Money in Search Equilibrium

Diamond (1984)
Introduction: Markets with Search Frictions

- Most real-world transactions involve various forms of impediments to trade, or “frictions”. Buyers may have trouble finding the goods they are looking for and sellers may not be able to find buyers for the goods they have to offer.

- These frictions can take many forms and may have many sources, including worker and firm heterogeneity, imperfect information, and costs of transportation.

- How are market outcomes influenced by such frictions? That is, how should we expect prices to form and–given that markets will not clear at all points in time–how are quantities determined? Do these frictions motivate government intervention?
This year’s Prize is awarded for fundamental contributions to search and matching theory. This theory offers a framework for studying frictions in real-world transactions and has led to new insights into the workings of markets. The development of equilibrium models featuring search and matching started in the early 1970s and has subsequently developed into a very large literature. The Prize is granted for the closely related contributions made by Peter Diamond, Dale Mortensen, and Christopher Pissarides. These contributions include the analysis of price dispersion and efficiency in economies with search and matching frictions as well as the development of what has come to be known as the modern search and matching theory of unemployment.
General Aspects of Search and Matching Markets: Price Formation

- Buyers and sellers face costs in their attempts to locate each other ("search") and meet pairwise when they come into contact ("matching").

- One of the main issues, therefore, is how price formation works in a market with search frictions. In particular, how much price dispersion will be observed, and how large are the deviations from competitive pricing?

- Peter Diamond addressed these questions in an important paper from 1971, where he showed,
  - first, that the mere presence of costly search and matching frictions does not suffice to generate equilibrium price dispersion.
  - Second, and more strikingly, Diamond found that even a minute search cost moves the equilibrium price very far from the competitive price: he showed that the only equilibrium outcome is the monopoly price. (The “Diamond paradox”)
Another important issue in search markets is whether there is too much or too little search, i.e., whether or not the markets deliver efficient outcomes.

Since there will be unexecuted trade and unemployed resources—buyers who have not managed to locate sellers, and vice versa—the outcome might be regarded as necessarily inefficient. However, the appropriate comparison is not with an economy without frictions.

Given that the friction is a fundamental one that the economy cannot avoid, the relevant issue is whether the economy is constrained efficient, i.e., delivers the best outcome given this restriction. It should also be noted that aggregate welfare is not necessarily higher with more search since search is costly.

A generic result is that efficiency cannot be expected and policy interventions may therefore become desirable.
Along similar lines, Diamond argued that a search and matching environment can lead to macroeconomic unemployment problems as a result of the difficulties in coordinating trade.

This argument was introduced in a highly influential paper, Diamond (1982b), where a model featuring multiple steady-state equilibria is developed.

The analysis provides a rationale for “aggregate demand management” so as to steer the economy towards the best equilibrium.

The key underlying this result is a search externality, whereby a searching worker does not internalize all the benefits and costs to other searchers.
Overview of the Model
Diamond (1984)

<table>
<thead>
<tr>
<th>States</th>
<th>Number of individuals</th>
<th>Value Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>with money</td>
<td>( m )</td>
<td>( w_m )</td>
</tr>
<tr>
<td>employed in trade process</td>
<td>( e )</td>
<td>( w_e )</td>
</tr>
<tr>
<td>unemployed</td>
<td>( n - e - m )</td>
<td>( w_u )</td>
</tr>
</tbody>
</table>

Assumptions

The utility of consumption coming from a completed purchase is \( u \).

Each production opportunity involves a labor cost, \( c \), which is an independent draw from the exogenous distribution \( G(c) \).
The technology controlling the matching of buyers and sellers, is made up of two parts,

- a determinate aggregate trade function relating the flow number of trades to the stocks of buyers and sellers, $f(e, m)$, and
- Poisson processes giving the stochastic rates of transactions for each individual.
Trade Process:
Aggregate Trade Technology

<table>
<thead>
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<th>Assumptions</th>
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</thead>
<tbody>
<tr>
<td>$f_e &gt; 0$, $f_m &gt; 0$ when $m &gt; 0$, $e &gt; 0$</td>
</tr>
<tr>
<td>$ef_e + mf_m &gt; f$</td>
</tr>
<tr>
<td>$f \geq ef_e$, $f \geq mf_m$</td>
</tr>
<tr>
<td>$\lim_{m \to \infty} \frac{f}{m} = \lim_{e \to 0} \frac{f}{m} = 0$</td>
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</table>
### Individual Experiences of the Trade Process

<table>
<thead>
<tr>
<th>Each seller experiences the arrival of buyers as a Poisson process with arrival rate $b$</th>
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</thead>
<tbody>
<tr>
<td>Each buyer experiences the arrival of sellers as a Poisson process with arrival rate $s$</td>
</tr>
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</table>
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- a determinate aggregate trade function relating the flow number of trades to the stocks of buyers and sellers, \( f(e, m) \), and
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The technology controlling the matching of buyers and sellers, is made up of two parts,

- a determinate aggregate trade function relating the flow number of trades to the stocks of buyers and sellers, $f(e,m)$, and
- Poisson processes giving the stochastic rates of transactions for each individual.

For micro-macro consistency, the sum of individual experiences must equal the aggregate experience

$$be = sm = f(e,m)$$
Production

- Production as instantaneous, with the search for production opportunities as time consuming.

<table>
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<th>Production Opportunities</th>
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<tr>
<td>The arrival of production opportunities is a Poisson process with arrival rate $a$.</td>
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</table>

- Each opportunity involves a labor cost, $c$, which is an independent draw from the exogenous distribution $G(c)$.
  - The support of $G$ is bounded below by $\underline{c}$. 
If all projects costing less than $c^*$ are taken, the rate of change in the stock of inventories satisfies,

$$\dot{e} = aG(c^*) (n - e - m) - f(e, m)$$

We will be concentrating on steady state equilibria where

- $\dot{e} = 0$, and
- with constant prices.

The optimal cut off rule for production

$$c^* = w_e - w_u$$
Individual Choices

- The rate of discount times the value of being in a position is equal to the sum of expected gains from instantaneous utility and from a change in status:
  \[
  rw_e = b(w_m - w_e)
  \]
  \[
  rw_m = s(u + w_u - w_m)
  \]
  \[
  rw_u = a \int_{0}^{c^*} (w_e - w_u - c)dG(c)
  \]

- Defining \( t \equiv \frac{bs}{r+b+s} = \frac{f^2}{r.e.m+f(e+m)} \), we have
  \[
  (r + t)c^* = tu - a \int_{0}^{c^*} (c^* - c)dG(c)
  \]
  \[
  (r + t + aG(c^*))c^* = u - c^*
  \]
Individual Choices: Interpretation of $t$

- Consider an individual with a unit of inventory. Then, the expected present discounted value of utility from the sale of that unit followed by the purchase of a unit of consumption is

$$\frac{bsu}{(r + b)(r + s)}$$

- The expected present discounted value of utility from a position with a unit of inventory is,

$$rw_e = t(u - c^*)$$

$$(u - c^*) \sum_{i=1}^{\infty} \left( \frac{bs}{(r + b)(r + s)} \right)^i = t(u - c^*)/r$$
Assume that prices are chosen so that the utility gain to the buyer equals the utility gain to the seller, then

\[ w_m - w_e = u + w_u - w_m \Rightarrow r + b = s \text{ or } r + \frac{f}{e} = \frac{f}{m} \]
Price Determination: Equal Share of Trade Surplus

- Using \( r + f/e = f/m \) to make \( m \) a function of \( e \), we can examine optimal choice, \( c^* \), and the steady state condition, \( \dot{e} = 0 \), in a single diagram in \((c^*, e)\) space.

- For \( e > e_\text{c} \), we have
  \[
  t = b/2 = f/2e
  \]

\[
\left. \frac{dt}{de} \right|_{r+b=s} = \frac{(f/m^2)(ef_e + mf_m - f)}{2[ef_m + (e/m)^2(f - mf_m)]} > 0
\]
Price Determination:
Multiple Equilibria

\[ aG(c^*)(n - e - m) = f(e, m) \]

\[ c^*(t) = c^*(f/2e) \]
If the government controls prices, but not production decisions, the possible steady state equilibria are solutions to the equation $\dot{e} = 0$, evaluated at $c^*(t)$.

$$aG(c^*)(n - e - m) = f(e, m)$$

$$c^* = c^*(t) = c^* \left( \frac{f^2}{rem + f(e + m)} \right)$$

We will analyze the set of solutions to these equations, considering the case where $f$ is homothetic.
Price Controls:
Loci where $\dot{e} = 0$

\[ aG(c^*)(n - e - m) = f(e, m) \]
Equilibrium with Price Controls: The $c^*(e)$ Curves

$$c^* = c^*(t) = c^* \left( \frac{f^2}{rem + f(e + m)} \right)$$
Equilibrium with Price Controls: Multiple Equilibria
Denote by $W(e_0; m, c^*)$ the aggregate present discounted value of utility for an economy with an initial level of inventories $e_0$ and constant levels of real money, $m$, and willingness to produce, $c^*$.

\[
W(e_0; m, c^*) = \int_0^\infty e^{-rt} \left[ uf(e, m) - a(n - e - m) \int_0^{c^*} c dG \right] dt
\]

subject to
\[
\dot{e} = aG(c^*)(n - e - m) - f(e, m)
\]
\[
e(0) = e_0
\]

Then,
\[
\frac{\partial W}{\partial c^*} = \frac{aG' (n - e - m) (u - c^*) (f_e - t)}{r + f_e + aG} \frac{(u - c^*) (f_e - t)}{r}
\]
Local Efficiency

- In an equilibrium satisfying the equal utility gain pricing condition, \( t = f/2e \), we have

\[
\text{sign} \left( \frac{\partial W}{\partial c^*} \right) = \text{sign} \left( f_e - \frac{f}{2e} \right)
\]

\[
\frac{\partial W}{\partial c^*} \leq 0 \quad \text{implies} \quad \frac{\partial W}{\partial m} > 0
\]

- Thus, none of the internal equilibria of the economy with endogenous prices are efficient relative to policies which could vary the incentive to produce and the real money supply.
SOME ECONOMISTS ATTRIBUTE fluctuations in unemployment to misperceptions of prices and wages. Others attribute such fluctuations to lags in adjustment of prices and wages (including staggered contracts). It seems to be a shared view that there would be no macroeconomic unemployment problems if prices and wages were fully flexible and correctly perceived. This paper continues the examination of a third cause of macro unemployment problems, the difficulty of coordination of trade in a many person economy. That is, once one drops the fictional Walrasian auctioneer and introduces trade frictions, one can have macro unemployment problems in an economy with correctly perceived, flexible prices and wages. This proposition is analyzed in a model where money plays a critical role in coordinating transactions.

For a given constant nominal money supply, it is found that there are multiple equilibria, with equilibrium at a higher level of production associated with lower prices and a higher real money supply. That is, an economy with this structure of transaction costs has multiple natural rates of unemployment. This suggests that in the presence of macro shocks, macro policy is needed to keep the economy from spending long periods of time in a low level equilibrium. Even with price controls, and so a fixed real money supply, there are multiple equilibria.
Thanks for Your Attention!