, the overall effect is \(-\frac{1}{\sigma_s - 1}\) \( \cdot \frac{\sigma_s - 1}{\sigma_s} = -\frac{1}{\sigma_s} \). Moreover, changes in entries to other industries affect the price index throughout the changes in demand that one industry faces for it’s products. The elasticity of change in cut-off productivity of export with respect to change in demand is \( \frac{\sigma_s - 1}{\sigma_s} \). Change in demand affect the price index throughout the change in cut-off productivity. As it is stated before, a reduction in cut-off productivity reduces the price index by changing the entry and the average productivity of firms that export. The overall effect is \( \frac{\sigma_s - 1}{\sigma_s} \cdot (-1 + \frac{\theta_s}{\sigma_s - 1}) \cdot \frac{\sigma_s - 1}{\sigma_s} = \frac{\sigma_s - 1}{\sigma_s} \).

As \( \hat{W}_j = \hat{w}_j / \prod_{s=1}^{S} (\hat{P}_{js})^{\hat{u}_{js}} \), the log-linear approximation around factual yields:

\[
1 - (1 + \frac{\theta_s}{\sigma_s - 1}) - (1 + \frac{\theta_s}{\sigma_s - 1})^2 - \cdots = \frac{2\theta_s}{\sigma_s - 1}
\]

\[
2(1 - \frac{\theta_s}{\sigma_s - 1})^2 = 1
\]
\[
\frac{\Delta W_j}{W_j} = \sum_{i=1}^{N} \sum_{s=1}^{S} \frac{\mu_{js}}{S_{m=1}^{N} T_{mjs}} \left[ \frac{\Delta w_j}{w_j} - \frac{\Delta \bar{c}_{js}}{c_{js}} - \frac{\Delta \bar{c}_{is}}{c_{is}} \right] + \frac{1}{\theta_s} \frac{\Delta M^i_{js}}{M^i_{js}} + \frac{1}{\sigma_s - 1} \frac{1}{\theta_s} \left( \sum_{r=1}^{S} \sum_{m=1}^{N} T_{rjs} \frac{T_{mjs}}{M^r_{js}} \right) \sum_{k=1}^{S} \eta_{sk} \Delta P_{jk} / P_{jk}
\]

Direct Income effect  Direct Cost of production effect  Term of trade effect  Home market effect

The first term is direct effect of change in income of labor has on welfare. The second term is direct effect of changes in cost of production. In models without intermediate goods for production, these two effects cancel out.

The third term is the traditional terms of trade effect and equals to \(-\Delta \tau_{ijs} / \tau_{ijs} + (\beta_{js} \Delta w_j / w_j - \beta_{is} \Delta w_i / w_i) + (1 - \beta_{js}) \sum_{k=1}^{S} \eta_{sk} \Delta P_{jk} / P_{jk} - (1 - \beta_{is}) \sum_{k=1}^{S} \eta_{sk} \Delta P_{ik} / P_{ik}\). It captures the direct effect changes in cost of inputs of production (wages and price indexes) have on the prices of the goods exported by country \(j\) relative to the direct effect changes in tariffs and cost of inputs of production have on the prices of the goods imported by country \(j\). Higher term of trade would be beneficial for a country, since it consumes cheaper imports and sales more expensive exports. In the models without intermediate goods only tariffs and wages determine term of trade, so the effect of changes in wages on term of trade effect would be overestimated.

The forth term is is the new trade home market effect emphasized by Venables (1987). It captures the indirect effect adjustments in entry and exit have on the aggregate price index in country \(j\). Changes in entry of an industry and changes in entry of other industries affect price index from two different channel.

### 2.7 Welfare gain of international trade

In the same way as Arkolakis et al. (2012), we will show that if the share of expenditure on domestic product and share of expenditure of consumers in a country is known in each industry, there is no need to solve the equilibrium in section 2.5 to evaluate change in welfare of trade shocks.

From (19), the share of expenditure on domestic product in industry \(s\) of country \(j\) equals to:

\[
\lambda_{jjs} = \frac{f_{jjs} \frac{\sigma_s - 1}{\sigma_s - 1} \frac{1}{\theta_s} \frac{b_{js}}{d_{jjs} \eta_{jjs} \beta_{js} c_{js}} }{\sum_{m=1}^{N} M_{ems} \frac{\sigma_s - 1}{\sigma_s - 1} \frac{1}{\theta_s} \frac{b_{ms}}{d_{mjs} \eta_{mjs} \beta_{ms} c_{ms}}} \theta_s .
\]

Substituting (27) in (18) we get:

\[
P_{js} = \gamma'_s \left( \frac{\sigma_w w_j f_{jjs} \gamma_{js} \beta_{js} \lambda_{jjs} \gamma_{js}}{R_{jjs}} \right) \frac{\sigma_s - 1}{\sigma_s - 1} b_{js} \frac{1}{\theta_s} \frac{\sigma_s - 1}{\sigma_s - 1} b_{js} \frac{1}{\theta_s} \frac{1}{\theta_s} \sum_{k=1}^{S} P_{jk} \eta_{sk} \beta_{js} (1 - \beta_{js}) \gamma_{js} .
\]

where \(\gamma'_s\) is constant. After some matrix algebra it can be shown that:

\[
\gamma'_s = \beta_{js} (1 - \beta_{js}) \left( 1 - \beta_{js} \right) \prod_{k=1}^{S} \eta_{sk} \beta_{js} (1 - \beta_{js}) \gamma_{js}.
\]
\[ W_j = \prod_{s=1}^{S} \prod_{k=1}^{S} \left( \frac{1}{\gamma_s} \left( \frac{\tau_s W_f j \mathbb{I}_{jjs} (\sigma_s)_{1-\theta_s} b_{jjs} \lambda_{jjs}}{R_{jjs}} \right) \right)^{a_{js}^{j\mu js}} \]  

(29)

Where \( a_{js}^{j\mu js} \) is the \((s,k)\) element of the leontief inverse matrix \((I - A_j)^{-1}\), where \( A_j \) is the matrix whose its \((s,k)\) element is \( \eta_{sk} (1 - \beta_{js}) \). \( a_{js}^{j\mu js} \) is the elasticity of price index of industry \( s \) with respect to price index of industry \( k \) in country \( j \).

Defining \( \hat{\pi}_{cjs} \) share of expenditure of consumers in industry \( s \), the welfare change in response to trade shocks is:

\[ \hat{W}_j = \prod_{s=1}^{S} \prod_{k=1}^{S} \left( (\hat{\pi}_{cjs}) \left( \frac{\sigma_s - 1}{\theta_s} \right) - \theta_s \frac{1}{\sigma_s - 1} \lambda_{jjs}^{1-\theta_s} \right)^{a_{js}^{j\mu js}} \]  

(30)

Since in autarky \( \lambda_{jjs} = 1 \) and the total expenditure on each industry equals to its revenue, form (11) it can be shown that \( \pi_{Ajs} = (\sum_{k=1}^{S} a_{sk}^{i})^{-1} = \Delta_{js}^{3} \). The welfare of gain of moving from observed equilibrium to autarky is:

\[ \hat{W}_j^{OE \rightarrow A} = 1 - \prod_{s=1}^{S} \prod_{k=1}^{S} \left( \pi_{cjs}^{A} \frac{1}{\sigma_s - 1} \lambda_{jjs}^{1-\theta_s} \right)^{a_{js}^{j\mu js}} \]  

(31)

3 Bilateral Trade Agreement

Consider a trade agreement between two countries (i and j) that have same tariffs on each other. Defining \( \tau \) the vector of tariffs in all sectors, a trade agreement between two countries is shown by moving from \( \tau \) to \( \tau' \). As trade agreement means reduction in trade barriers: \( \tau' \leq \tau \).

We used the definition of intensive and extensive margin same as Chaney (2008). The difference here is that not only the volume of trade in one industry depend on the trade costs of that industry, but also depends on the trade costs of other sectors due to sectoral linkage. So, we also have cross-industry margins.

Differentiating from aggregate export \( X_{ijs} = M_{is}^{c} \int_{\phi_{ijs}}^{\infty} x_{ijs} dG(\phi) \), we have

\[ dX_{ijs} = \sum_{r=1}^{S} \left( \int_{\phi_{ijs}}^{\infty} x_{ijs} dG(\phi) \right) \frac{\partial M_{is}^{c}}{\partial \tau_r} \]  

(Entry)

\[ + \left( M_{is}^{c} \int_{\phi_{ijs}}^{\infty} \frac{\partial x_{ijs}}{\partial \tau_r} dG(\phi) \right) \]  

(Intensive Margin)

\[ + \left( M_{is}^{c} \frac{\partial x_{ijs}}{\partial \tau_r} G(\phi_{ijs}) \frac{\partial \phi_{ijs}}{\partial \tau_r} \right) \]  

(Extensive Margin)

(32)

When tariff in one industry changes, it affect volume of trade in all industries due to sectoral linkage. The intensive margins are the change in export of each existing exporter \( (\phi > \phi_{x,s}) \). The extensive margins are the changes in the number of exporters. We show the elasticities of intensive and extensive margins in industry \( s \) with respect to tariffs in industry \( r \) by \( \zeta_{ij,s,r} \) and \( \xi_{ij,s,r} \) respectively.

\[ \text{from (11) in autarky } \frac{1}{\pi_{cjs}} = 1 + \sum_{r=1}^{S} (1 - \beta_{ir}) \eta_{ir} \frac{1}{\pi_{cjs}}. \]  

It is easy to find \( \pi_{Ajs}^{A} \) by some matrix algebra.
It can be shown that the intensive and extensive margins of trade in each industry are:

\[ \zeta_{ijs,r} = (\sigma - 1)(1 + (1 - \beta_{is}) \sum_{k=1}^{S} \eta_{sk} \epsilon_{sk} - \epsilon_{sr}) \] (33)

\[ \xi_{ijs,r} = (\theta - (\sigma - 1))(1 + (1 - \beta_{is}) \sum_{k=1}^{S} \eta_{sk} \epsilon_{sk} - \epsilon_{sr}) \] (34)

Where \( \epsilon_{sr} \) is the elasticity of price index of an industry with respect to tariff imposed on another industry. Note that a change in price index of an industry may change price indices of other industries due to sectoral linkage; so \( \epsilon_{sr} \) takes into account the general equilibrium effect of change in price indices.

The margins of trade have two component. The first term shows how much does change in the relative price, changes the volume of export of each exporter (in intensive margin) and the total number of exporters (in extensive margin). The second term shows how much does relative price of export changes in target market in response to a change in tariff. The second term is the elasticity of relative price of exports to price index of target market with respect to trade barrier.

Changes in tariffs affect relative prices of export from three channels. First, reduction in tariff directly lower the relative price of producers in that industry. However, second, it also reduces the price index of that industry. The more is the share of trade with its trade partner, the higher is the reduction in price index. Reduction in price index of industry has negative effect on export by making competition tougher. Third, reduction in tariff for import in an industry, reduces the marginal cost of production in industries that uses products of that industry as an input. The overall effect of changes in tariffs on relative price of export depends on share of foreign product in the total goods that demand by each country and industry and the sectoral linkage between industries. If a country is small, change in tariffs don’t change the price index in other country, but it gain from reduction in cost of production.

Higher the elasticity of relative price of export with respect to tariff means with same reduction in trade barrier the relative price of exports to price index of target market decreases more. So, reducing trade barrier has more effect on volume of export and number of exporters and both margins would be higher.

Higher \( \sigma \) makes the competition tougher and the price advantage more important. It has two opposing effect on two margins. In the hand, reduction in tariff has more effect on volume of export (more intensive margin). In the other hand, it decrease number of new low productivity firms to export market due to less reduction in cut-off productivity of export market (less extensive margin).

Higher \( \theta \) means more homogeneity in productivity of firms. So, by a reduction in cut-off productivity more firms will enter the export market and extensive margin would be higher.
3.1 Similar countries

In general, it is only possible to calculate the effects of changes in tariffs numerically. But, when the two countries and all industries are similar, we can solve the equilibrium given tariffs.

Since two countries are similar and wage is normalized to 1, the free entry condition in each industry is same for all countries. It can be shown that the number of potential producers in each sector is same for all countries and doesn’t change by change in \( \tau \). Solving the equilibrium defined in section 2, the price index in each sector and the marginal cost of production for a firm with productivity of 1 are:

\[
P_s = \gamma \left( f_s \left( \frac{(\sigma-1)-\theta}{\sigma-1} \right) + (d\tau_s)^{-\theta} f_s \left( \frac{(\sigma-1)-\theta}{\sigma-1} \right) \right)^{-\frac{1}{\theta}} \bar{c}
\]

\[
\bar{c} = \beta - \beta \left( \frac{1-\beta}{S} \right)^{(1-\beta)} \gamma (1-\beta) \prod_{r=1}^{S} \left( f_r \left( \frac{(\sigma-1)-\theta}{\sigma-1} \right) + (d\tau_r)^{-\theta} f_r \left( \frac{(\sigma-1)-\theta}{\sigma-1} \right) \right)^{-\frac{1}{\theta}} \beta.
\]

Where \( \alpha \) and \( \gamma \) are constants.

Defining \( \epsilon_{ss} \) elasticity of price index of an industry with respect to tariff imposed on that industry and \( \epsilon_{sr} \) (\( r \neq s \)) the elasticity of price index of an industry with respect to tariff imposed on another industry, from (35) we have:

\[
\epsilon_{ss} = \frac{\partial P_s}{\partial \tau_s} \frac{\tau_s}{P_s} = (1 + \frac{1-\beta}{S\beta}) \frac{X_s}{X_s}
\]

\[
\epsilon_{sr} = \frac{\partial P_s}{\partial \tau_r} \frac{\tau_r}{P_s} = \frac{1-\beta}{S\beta} \frac{X_r}{X_r}
\]

We can interpret these elasticities as the sum of a geometric series.

\[
\epsilon_{ss} = (1 + \frac{1}{S} (1-\beta) + \prod_{s=1}^{S} \left( 1 + \frac{1-\beta}{S} \right)) \frac{X_s}{X_s}
\]

\[
= (1 + \frac{1}{S} \frac{1-\beta}{S-1}) \frac{X_s}{X_s} = (1 + \frac{1-\beta}{S\beta}) \frac{X_s}{X_s}
\]

The first term shows the direct effect of change in price index of an industry resulted from change in tariff of that industry modified by ratio of imported good in that industry. Further, the change in price index changes the marginal cost of production. The second term shows the initial change in \( P_s \) resulted from change in marginal cost of production \( \bar{c} \). Moreover, This change in \( \bar{c} \) not only changes \( P_s \) but also changes the price indeces of all industries. The third term shows the effect of change in prices indeces of all industries in \( \bar{c} \) and hence on \( P_s \). This effect amplifies geometrically and reduces more the price index of industry \( s \). The overall effect is showed in (40).

This also true for change in tariff of another industry, but, the reduction in \( \tau_r \) doesn’t have direct effect in price index of industry \( s \) (the first term). Hence, the overall effect would be\( (1-\beta) \frac{X_r}{X_r} \).

\[\alpha = \frac{\theta(1+\beta) \sigma-1}{\sigma-1} \text{ and } \gamma = \left[ \beta - \beta \left( \frac{1-\beta}{\sigma-1} \right)^{(1-\beta)} f_s \left( \frac{\sigma-1}{\sigma-1} \right) \right] \left( \frac{\theta-1}{\sigma-1} \right) \frac{1}{\gamma} \right] \left[ \frac{\gamma}{\alpha + (1-\beta) \sigma} \right] \left( \frac{1}{\gamma} \right) \]

\(12\)
Consider the case when $\beta = 1$ and the only input of production is labor. The elasticity of price index of an industry with respect to trade barriers of other industries are zero and with respect to its own tariff is $X^s_t / X_s$ (the only effect is the direct effect of change in its own tariff). As the price of input doesn’t change by changes in tariffs, $\epsilon_{ss} = 1 - X^s_t / X_s$. If share of export is small, then change in trade barrier doesn’t change price index of target market so $\epsilon_{ss} = \epsilon_{sr} = 0$.

When countries are similar, by changes in tariffs in other industries, reduction in price indices and marginal cost of production are same in both countries. Consider a reduction in $\tau_r$, it lowers the relative price of industry $s$ by decreasing the price of intermediate goods used in production same as it increases the relative price of industry $s$ by lowering the price index in target market. So if price indices in other industries reduce, reduction in the marginal cost of production and price index of the industry cancel out each other. Therefore, the relative price of export good in target market in that industry doesn’t change.

It can be shown that the intensive and extensive margins of trade in each industry for changes in tariff of that industry are:

$$\zeta_{ijs,s} = (\sigma - 1)(1 - X^s_t / X_s) \quad (40)$$

$$\xi_{ijs,s} = (\theta - (\sigma - 1))(1 - X^s_t / X_s) \quad (41)$$

As changes in price index of other industries don’t affect the price index of this industry, effect of reduction in tariff would be same as the models without sectoral linkage. When countries are same, sectoral linkage does change price indices, but it doesn’t change margins of trade.

4 Trade policies on intermediate goods

The model can be easily used to evaluate trade policies that specially target intermediate goods. Setting different tariffs on intermediate for firms is common policy in developing countries. In the model in section 2, it is assumed that consumers and producers demand the same final good. However, it can be assumed that there are two final goods in each industry and each country. One final good is used for consumption and only demand by consumers, and one final good is used for production by producers of consumption and intermediate goods. Each industry can be divided to two sectors: the sector that produce consumption goods that are aggregated to final consumption good, and the sector that produce intermediate goods that are aggregated to final intermediate good. Producers in both section of an industry demand the final intermediate goods for production.

Ossa (2015) emphasized that variation in trade elasticities between industries greatly increases the welfare gain from trade. In the same way, as the elasticities of substitution and trade elasticities differ significantly in consumption goods and intermediate goods, accounting for this difference can magnify the estimated welfare gain from trade.
4.1 equilibrium given tariffs

We rewrite the equilibrium given tariffs in section 2.5, assuming different goods are demanded for consumption and production of final goods. The only difference between these two sectors that we consider here are tariffs and elasticity of substitution. Superscript I and C denote intermediate goods and consumption goods, respectively. Setting $\eta_{is} = 0$ for all $r$ in (11), the total revenue spent on final consumption good is:

$$R^C_{is} = w_i \mu_{is} L_i$$

(42)

Similarly, setting $\mu_{is} = 0$ in (11), the total revenue spent on final intermediate good is:

$$R^I_{is} = w_i \sum_{r=1}^{S} \sum_{\delta \in \{C, I\}} (1 - \beta_{ir}) \eta_{is} \theta_r M^r,\delta f^r_{ir}$$

(43)

From (??) the labor market equilibrium is:

$$\alpha_i L_i = \sum_{s=1}^{S} \sum_{\delta \in \{C, I\}} \frac{1}{\sigma^S_{i}} (1 + \theta_s)(1 + \beta_{is}(\sigma^S_{i} - 1)) M^r,\delta f^r_{is}$$

(44)

From (19), the free entry condition for both sectors of each industry are:

$$\sigma^S_{i} \theta_s \theta_{i} \frac{\tilde{c}^S_{is}}{\sigma^S_{i} - 1} = \sum_{v=1}^{N} \left( \gamma_s \left( \sum_{k=1}^{S} \frac{f^l_{is}}{R^l_{is}} \right)^{\frac{\sigma^S_{i}}{\sigma^S_{i} - 1}} \left( \frac{b_{is}}{\alpha_i \sigma^S_{i} - 1} \right)^{\theta_s} \right)^{\sigma^S_{i} - 1} f^l_{is}$$

(45)

Where $\tilde{c}$ is determined by price indeces of final intermediate goods:

$$\tilde{c}^S_{is} = \left( \frac{w_i}{\beta_{is}} \right)^{\theta_{is}} \left( \prod_{k=1}^{S} \frac{P^l_{is}}{P^l_{ik}} \right)^{1 - \beta_{is}}$$

(46)

Where from (18), $P^l_{is}$ is:

$$P^l_{is} = \gamma_s \left( \frac{\sigma^S_{i} w_i}{R^l_{is}} \right)^{\theta_{is}} \left( \sum_{v=1}^{N} \left( \frac{b_{is}}{\alpha_i \sigma^S_{i} - 1} \right)^{\theta_s} \right)^{\sigma^S_{i} - 1} \tilde{c}^S_{is}$$

(47)

The equilibrium for given tariffs is $N w_i$, $M^l, M^C_{is}, R^C_{is}, R^I_{is}, \tilde{c}_{is}$, and $P^l_{is}$ that satisfy system of N equations of (44), 2NS equations of (45), and NS equations of (43), (42), (46), (47).

If a country changes tariffs imposed to exporting intermediate goods to that country, how this affects different industries? Consider a reduction in tariffs of imported intermediate goods. Beside the direct reduction in price of imported intermediate goods, it lowers the cut-off productivity for export to the country. So, more firms start to exporting to the country. It also lower the average productivity of firms that export to the country. As explained in sector 2.6, the overall effect of change in tariffs of imported intermediate goods on price index of imported products is positive. Domestic firms in intermediate goods sector will lose some of their sell because of tougher competition, but other sectors that use these intermediate goods for production will gain from lower marginal cost of production. Hence, the cut-off productivity for production and exports to other countries become lower in those industries.
4.2 Welfare gain of international trade

In the same way as section 2.7, Price index of final consumption good equals to

\[
P_{j}s = \gamma_s' \left( \sigma_s^{-1} - \eta_s \right) b_{js} \lambda_{jjs}^C \prod_{k=1}^S P_{jk}^I \eta_{sk} (1 - \beta_{js}). \tag{48}\]

After some matrix algebra it can be shown that:

\[
W_j = \prod_{s=1}^S \left( \frac{1}{\gamma_s' \eta_s} \right) b_{js} \lambda_{jjs}^C \prod_{k=1}^S \left( \frac{1}{\gamma_s' \eta_s} \right) b_{js} \lambda_{jjs}^I \tilde{a}_{js}^{I,j,s} \tag{49}\]

Where \(\tilde{a}_{js}^{I,j,s}\) is the \((s,k)\) element of the adjusted leontief inverse matrix \(A_j(1 - A_j)^{-1}\), where \(A_j\) is the matrix whose its \((s,k)\) element is \(\eta_{sk}(1 - \beta_{js})\). \(\tilde{a}_{js}^{I,j,k}\) is the elasticity of price index of final consumption good in industry \(s\) with respect to price index of final intermediate good in industry \(k\) in country \(j\).

The welfare change in response to trade shocks is:

\[
\tilde{W}_j = \prod_{s=1}^S \left( \frac{1}{\gamma_s' \eta_s} \right) b_{js} \lambda_{jjs}^C \prod_{k=1}^S \left( \frac{1}{\gamma_s' \eta_s} \right) b_{js} \lambda_{jjs}^I \tilde{a}_{js}^{I,j,s} \tag{50}\]

Same as section 2.7, it can be shown that the welfare of gain of moving from observed equilibrium to autarky is:

\[
\tilde{W}_j^{OE \rightarrow A} = 1 - \prod_{k=1}^S \left( \lambda_{jjs}^C \right) \prod_{k=1}^S \left( \lambda_{jjs}^I \right) \left( \frac{\tilde{a}_{js}^{I,j,s}}{\Delta_{js}} \right) \eta_{js} \tag{51}\]

Where \(\Delta_{js} = \sum_{k=1}^S \tilde{a}_{sk}^{I,j,s} - 1\). Note the difference between (31) and (51). In (31), it is the share of expenditure in domestic good that determines welfare gain from trade. In (51) both shares of domestic goods in consumption goods and intermediate goods are determining the welfare gain from trade. However, what is more important is share of domestic good in intermediate good, because its effect magnifies with sectoral linkage. Also, here, only the elasticity of substitution of intermediate goods appear in welfare change formula. Therefore, the estimated welfare gain from trade would be higher.
5 Conclusion

We introduce a general equilibrium model of international trade that takes into account the endogenous entry and exit of intermediate goods producers and the industry linkages to quantitatively assess the export and import subsidy programs as in China. Our model extends the inter-industry trade model Eaton and Kortum (2002), intra-industry model of Krugman (1980), and firm heterogeneous model of Melitz (2003). We show how the model can be easily used to evaluate the welfare gains of trade policies that specifically target intermediate goods like subsidies to firms for importing intermediate goods. We show how changes in tariff of intermediate goods spread out to other sectors. To decompose the effects of changes in tariffs on welfare, we log-linearized the change in welfare around the factual. We show that change in tariffs affect welfare through four channels: first, it has direct effect on wage of a country, second, it affect the cost of input bundle of production (labor and intermediate goods), third, it changes the term of trade of country, forth, it causes entry and exit in industries. In the quantitative trade models surveyed by Arkolakis et al. (2012), it is the share of expenditure in domestic good that determines welfare gain from trade. We show that both shares of domestic goods in consumption goods and intermediate goods together determine the welfare gain from trade. However, what is more important is share of domestic good in intermediate good, because its effect magnifies with sectoral linkage. Also, because the elasticity of substitution of intermediate goods in lower than consumption goods the estimated welfare gain from trade would be higher.
References


