Krugman Monopolistic Competition Trade Model

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Introduction

- Dixit and Stiglitz (1977): brought monopolistic competition into a GE framework
- Economy of scale motive of trade
- No differences in factor intensities
- One factor of production: Call it labor $L$ with wage $w$
- Labor moves across producers but not across countries
Model

- Same efficiencies for all the labor and producers $z$
- Production setup requires a fixed cost (additional labor) $F$
- The range of goods to be produced are endogenous
  - It is determined by the free entry condition
- Each producer makes a different good: so it sells it monopolistically.
- The space of goods is modeled as a continuum
  - Index the goods by $j$
- For now consider a closed economy
Preferences: Spence-Dixit-Stiglitz (SDS)

A CES over the continuum of goods:

\[ U = \left( \int y(j)^{(\sigma-1)/\sigma} \, dj \right)^{\frac{\sigma}{\sigma-1}} \]

\( \sigma > 1 \)

Budget Constraint:

\[ \int p(j) y(j) \, dj = X = wL + \Pi \]

\( X : \) Total Spending
Demand for good $j$:

$$x(j) = \left( \frac{p(j)}{P} \right)^{-(\sigma - 1)} X$$

where

$$P = \left( \int p(j)^{-(\sigma - 1)} dj \right)^{-1/(\sigma - 1)}$$
• The market structure is monopolistic competition.
• Each good is produced by a separate monopolist who takes total spending $X$ and the price index $P$ in each market as given.
• Markets are segmented so that producers can set a different price in each national market.
Profit maximization \( \Rightarrow \) Constant Markup \( \bar{m} = \frac{\sigma}{\sigma - 1} \) over price:

\[
p = \bar{m} \frac{w}{z}
\]

Revenue of a representative firm:

\[
x = \left( \frac{\bar{m}w}{zP} \right)^{-(\sigma - 1)} X
\]

Profit in market \( n \):

\[
\Pi = \frac{x}{\sigma} - wF
\]
Suppose $J$ is the measure of producers of goods

Price Index:

\[
P = \left( \int_{0}^{J} p(j)^{-(\sigma - 1)} \, dj \right)^{-1/(\sigma - 1)}
\]

\[
= J^{-1/(\sigma - 1)} p
\]

\[
= J^{-1/(\sigma - 1)} \bar{m}^{\frac{W}{Z}}
\]
Market Equilibrium

- Firm's revenue in equilibrium:

\[ x = \left( \frac{\bar{m}w}{zP} \right)^{-(\sigma-1)} X \]

\[ = \left( \frac{\bar{m}w}{zJ^{-1}/(\sigma-1) \bar{m}w/z} \right)^{-(\sigma-1)} X \]

\[ = \frac{X}{J} \]
Equilibrium Conditions

- Income = Expenditure

\[ Jx = X = wL + \Pi \]

- Free Entry

\[ \Pi = 0 \]

- Unknowns: \( \frac{w}{P}, J \)
Equilibrium Conditions

- \( \Pi = 0 \Rightarrow \)

\[
w \sigma F = x = \frac{X}{J}
\]

\[
= \frac{wL}{J}
\]

Therefore the endogenous number of firms becomes:

\[
J = \frac{L}{\sigma F}
\]
Equilibrium Conditions

- Real Wage

\[
\frac{w}{P} = \frac{w}{J^{1/(\sigma-1)} \bar{m}^{\frac{w}{z}}} = \frac{zJ^{1/(\sigma-1)}}{\bar{m}} = \frac{z \left( \frac{L}{\sigma F} \right)^{1/(\sigma-1)}}{\bar{m}}
\]

- Aggregate Welfare

\[
W = \frac{wL}{P} = \frac{zL^{\sigma/(\sigma-1)}}{\bar{m} \left( \sigma F \right)^{1/(\sigma-1)}}
\]
Consider $N$ similar countries and no trade costs

$L \rightarrow NL \Rightarrow$

\[
\frac{w}{P} = \frac{z \left( \frac{NL}{\sigma F} \right)^{1/(\sigma-1)}}{\bar{m}}
\]

\[
W = \frac{z (NL)^{\sigma/(\sigma-1)}}{\bar{m} (\sigma F)^{1/(\sigma-1)}}
\]

Total measure of goods $= NJ = \frac{NL}{\sigma F} \Rightarrow$

\[
J = \frac{L}{\sigma F} \text{ in each country}
\]
Consider 2 similar countries and trade costs $d_{HF} = d_{FH} = d$

Revenue of a representative firm in $i$ selling to $n$:

\[ r_{ni} = \left( \frac{\bar{m}wd}{zP} \right)^{-(\sigma - 1)} \quad X : (n \neq i) \]

\[ r_{ii} = \left( \frac{\bar{m}w}{zP} \right)^{-(\sigma - 1)} \quad X : (n = i) \]
Total sales of firms in a country:

\[
R = J \left( \left( \frac{\bar{m}w}{zP} \right)^{-(\sigma-1)} X + \left( \frac{\bar{m}wd}{zP} \right)^{-(\sigma-1)} X \right)
\]

\[
= J \left( \frac{\bar{m}w}{z} \right)^{-(\sigma-1)} \left( P^{\sigma-1} X + d^{-(\sigma-1)} P^{\sigma-1} X \right)
\]

\[
= J \left( \frac{\bar{m}w}{z} \right)^{-(\sigma-1)} \left( 1 + d^{-(\sigma-1)} \right) P^{\sigma-1} X
\]
Total Profits of firms in a country:

\[ \Pi = \frac{R}{\sigma} - JwF \]
Costly Trade

- Price index in a country:

\[
P = \left( \int_0^1 p(j)^{-(\sigma-1)} \, dj \right)^{-1/(\sigma-1)}
\]

\[
= \left( J \left( \frac{\bar{mw}}{z} \right)^{-(\sigma-1)} + J \left( \frac{\bar{mwd}}{z} \right)^{-(\sigma-1)} \right)^{-1/(\sigma-1)}
\]

\[
= \frac{1}{J^{1/(\sigma-1)}} \frac{\bar{mw}}{z} \left( 1 + d^{-(\sigma-1)} \right)^{-1/(\sigma-1)}
\]
Costly Trade Equilibrium Conditions

\[ R = wL + \Pi \]
\[ \Pi = 0 \]

\[ JwF = \frac{R}{\sigma} \]

\[ = \frac{wL}{\sigma} \]

\[ J = \frac{L}{\sigma F} \]
Costly Trade Equilibrium Conditions

\[ P = \frac{1}{\left( \frac{L}{\sigma F} \right)^{1/(\sigma-1)}} \frac{\bar{m}w}{z} \left( 1 + d^{-1/\sigma} \right)^{-1/(\sigma-1)} \]

or

\[ \frac{w}{P} = \left( \frac{L}{\sigma F} \right)^{1/\sigma} z \left( 1 + d^{-1/\sigma} \right)^{1/(\sigma-1)} \]
Gains from trade: Moving from Autarky to costly trade

\[
\frac{(w/P)^d}{(w/P)^A} = \left(1 + d^{-(\sigma-1)}\right)^{1/(\sigma-1)}
\]
Costly Trade

- Trade Elasticity:

\[
\frac{X_{FH}}{X_F} = \bar{J}_H \left( \frac{\bar{mw}_H d_{FH}}{zP_F} \right)^{-(\sigma-1)}
\]
References

- Eaton Kortum Book chapter 3.
- Krugman 1979, 1980
- Krugman & Helpman (1985)