1. **Os, Exercise 282.3 (Adverse Selection)** Firm A (the *acquirer*) is considering taking over firm T (the *target*). It does not know firm T’s value; it believes that this value, when firm T is controlled by its own management, is at least $0 and at most $100, and assigns equal probability to each of the 101 dollar values in this range. Firm T will be worth 50% more under firm A’s management than it is under its own management. Suppose that firm A bids $y$ to take over firm T; and firm T is worth $x$ (under its own management). Then if T accepts A’s offer, A’s payoff is $\frac{3}{2}x - y$ and T’s payoff is $y$; if T rejects A’s offer, A’s payoff is 0 and T’s payoff is $x$. Model this situation as a Bayesian game in which firm A chooses how much to offer and firm T decides the lowest offer to accept (you consider this decision making to be simultaneous; however, the outcome will not change if the game is played sequentially either). Find the Nash equilibria of this game. Explain why the logic behind the equilibrium is called *adverse selection*.

2. **(Hierarchical Group of Lions)** A group of $n > 2$ hierarchically ordered lions faces a piece of prey. If lion 1 does not eat the prey, the prey escapes and the game ends. If it eats the prey, it gets tired and lion 2 can eat it. If lion 2 does not eat lion 1, the game ends; if it eats lion 1, then it gets tired and lion 3 can eat it; and so on. Formulate the interaction among the lions as an extensive-form game and find its solution using backward induction. For $N=3$, reformulate the game in the strategic form and find all pure Nash equilibria. Are there any Nash equilibrium which is not sequentially rational?

3. **(Sequential Battles)** Armies 1 and 2 are fighting over an island initially held by a battalion of army 2. Army 1 has $K$ battalions and army 2 has $L$. Whenever the island is occupied by one army the opposing army can launch an attack. The outcome of the attack is that the occupying battalion and one of the attacking battalions are destroyed; the attacking army wins and so long as it has battalions left, occupies the island with one battalion. The commander of each army is interested in maximizing the number of surviving battalions but also regards the occupation of the island as worth more than one battalion but less than two (if, after an attack, neither army has any battalions left, then the payoff of each commander is 0.) Analyse this situation as an extensive game and, using the notion of subgame perfect equilibrium, predict the winner as a function of $K$ and $L$.

4. **(A market game)** A seller owns one indivisible unit of a good, which she does not value. Several potential buyers, each of whom attaches the same positive value $v$ to the good, simultaneously offer prices they are willing to accept. If she does not accept any offer, then no transaction takes place, and all payoffs are 0. Otherwise, the buyer whose offer the seller
accepts pays the amount p she offered and receives the good; the payoff of the seller is p, the payoff of the buyer who obtained the good is v-p, and the payoff of every other buyer is 0. Model this situation as an extensive game with perfect information and simultaneous moves and find its subgame perfect equilibria. Show, in particular, that in every subgame perfect equilibrium every buyer’s payoff is zero.

**Recommended Exercises:**

**Os. Exercise 177.1 (Firm-union bargaining)**

**Os. Exercise 192.1 (Sequential variant of Bertrand’s duopoly game)**

*(Dollar auction)* An object that two people each value at v (a positive integer) is sold in an auction. In the auction, the people alternately have the opportunity to bid; a bid must be a positive integer greater than the previous bid. (In the situation that gives the game its name, v is 100 cents.) On her turn, a player may pass rather than bid, in which case the game ends and the other player receives the object; both players pay their last bids (if any). (If player 1 passes initially, for example, player 2 receives the object and makes no payment; if player 1 bids 1, player 2 bids 3, and then player 1 passes, player 2 obtains the object and pays 3, and player 1 pays 1.) Each person’s wealth is w, which exceeds v; neither player may bid more than her wealth. For v = 2 and w = 3 model the auction as an extensive game and solve it by backward induction.