On Money As A Medium Of Exchange
Kiyotaki, Wright (1989)

S.Ekbatani, S.Ahmadi-Renani

Sharif University of Technology
Graduate School of Management and Economics

3 Khordad 1393
Abstract

- Nash equilibria in trading strategies: Certain goods emerge endogenously as media of exchange, or commodity money.
- Equilibria with genuine fiat currency circulating as the general medium of exchange.
- That equilibria are not generally Pareto optimal.
- Introducing fiat currency into a commodity money economy may unambiguously improve welfare.
Introduction

- Analyze a simple general equilibrium matching model
- Objects that become media of exchange will be determined endogenously as part of the noncooperative equilibrium.
- Different commodities potentially playing this role depending both on their intrinsic properties and on extrinsic beliefs.
- Genuine fiat currency may or may not circulate in the economy, depending on extrinsic beliefs, or social custom, preferences and technology.
- Overlapping generations models basically ignore the medium of exchange role, concentrating on money’s store of value function.
- Cash-in-advance models simply impose the medium of exchange role by an ad hoc restriction that goods can be acquired only using money.
Introduction

- Ideas going back at least to Adam Smith (1776), the driving force behind the use of money is specialization.
- Agents do not necessarily consume what they produce.
- Trade must be bilateral and quid pro quo.
- Jevons’s (1875) “double coincidence of wants” problem.
- A critical factor in determining if an object can serve as a medium of exchange is whether or not agents believe that it will.
- Involves strategic elements.
The Goal

- To use the sequential matching model to derive commodity and/or fiat money endogenously
Three indivisible commodities called goods 1, 2, and 3

Continuum of infinitely lived agents with unit mass, with equal proportions of types I, II, and III

Specialize in both consumption and production: type i agents derive utility only from the consumption of good i and are able to produce only good $i^* \neq i$

All goods are storable at a cost, but agents can store only one unit at a time

Let $c$ denote the cost to type $i$ of storing good $j$. assume $c_{i3} > c_{i2} > c_{i1} > 0$ for all $i$
The Economy

- $U_i$ denote the instantaneous utility from consuming good $i$
- $D_i$ denote instantaneous disutility from producing good $i^*$
- $i$’s expected discounted lifetime utility:

$$E \sum_{t=0}^{\infty} \beta^t [I_{iU}(t)U_i - I_{i^*D}(t)D_i - I_{ij}^c(t)c_{ij}]$$

- $I_{iU}(t)$ is a (random) indicator function that equals one if the agent eats his consumption good $z$, zero otherwise
- $I_{i^*D}(t)$ equals one if he produces his production good $i^*$, zero otherwise
- $I_{ij}^c(t)$ equals one if he stores any good $j$, zero otherwise
The Economy

- Net utility $u_i = U_i - D_i$ is large enough that agent will not want to drop out of the economy
- Assumption A. For all $i$, $u_i > (c_{ii^*} - c_{ik})/(1 - \beta)$, for all $k$
- Type $i$ acquires his consumption good $i$, he will consume it and produce a new unit of $i^*$
- Each type $i$ always has an inventory of exactly one unit of one good other than good $i$
- Each period, agents are matched randomly in pairs and must decide whether or not to trade bilaterally
The distribution of potential matches can be characterized by the time path of $p(t) = \ldots p_{ij} \ldots$

$p_{ij}$ is the proportion of type $i$ agents holding good $j$ in inventory at date $t$

steady-state equilibria, $p(t) = p$ for all $t$

$\tau_i(j, k) = 1$ if $i$ wants to trade $j$ for $k$, and zero otherwise

Type $i$ with good $j$ and type $h$ with good $k$ trade if $\tau_i(j, k) \cdot \tau_h(k, j) = 1$
The Economy

- **Definition.** A steady-state Nash equilibrium is a set of trading strategies $\tau_i$, one for each type $i$, together with a steady-state distribution of inventories $p$, that satisfies
  - Maximization: each individual $i$ chooses $\tau_i$ to maximize expected utility given the strategies of others and the distribution $p$
  - Rational expectations: given $\tau_i$, $p$ is the resulting steady-state distribution
The Economy

- Let $V_i(j)$ be the expected discounted utility for type $i$ when he exits a trading opportunity with good $j$.

- When $i$ exits with his own consumption good $i$, he yields the instantaneous utility $u_i = U_i - D_i$ plus the indirect utility of storing a $i^*$. Therefore, $V_i(i) = u_i + V_i(i^*)$.

- The indirect utility of storing good $j \neq i$ described by Bellman’s equation of dynamic programming:

$$V_i(j) = -c_{ij} + \max \beta E[V_i(j')|j]$$

- Where $E[V_i(j')|j]$ is the expectation of $V_i$ at the next period’s random state $j'$.
With shorthand notation $V_{ij} \equiv V_i(j)$

$$\tau_i(j, k) = 1 \quad \text{iff} \quad V_{ik} > V_{ij}$$

In equilibrium, agents of the same type never trade since both cannot prefer what the other has.
Lemma 1. Under assumption A, each type \( i \) will accept good \( i \), eat it, and produce a new unit of good \( i^* \) whenever he has the opportunity. That is, for all \( i \), \( \max_j V_{ij} = V_{ii} = u_i + V_{ii^*} \)

Proof. Suppose that some \( i \) prefers \( k \neq i \) to all other goods:

\[
V_{ik} = -\frac{c_{ik}}{1-\beta} \geq V_{ii} \geq u_i - \frac{c_{ii^*}}{1-\beta}
\]

If he does not consume it:

\[
V_{ii} = -\frac{c_{ii}}{1-\beta} \geq u_i + V_{ii^*} \geq u_i - \frac{c_{ii^*}}{1-\beta}
\]

Both contradicting Assumption A.
Trade always occurs when a double coincidence emerges.
Equilibrium: Model A

- Type I produces good 2, II produces good 3, and III produces good 1
- **Fundamental** equilibrium: Agents always prefer a lower-storage-cost commodity to a higher-storage-cost commodity unless the latter is their own consumption good
- **Speculative** equilibrium: Sometimes agents trade a lower- for a higher-storage-cost commodity not because they wish to consume it, but because it is more marketable
The fundamental strategies are described by $V_{ii} = \max_j V_{ij}$ for all $i$ and the inequalities $V_{12} > V_{13}$, $V_{21} > V_{23}$, and $V_{31} > V_{32}$.
Equilibrium: Model A

- A typical type I agent when he exits a match with good 2, \( b = \beta/3 \)
  \[
  V_{12} = -c_{12} + b[V_{12} + p_{21}(u_1 + V_{12}) + p_{23} \max(V_{12}, V_{13}) + V_{12}]
  \]

- A similar story when I exits a match with good 3 implies
  \[
  V_{13} = -c_{13} + b[V_{13} + V_{13} + p_{31}(u_1 + V_{12}) + p_{32} \max(V_{12}, V_{13})]
  \]

- It is easy to show \( V_{12} > V_{13} \) iff \( c_{13} - c_{12} > (p_{31} - p_{21})bu_1 \)
Equilibrium: Model A

- A typical type II agent
  \[ V_{21} = -c_{21} + b[p_{12}(u_2 + V_{23}) + p_{13}\max(V_{21}, V_{23}) + V_{21} + p_{31}V_{21} + p_{32}\max(u_2, V_{23})] + V_{21} \]
  \[ V_{23} = -c_{23} + b[V_{23} + V_{23} + p_{31}\max(V_{21}, V_{23}) + p_{32}(u_2 + V_{23})] \]

- It is easy to show \( V_{21} > V_{23} \) for all parameter values and \( p_{ij} \)

- The same sort of arguments imply \( V_{31} > V_{32} \)
Equilibrium: Model A

- For these fundamental strategies, the steady-state inventory distribution $\mathbf{p}$ is given by $(p_{12}, p_{23}, p_{31}) = (1, .5, 1)$, and these strategies constitute equilibrium iff $c_{13} - c_{12} > .5b u_1$
- Type I and type III always keep their production goods until they can trade directly for their consumption goods.
- Type II agents trade their production good 3 for good 1 whenever possible. They thereby act as *middlemen* transferring good 1 from type III to type I.

![Diagram of trade routes between type I, II, and III agents]

- Good 1 is the unique medium of exchange, or commodity money.
Equilibrium: Model A

- If \( c_{13} - c_{12} < (p_{31} - p_{21})bu_1 \), the best response by type I to fundamental play in this case is to speculate by attempting to trade good 2 for good 3, which has a higher storage cost but is also more marketable.
- Fundamental play is still the best response by II and III.
- The inventory distribution implied by these strategies is given by \((p_{12}, p_{23}, p_{31}) = (.5\sqrt{2}, \sqrt{2} - 1, 1)\), and so speculative equilibrium obtains iff \( c_{13} - c_{12} < (\sqrt{2} - 1)bu_1 \).
- Type I agents now also play the role of middlemen in some trades, while uses good 3 as a medium of exchange.
Equilibrium: Model A

- Dual commodity monies, with both the most storable and the least storable objects
- An example of an object used as a medium of exchange in spite of the fact that it is dominated in rate of return by another object
- No other pure equilibrium exists, but there are mixed strategy equilibria
Equilibrium: Model B

- Type I produces good 3, II produces good 1, and III produces good 2.
- There exists an equilibrium with all agents playing fundamental, and the distribution is given by \((p_{12}, p_{23}, p_{31}) = (0.5\sqrt{2}, 1, \sqrt{2} - 1)\).
- Type II agents store their production good until they can buy their consumption good directly, while types I and III trade for more storable commodities.

Both goods 1 and 2 serve as media of exchange.
Equilibrium: Model B

- In speculative equilibrium \((p_{12}, p_{23}, p_{31}) = (\sqrt{2} - 1, .5\sqrt{2}, 1)\) type III agents speculate by not trading their higher-storage-cost good 2 when offered good 1. Type II speculates too by acquiring the costly good 3 from type I to facilitate trade with III. Type I buys good 2 from type III to reduce his storage cost and also to facilitate trade with type II.
- Both goods 2 and 3 serve as commodity money while, the most storable good 1 does not.

- In some nonempty region of parameter space these two equilibria coexist. Either goods 1 and 2 or goods 2 and 3 may end up as commodity monies, depending solely on extrinsic beliefs.
Fiat Money

- Fiat money is by definition an object that is intrinsically worthless (does not appear in any utility or production function).
- Economy is endowed with a fixed quantity of a new object called good 0. No one will derive utility from it and it is no help in production. It is, by definition, fiat money.
- $c_i 0 = 0$ but one can not hold both fiat currency and real commodities at the same time (only one storage).
- If $P$ units of good 0 are required to buy one unit of each of real commodities, then $S = M/P$ will be the quantity of real balances in circulation.
- Given that each agent holding fiat money will have exactly $P$ units of the stuff, $S$ will also equal the proportion of all agents holding good 0, $S = \sum_i p_{i0}/3$. 

S.Ekbatani, S.Ahmadi-Renani (GSME)
Fiat Money

- *General* medium of exchange, which is by definition an object "which is habitually, and without hesitation, taken by anybody in exchange for any commodity" (Wicksell 1967)

- There exists equilibria in which fiat money does not circulate, $V_{i0} = 0$

- Everyone believes that others will accept fiat money and ask if this could be an equilibrium

- Good 0 is preferred to the other goods

- For type I, good 1 is best and good 0 is second-best, but what about goods 2 and 3? It depends on the quantity of real balances in circulation
Fiat Money

- Choose S, determining \( \pi = \pi(S) \), with following conditions are satisfied, there exists an equilibrium in which all agents play fundamental strategies.

\[
\begin{align*}
(i) \quad & [1 - 2b + b\pi^3(1 + \pi)^{-1}(1 + \pi - \pi^2)^{-1}](c_{13} - c_{12}) \\
& > b\pi(1 + \pi)^{-1}(1 + \pi - \pi^2)^{-1}\left[\pi(1 - \pi)c_{12} + u_1\left(1 - 2b + \frac{b\pi^2}{1 + \pi}\right)\right], \\
(ii) \quad & \left(1 - 2b - \frac{b\pi^2}{1 + \pi}\right)(c_{32} - c_{31}) > b\pi^3(1 - \pi)(1 + \pi)^{-1} \\
& \quad \cdot (1 + \pi - \pi^2)^{-1}c_{31},
\end{align*}
\]

- Condition (i) rules out speculation by type I
- Condition (ii) is less easy to interpret; However it is redundant for small S
- These conditions hold for any value of S in \([0,1]\) if \(c_{13}\) and \(c_{32}\) are sufficiently large
Fiat Money

- $S = 0$, we are back to commodity money
- $S = 1$ there is nothing but fiat money in circulation
- $0 < S < 1$ both real and fiat money; However, fiat money is the only general medium of exchange: no agent ever offers good 0 for good $j$ and gets refused

![Fig. 7.—Fiat money equilibrium](image-url)
Fiat Money

- (a) stocks of good j,
- (b) the number of times it gets traded per period,
- (c) its velocity,
- (d) the probability it gets accepted
Utility levels, given by $W_i = (1 - \beta) \sum_j p_{ij} V_{ij}$

Are the equilibrium outcomes optimal relative to other sets of trading strategies?

If all use the $\tau \equiv 1$ strategy, we conclude that equilibria are not generally optimal.

He has incentive to reject offers of high-storage-cost goods even though when everyone behaves so "selfishly" they will all be worse off in the long run.
The fundamental commodity money equilibrium for model A is actually a special case of fiat money equilibrium with $S = 0$

$\partial W_i / \partial S > 0$ for all $i$ as long as the $u_i$ are not too large

using fiat money reduces the inefficient storage of real commodities
Welfare

- Fiat money is neutral here

- Welfare depends solely on real balances, $S = \frac{M}{P}$, not nominal balances, $M$
Questions?