Armington Model

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Armington Model: Home bias
Consumers’ desire for goods from different countries
A homogenous good model
Easy to calibrate
Model

- $N$ countries.
- Each Country: $y_i$ units of unique output: exogenously
  - Simply as endowment
- Preferences: CES over countries’ goods, $\sigma > 1$

$$U_n = \left( \sum_{i=1}^{N} \frac{\alpha_i^{1/\sigma} y_{ni}^{(\sigma-1)/\sigma}}{\sigma-1} \right)^{\sigma/(\sigma-1)}$$

- Iceberg trade costs $d_{ni}$
resource constraint for each good $i$

$$y_i = \sum_{n=1}^{N} d_{ni} y_{ni}$$

Due to bilateral resistance, the law of one price will not hold, i.e. the price of good $i$ will differ across markets $n$.

$p_{ni}$ price of good $i$ in country $n$

$$p_{ni} = d_{ni} p_{ii}$$
country $i$’s total income

\[ Y_i = \sum_{n=1}^{N} p_{ni} y_{ni} \]

Budget constraint in $n$

\[ X_n = \sum_{i=1}^{N} p_{ni} y_{ni} \]
Equilibrium

- Consider a competitive Equilibrium:
  - Given prices, each country $i$ sells its endowment so as to maximize its income $Y_i$ subject to the resource constraint.
  - Given income $Y_l$ and prices, the representative consumer in each country $n$ allocates spending across goods $i$ so as to maximize utility $U_n$ subject to its budget constraint.
  - No deficit:
    \[ D_n = X_n - Y_n = 0 \]
Consumers:

\[ X_{ni} = \alpha_i \left( \frac{p_{ni}}{P_n} \right)^{-\sigma} X_n \]

Where

\[ P_n = \left( \sum_{k=1}^{N} \alpha_k \left( p_{nk} \right)^{-\sigma} \right)^{-1/\sigma} \]
Therefore:

\[
\frac{X_{ni}}{X_n} = \frac{\alpha_i \left(p_{ii} d_{ni}\right)^{-(\sigma-1)}}{\sum_{k=1}^{N} \alpha_k \left(p_{kk} d_{nk}\right)^{-(\sigma-1)}}
\]
Country $i$’s trade share in $n$ is its contribution to the sum
$$\sum_{k=1}^{N} \alpha_k \left( p_{kk} d_{nk} \right)^{-(\sigma-1)}.$$ 

Its contribution reflects
- Its importance in preferences $i$
- The local price of its goods $p_{ii}$
- The cost of getting the goods from $i$ to $n$; as determined by $d_{ni}$.

Elasticity of substitution $\sigma$ governs the sensitivity of trade shares to trade costs.
If $\sigma < 1$: Contradicts with stylized facts.

So $\sigma > 1$
Equilibrium conditions:

\[ p_{ii}y_{ii} = \sum_{n=1}^{N} p_{ni}y_{ni} = Y_i \]

This equation states that country \( i \)'s income is simply the value of its endowment at local prices or, equivalently, its sales around the world.
Since $X_{ni} = p_{ni}y_{ni}$ ⇒

$$Y_i = \sum_{n=1}^{N} \frac{\alpha_i (p_{ii}d_{ni})^{-(\sigma-1)}}{\sum_{k=1}^{N} \alpha_k (p_{kk}d_{nk})^{-(\sigma-1)}} X_n$$

if $D_n = 0$ ⇒

$$p_{ii}y_i = \sum_{n=1}^{N} \frac{\alpha_i (p_{ii}d_{ni})^{-(\sigma-1)}}{\sum_{k=1}^{N} \alpha_k (p_{kk}d_{nk})^{-(\sigma-1)}} p_{nn}y_n$$
Define

\[ s_i = \alpha_i^{1/(\sigma-1)} y_i \]

Therefore:

\[ Y_i = \sum_{n=1}^{N} \frac{s_i^{\sigma-1} (Y_i d_{ni})^{-(\sigma-1)}}{\sum_{k=1}^{N} s_k^{\sigma-1} (Y_k d_{nk})^{-(\sigma-1)}} Y_n \]

Numerically, we can solve for \( Y_n \) given \( s_n, d_{ni} \)

Take good \( N \) to be the numeraire: \( p_{NN} = 1, Y_N = y_N \)
\[ U_n = \frac{p_{nn} y_n}{P_n} = s_n \left( \frac{X_{nn}}{X_n} \right)^{-1/(\sigma - 1)} \]
Welfare: Case 1

- Case 1: $d_{ni} = 1$ for all $n, i \Rightarrow$

\[
p_{nn} = \left( \frac{\alpha_n / y_n}{\alpha_N / y_N} \right)^{1/\sigma}
\]

- Welfare:

\[
U_n = s_n \left[ \sum_{k=1}^{N} \left( \frac{s_k}{s_n} \right)^{(\sigma-1)/\sigma} \right]^{1/(\sigma-1)}
\]

- Country $n$’s welfare is increasing in its own size and in the size of its trading partners.

- Is it better to have more similar partners or less big ones?
Case 2: \( d_{ni} = d \) and \( s_n = s \) \( \Rightarrow \)

\[
U = \frac{Y}{P} = s \left[ 1 + (N - 1) d^{-(\sigma - 1)} \right]^{1/(\sigma - 1)}
\]

Welfare is decreasing in the trade cost
The Armington framework provides an excellent tool to focus purely on the role of trade costs without having to model the forces that shape specialization.

Given that much of the policy interest in trade concerns exactly issues of industrial structure, we turn to theoretical frameworks (Ricardian, DFS, EK,..) in which trade has nontrivial implications for who makes what.
Gravity Results

- We had \( Y_i = \sum_{n=1}^{N} \frac{\alpha_i (p_{ii} d_{ni})^{-(\sigma-1)}}{\sum_{k=1}^{N} \alpha_k (p_{kk} d_{nk})^{-(\sigma-1)}} X_n \implies \)

\[
Y_i = \alpha_i p_{ii}^{-(\sigma-1)} \sum_{n=1}^{N} \left( \frac{d_{ni}}{P_n} \right)^{-(\sigma-1)} X_n

= \alpha_i p_{ii}^{-(\sigma-1)} \Xi_i
\]

where \( \Xi_i = \sum_{n=1}^{N} \left( \frac{d_{ni}}{P_n} \right)^{-(\sigma-1)} X_n \)

- So

\[
X_{ni} = \frac{X_n Y_i}{\Xi_i} \left( \frac{d_{ni}}{P_n} \right)^{-(\sigma-1)}
\]