Dornbusch, Fischer, Samuelson (DFS) Model

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Introduction

- Ricardian Trade Theory
- Dornbusch, Fischer, Samuelson AER (1977)
- A convenient parameterization for multi-goods model
- Extension to transportation costs

Model

- Two countries H, F
- Same preferences
- Multi goods: Commodity space $j \in [0, 1]$
- Factor of production: Labor L_H, L_F
- CRS production
  - Labor productivity in producing good $j : z_H(j), z_F(j)$

$$A(j) \equiv \frac{z_H(j)}{z_F(j)}$$

- Order goods with decreasing $A(j)$; i.e. $A'(j) < 0$ (See picture) A continious and strictly increasing.

Model

- Competitive equilibrium
- Costless trade: no transportation costs, tariffs
- Wage rates: w_H, w_F
- Buy j from vendor in H if

$$p_H(j) = \frac{w_H}{z_H(j)} \leq \frac{w_F}{z_F(j)} = p_F(j)$$

or if

$$\omega = \frac{w_H}{w_F} \leq \frac{z_H(j)}{z_F(j)} = A(j)$$

- H has comparative advantage in goods where $A(j) \geq \omega$.(See picture)

Model

- For a given ω, define the cutoff point $j^* = \phi(\omega) = A^{-1}(\omega)$
- Good j where $j \leq j^*$ ar eproduced in H the rest in F.(See picture)
  - $j \in [0, \phi(\omega)]$ are produced in H
  - $j \in [\phi(\omega), 1]$ are produced in F

Demand

- HH's problem:

$$\max_{Q(j)} \int_0^1 \log Q\left(j\right) dj$$

- Budget constraint

$$\int_0^1 p\left(j\right) Q\left(j\right) dj = M = wL$$

- Demands

$$p\left(j\right) Q\left(j\right) = M$$

Prices

$$
\begin{aligned}
p(j) &= \frac{w_H}{z_H(j)} \text{ if } j \in [0, \phi(\omega)] \\
p(j) &= \frac{w_F}{z_F(j)} \text{ if } j \in [\phi(\omega), 1]
\end{aligned}
$$

Equilibrium

- Labor market equilibrium in H :

$$
\begin{aligned}
L_H &= \int_0^{j^*(\omega)} \left(\underbrace{\frac{M_H}{p(j)}}_{=Q_H(j)} + \underbrace{\frac{M_F}{p(j)}}_{=Q_F(j)} \right) `dj \\
&= \int_0^{\phi(\omega)} \left(\frac{M_H}{w_H} + \frac{M_F}{w_H} \right) dj \\
&= \int_0^{\phi(\omega)} \left(L_H + \frac{L_F}{\omega} \right) dj
\end{aligned}
$$

$$\omega L_H = \phi(\omega) \left(\omega L_H + L_F \right)$$

Solution

- Fixed point problem (See picture)

$$\omega = \frac{\phi(\omega)}{1 - \phi(\omega)} \frac{L_F}{L_H}$$

Alternative solution 1

- Fraction of world income spent on a's goods

$$\int_0^{\phi(\omega)} \frac{p(j)\, Q(j)}{M} dj = \int_0^{\phi(\omega)} dj = \phi(\omega)$$

- Solve for equilibrium relative wage

$$\begin{aligned} w_H L_H &= \phi(\omega) * \text{ world Income} \\ &= \phi(\omega)(w_H L_H + w_F L_F) \end{aligned}$$

$\Rightarrow$

$$\omega = \frac{\phi(\omega)}{1 - \phi(\omega)} \frac{L_F}{L_H}$$

Alternative solution 2

- Equilibrium: Income = Expenditure
- Income and expenditure in each country n is $w_n L_n$
- Thus H's sales at home are just $j^* w_H L_H$ while its export revenues are $j^* w_F L_F$.
- Full employment in H thus requires that

$$w_H L_H = j^*. \left(w_H L_H + w_F L_F \right)$$

- Solve for equilibrium relative wage $\Rightarrow$

$$\omega = \frac{\phi\left(\omega\right)}{1 - \phi\left(\omega\right)} \frac{L_F}{L_H}$$

An example

- Let's take

$$A\left(j\right)=\left(\frac{T_H}{T_F}\right)^{\frac{1}{\theta}}\left(\frac{j}{1-j}\right)^{-\frac{1}{\theta}}$$

- Explanation!
- Result:

$$\omega=\left(\frac{T_H/L_H}{T_F/L_F}\right)^{\frac{1}{1+\theta}}$$

Transportation costs

- d_{ij} is the iceberg-cost of sending a good to from j to i.
- F buys good j from H if

$$\frac{w_H d_{FH}}{z_H(j)} \leq \frac{w_F}{z_F(j)}$$

  or if

$$\omega \leq \frac{z_H(j)}{z_F(j)} \frac{1}{d_{FH}}$$

- So there is a j' such that $\omega = \frac{z_H(j')}{z_F(j')} \frac{1}{d_{FH}}$

Transportation costs

- H buys good j from H if

$$\frac{w_H}{z_H(j)} \le \frac{w_F d_{HF}}{z_F(j)}$$

or if

$$\omega \le \frac{z_H(j)}{z_F(j)} d_{HF}$$

- So there is a j'' such that $\omega = \frac{z_H(j'')}{z_F(j'')} d_{HF}$.(See picture)
- Goods $j \in (j', j'')$ are produced for domestic consumption only.

Transportation costs: Equilibrium

- In equilibrium: Income $=$ Expenditure

$$w_H L_H = j' w_F L_F + j'' w_H L_H$$

or

$$\omega L_H = A^{-1}\left(\omega d_{FH}\right) + A^{-1}\left(w/d_{HF}\right)\omega L_H$$

Transportation costs: Equilibrium

- In the example above we get

$$j'' = \frac{T_H w_H^{-\theta}}{\Phi_H}$$

where $\Phi_H = T_H w_H^{-\theta} + T_F \left(d_{HF} w_F\right)^{-\theta}$
and

$$j' = \frac{T_H \left(d_{FH} w_H\right)^{-\theta}}{\Phi_F}$$

where $\Phi_F = T_H \left(d_{FH} w_H\right)^{-\theta} + T_F w_F^{-\theta}$

Welfare Gains:

- Price index

$$
\begin{aligned}
P_H &= \exp\left\{\int_0^1 \log p\left(j\right) dj\right\} \\
&= \exp\left\{\int_0^{j''} \log w_H - \log z_H\left(j\right) dj + \int_{j''}^1 \log d_{HF} w_F - \log z_F\left(j\right) dj\right.
\end{aligned}
$$

- Under Autarky

$$
P_H^A = \exp\left\{\int_0^1 \log w_H^A - \log z_H\left(j\right) dj\right\}
$$

- It can be shown that:

$$
\frac{w_H / P_H}{w_H^A / P_H^A} = \left(\frac{T_H w_H^{-\theta}}{\Phi_H}\right)^{-1/\theta} = \left(1 + \frac{T_F}{T_H}\left(\frac{d_{HF} w_F}{w_H}\right)^{-\theta}\right)^{1/\theta}
$$

Welfare Gains:

- If $d_{HF} = d_{FH} = 1 \Rightarrow$

$$\frac{w_H / P_H}{w_H^A / P_H^A} = \left(1 + \left(\frac{T_F}{T_H}\right)^{1/(1+\theta)} \left(\frac{L_F}{L_H}\right)^{\theta/(1+\theta)}\right)^{1/\theta}$$

- If $d_{HF} = d_{FH} = d$ and $T_H = T_F$ and $L_H = L_F \Rightarrow w_H = w_F \Rightarrow$

$$\frac{w_H / P_H}{w_H^A / P_H^A} = \left(1 + d^{-\theta}\right)^{1/\theta}$$

Extension (Eaton Kortum 2002 Framework)

- To extend, first we need to extend the framework to a new commodity space.
- Think of each country i's efficiency at making any good j as the realization of a random variable Z_i drawn independently from a probability distribution $F_i(z)$. Or by the pair (x_1, x_2) of labor requirements.
- Describe joint distribution by measure μ on Borel sets of R^2_{++}.

Extension (Eaton Kortum 2002 Framework)

- Goods produced in H are

$$\begin{aligned} B_H(\omega) &= \left\{(x_1, x_2) \in R^2_{++} : \omega x_1 \leq x_2\right\} \\ \mu(B_H(\omega)) &= \text{Fraction of spending on } H\text{'s goods} \end{aligned}$$

- Equilibrium: Income = Expenditure

$$w_H L_H = \mu(B_H(\omega))(w_H L_H + w_F L_F)$$

or

$$\begin{aligned} \omega L_H &= \mu(B_H(\omega))(\omega L_H + L_F) \\ \omega &= \frac{\mu(B_H(\omega))}{1 - \mu(B_H(\omega))} \frac{L_F}{L_H} \end{aligned}$$

A convenient parameterization

- Based on exponential distribution on R_+, with cdf

$$\Phi(x) = \Pr(X \leq x) = 1 - e^{-\lambda x}$$

  and density

$$\phi(x) = \lambda e^{-\lambda x}$$

- Write $X \tilde{} \exp(\lambda)$

A convenient parameterization

- "Hazard rate" is constant for this distribution

$$\frac{\phi\left(x\right)}{1-\Phi\left(x\right)}=\lambda$$

- The mean and standar deviation are both $\frac{1}{\lambda}$
- If $X\tilde{}\exp\left(\lambda\right)$ and $y=x^{\theta}$ then y has *Frechet* Distribution (Eaton kortum 2002).

A convenient parameterization: Facts

1. If $X\tilde{}\exp(\lambda)$ and $y = kx$ then $y\tilde{}\exp\left(\frac{\lambda}{k}\right)$
2. If X and Y are independent, $X\tilde{}\exp(\lambda)$ and $Y\tilde{}\exp(\mu)$ then $Z = \min(X, Y)\tilde{}\exp(\lambda + \mu)$
3. If X and Y are independent, $X\tilde{}\exp(\lambda)$ and $Y\tilde{}\exp(\mu)$ then

$$\Pr(X < Y) = \frac{\lambda}{\lambda + \mu}$$

A convenient parameterization

- Apply to DFS
- Let the joint distribution of (x_H, x_F) be described by independent exponentials T_H, T_F.

$$\begin{aligned}\mu\left(B_H(\omega)\right) &= \Pr\left(\omega x_H \leq x_F\right) \\ &= \frac{T_H/\omega}{T_H/\omega + T_F}\end{aligned}$$

- Imposing into the equilibrium condtions:

$$\begin{aligned}\omega &= \frac{\mu\left(B_H(\omega)\right)}{1-\mu\left(B_H(\omega)\right)}\frac{L_F}{L_H} \\ &= \frac{T_H/\omega}{T_F}\frac{L_F}{L_H}\end{aligned}$$

Extension (Eaton Kortum 2002 Framework)

- Suppose $Z = X^{-1/\theta}$. Then Z has Frechet distribution.
- Suppose productivities are distributed with this Frechet distribution with parameter θ and T_i; i.e.

$$F_{Z_i}(z) = \Pr(Z_i \leq z) = \exp\left[-T_i z^{-\theta}\right]$$

Extension (Eaton Kortum 2002 Framework)

- $B_H(w_H, w_F) = \left\{(z_H, z_F) \in R^2_{++} : \frac{w_H}{z_H} \leq \frac{w_F}{z_F}\right\}$
- It can easily be shown that

$$\mu\left(B_H(w_H, w_F)\right) = \frac{T_H w_H^{-\theta}}{T_H w_H^{-\theta} + T_F w_F^{-\theta}} = \frac{T_H \omega^{-\theta}}{T_H \omega^{-\theta} + T_H}$$

- Thus:

$$\begin{aligned} \omega &= \frac{\mu\left(B_H(\omega)\right)}{1 - \mu\left(B_H(\omega)\right)} \frac{L_F}{L_H} \\ &= \frac{T_H \omega^{-\theta}}{T_F} \frac{L_F}{L_H} \end{aligned}$$

$$\omega = \left(\frac{T_H}{T_F} \frac{L_F}{L_H}\right)^{\frac{1}{1+\theta}}$$

Extension (Eaton Kortum 2002 Framework)

- Extending to N countries $1, 2, .., N$
- The probability that country i is the lowest cost supplier to country n

$$\mu\left(B_i\left(w_1, .., w_N\right)\right) \equiv \Pr\left[\frac{w_i d_{ni}}{z_i} \leq \min_{k \neq i}\left\{\frac{w_k d_{nk}}{z_k}\right\}\right] = \frac{T_i\left(w_i d_{ni}\right)^{-\theta}}{\sum_{k=1}^{N} T_k\left(w_k d_{nk}\right)^{-\theta}}$$

- Define $\Phi_n = \sum_{k=1}^{N} T_k\left(w_k d_{nk}\right)^{-\theta}$; therefore:

$$\frac{X_{ni}}{X_n} = \frac{T_i\left(w_i d_{ni}\right)^{-\theta}}{\Phi_n}$$