1- (25 Marks) Consider the following two-stage entry-and-competition game played by N firms in a market with one homogeneous good. In the first stage all firms simultaneously decide whether or not to enter. If a firm enters, it pays a setup cost $F > 0$. In the second stage all firms observe the decisions made by all firms in the first stage and then the entrants simultaneously compete in prices (Bertrand Competition). Total demand is $p(q) = a - bq$ and the total cost of producing quantity $q$ for each firm is $C(q) = cq$, with $a > c \geq 0$ and $b > 0$.

a) Find all pure strategy Sub-game Perfect Nash Equilibria (SPNE) when $N = 2$. (Hint: You should distinguish two cases, depending on how high monopoly profits are compared to entry cost.)

b) Find all pure strategy SPNE when $N > 2$.

c) Show that there exists a mixed strategy SPNE when $N = 2$. Find the probability with which a firm enters as a function of $F$ and the monopoly profit.

2- (25 Marks) Consider a game with two agents, 1 and 2; involved in a lawsuit where agent 1 is suing agent 2 for certain damages. Suppose that agent 1 knows the relevant evidence that allows her to be sure whether she would win the lawsuit. Formally, whether she would win or lose can be conceived as depending on the choice of Nature from the set $\{w, l\}$, where $w$ refers to “player 1 winning” and $l$ to “player 1 losing”. Player 2 does not have the information on the choice of Nature but believes that $w$ materializes with probability $2/3$.

Agent 1 (the informed party) can propose one of two agreements to agent 2: a low assessment of damages equal to $m = 1$ or a high one equal to $m = 2$. If player 2 accepts the assessment $m \in \{1, 2\}$, there is no trial and the payoff of agent 1 is $m$ and that of agent 2 is $-m$. However, if agent 2 rejects the proposed damages, the dispute goes to trial. In that case, the payoffs are as follows. If player 1 wins the trial (the choice of Nature is $w$), the payoff of player 1 is 3 and the payoff of 2 is $-4$, while in the other case the payoff of player 1 is $-1$ and the payoff of player 2 is 0.

a) Formulate the game as a signaling game, specifying precisely all its constituents items.

b) Argue rigorously that there is no separating equilibrium.

c) Find all the signaling equilibria in pure strategies, specifying explicitly the beliefs that may be used to support each of them.
3- (20 Mark) Consider the following Prisoners’ dilemma game:

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
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</thead>
<tbody>
<tr>
<td>C</td>
<td>2, 2</td>
<td>-1, 3</td>
</tr>
<tr>
<td>D</td>
<td>3, -1</td>
<td>0, 0</td>
</tr>
</tbody>
</table>

a) Can the cooperative outcome (C, C) be sustained as a subgame perfect equilibrium if this game repeated a finite number of times? Does your answer depend on whether there is discounting or not?

b) How does your answer change to part a if the game repeated infinitely and each player has a discount factor \( \delta < 1 \)? For what values of \( \delta \), is it possible to find a symmetric limited punishment strategy which be a subgame perfect equilibrium for this game. How does your answer depend on the length of the punishment period \( K \geq 1 \)?

4- (20 Marks) Consider 3 players who have access to one unit of output. Player 1 has one vote, player 2 has two votes, and player 3 has three votes. Any majority may control the allocation and in case of a tie the side with more players wins. Each person cares only about the amount of output she obtains.

a) Model this situation as a coalitional game.

b) Define the concept of the core for a coalitional game.

c) Find the core for this game.

5- (20 Marks) Two players, 1 and 2, bargain over how to split a pie of size 1. There are \( T \) periods (\( T \) is finite). In each odd period (i.e. \( t = 1, 3, ... \)), as long as there has been no agreement, player 1 makes a proposal and player 2 accepts or rejects it. In each even period (i.e. \( t = 2, 4, ... \)), as long as there has been no agreement, player 2 makes a proposal and player 1 accepts or rejects it. The game ends, with the agreed split, as soon as a proposal is accepted. If there is no agreement, the pie is wasted. There is no discounting but a player pays a cost \( c < 1 \) every time she makes a proposal, whether it is accepted or not (and she is always obliged to make a proposal when it is her turn). If a split \((x_1; x_2)\) is agreed then i’s payoff is \( x \), less her proposal costs.

a) Find a SPNE and show that it is unique (i) if \( T \) is even, and (ii) if \( T \) is odd.

b) Now suppose that the game is before except that \( T \) is infinite. Find a value \( x \) such that there is a stationary SPNE in which, in any period, the proposer always proposes \( x \) for herself and \( 1-x \) for the other player.

110 marks

Good luck – Farshad Fatemi