

In the Name of God
Sharif University of Technology
Graduate School of Management and Economics
Macroeconomics 2 - 2024
Problem Set 4

1 Lucas 1988

In this problem, we want to replicate the Lucas's 1988 paper (the human capital endogenous growth part). Consider the Lucas 1988 economy with endogenous human capital growth in continuous time.

1. Setup the problem for the social planner case.
2. Write down the FOCs.
3. Following Lucas's method, solve for the growth rates of capital, output, consumption and human capital. You can first take u as given and solve everything in terms of the growth rate of human capital and then solve for the optimal u .
4. How does u depend on the exogenous parameters. Explain the intuitions.
5. How do growth rates of human capital and output depend on the exogenous parameters of the model. Explain the intuitions.
6. Explain why γ shows up in the growth rates.
7. Explain how A does not show up in the growth rates. Any intuition?
8. Explain the role of δ and how it affects the growth rate.

2 Lucas 1988: A variation

In this problem, we want to setup the Lucas's 1988 paper (the human capital endogenous growth part) in discrete time.

Consider the Lucas 1988 economy with endogenous human capital growth in discrete time where $h_{t+1} = h_t(1 + \delta(1 - u_t))$ and $c_t + k_{t+1} = y_t = Ak_t^\alpha (u_t N_t h_t)^{1-\alpha}$ and $W = \sum \beta^t \frac{c^{1-\sigma} - 1}{1-\sigma}$

1. Setup the problem for the competitive equilibrium case.
2. Write down the FOCs. Use λ_t, μ_t in as the Lagrange Multipliers.
3. Eliminate λ_t, μ_t from the FOCs and then explain the intuition of the two equations. One of them is the Euler equation and the other is a similar one for human capital accumulation and hours of work. Explain how does a household decide about one more hour of work.

3 Lucas 1988: With Taxes

1. Consider the Lucas 1988 economy with endogenous human capital growth and focus on the competitive equilibrium case. Suppose the government impose the following taxes: τ_c consumption tax, τ_k capital tax (on the rental capital), and τ_l labor income tax on wages. Then it rebates all the taxes to households as lump-sum subsidies.
 - (a) Write down the household budget constraint.
 - (b) Write down the model FOCs.
 - (c) First, suppose the all taxes are zero. Solve for the growth rates of consumption, capital, output and human capital. Also solve for the interest rates and the fraction of time that HH works. What is the fraction of total consumption to output?
 - (d) Now, under the following scenarios: Solve for the BGP's equilibrium endogenous growth rate. Then solve for the relationship between the levels (K and H). Explain how government can affect the growth and levels through these taxes.
 - i. Suppose only consumption tax is non-zero.
 - ii. Suppose only labor income tax is non-zero.
 - iii. Suppose only capital tax is non-zero.
 - (e) (Optional) Can you design a tax system (τ_c, τ_l, τ_k) to achieve the first best allocation (Social planner). Note that negative taxes mean subsidies.