Real Business Cycle Model (RBC)

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Sharif University of Technology

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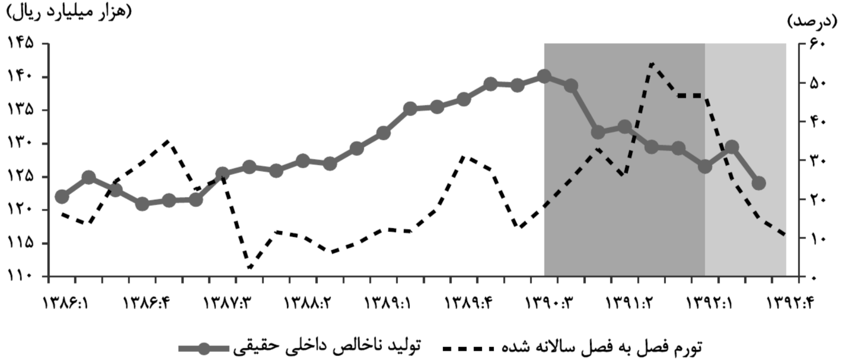
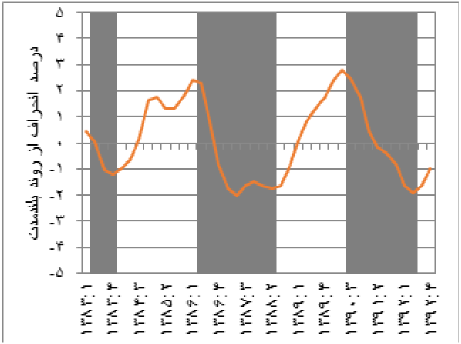
Business Cycles



شکل ۳.۱.۱. تورم و تولید ناخالص داخلی حقیقی

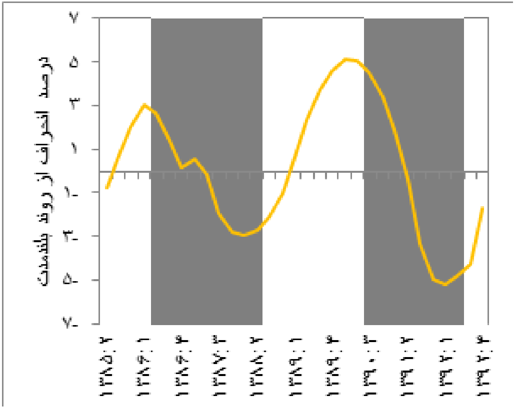
Business Cycles

GDP

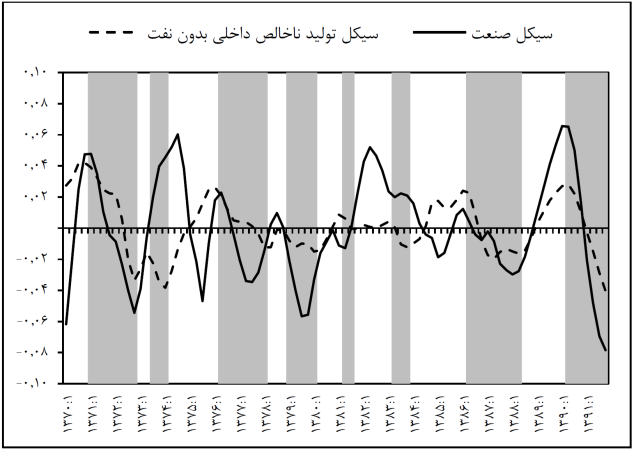


Business Cycles

Industrial Production

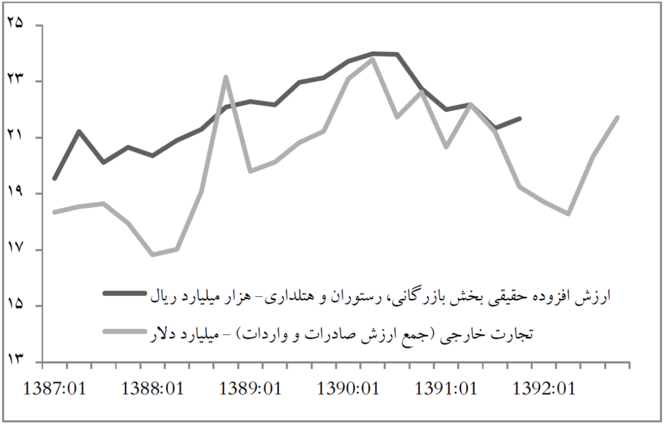


GDP and Industrial Production



شکل ۵.۲.۳. چرخه صنعت در مقایسه با چرخه تولید ناخالص داخلی بدون نفت

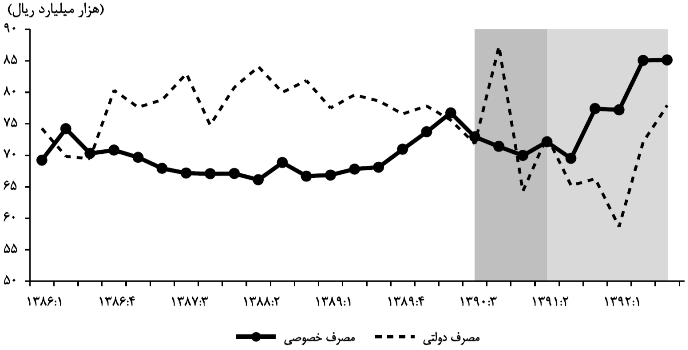
Service Sector Production



شکل ۳.۲.۸. مقایسه ارزش افزوده بخش بازرگانی و تجارت خارجی

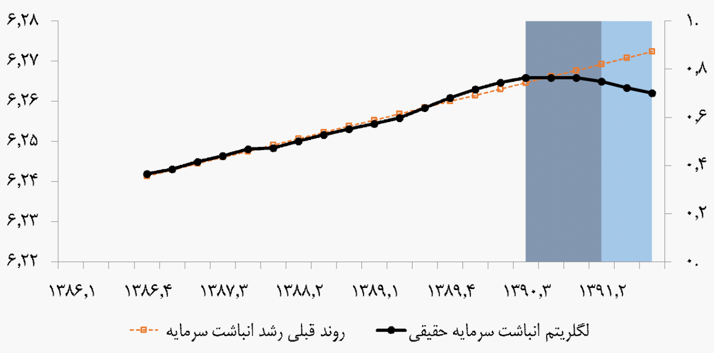
Business Cycles

Consumption

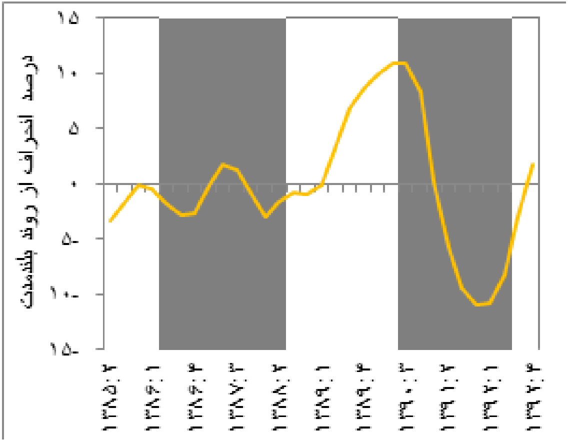


Business Cycles

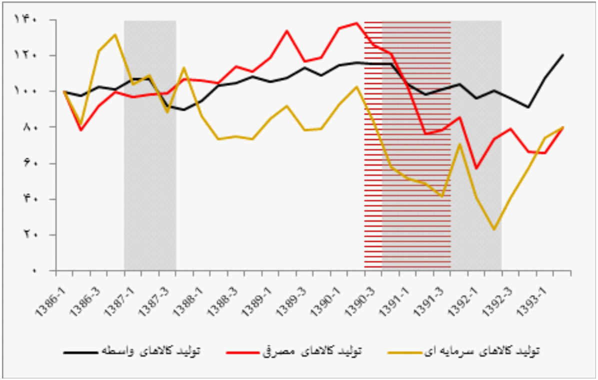
Capital formation (Investment)



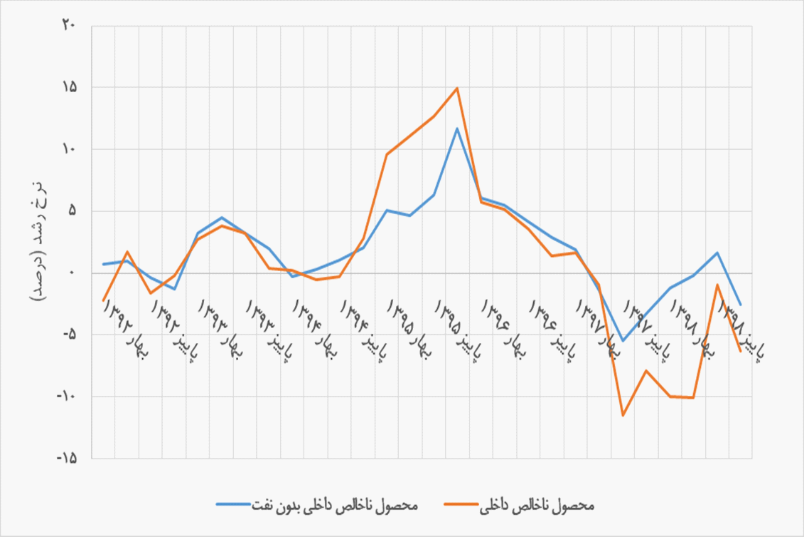
Capital formation (Investment)

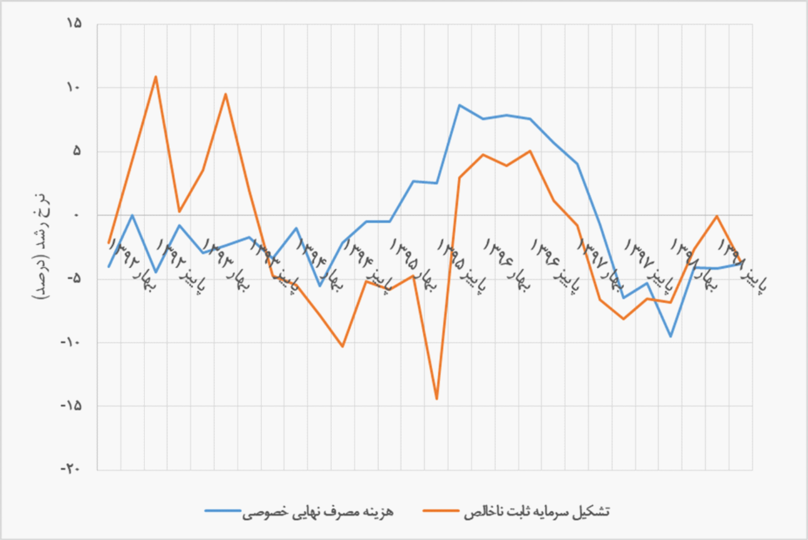


Business Cycles



Business Cycles





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-۲۰
نرخ رشد (درصد)
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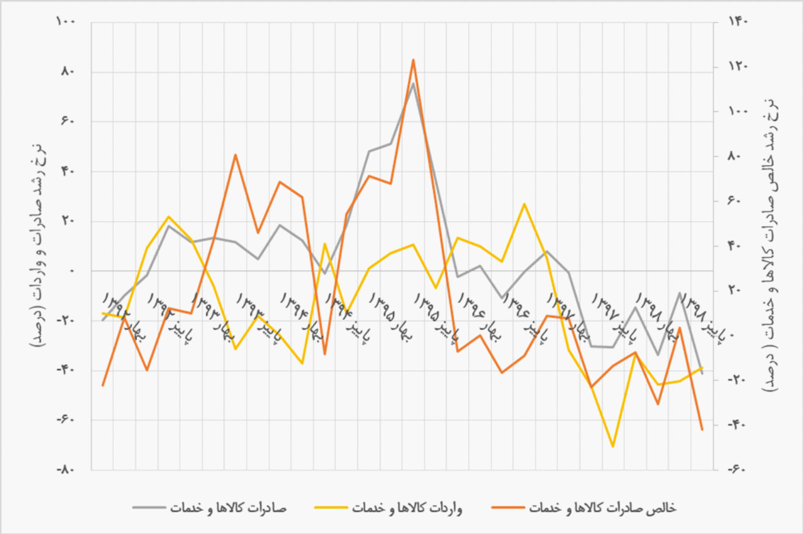
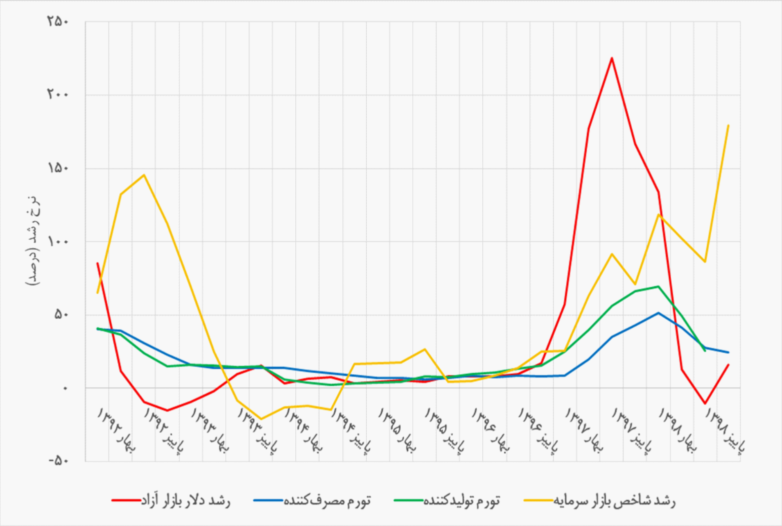
پاییز ۱۳۹۲
بهار ۱۳۹۳
پاییز ۱۳۹۳
بهار ۱۳۹۴
پاییز ۱۳۹۴
بهار ۱۳۹۵
پاییز ۱۳۹۵
بهار ۱۳۹۶
پاییز ۱۳۹۶
بهار ۱۳۹۷
پاییز ۱۳۹۷
بهار ۱۳۹۸
پاییز ۱۳۹۸
هزینه مصرف نهایی خصوصی
تشکیل سرمایه ثابت ناخالص

Business Cycles



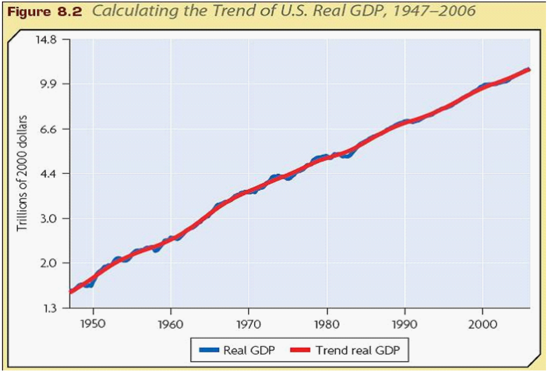
Business Cycles



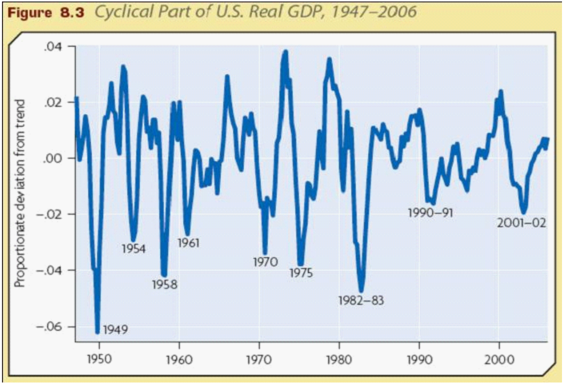
Business Cycles

نرخ رشد	۱۳۹۷	۱۳۹۸
سطح عمومی قیمتها	۲۶,۹	۳۴,۸
تولید ناخالص داخلی بدون نفت	– ۱,۳	– ۰,۶
تولید ناخالص داخلی	– ۴,۰	– ۷,۰
مصرف خصوصی	– ۲,۲	– ۵,۴
سرمایه گذاری	– ۵,۶	– ۳,۴
واردات کل	– ۲۸,۴	– ۳۹,۸
صادرات کل	– ۱۱,۴	– ۲۴,۵S

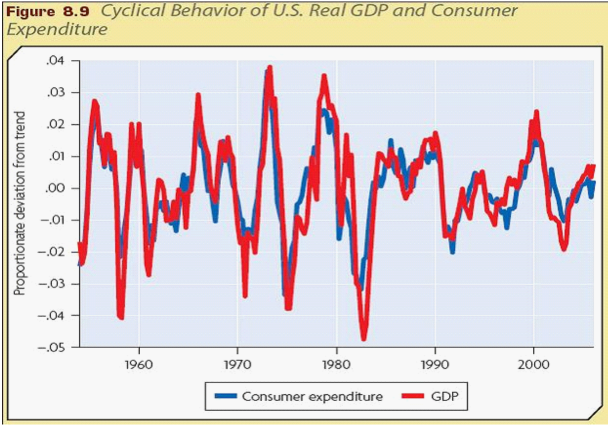
Business Cycles



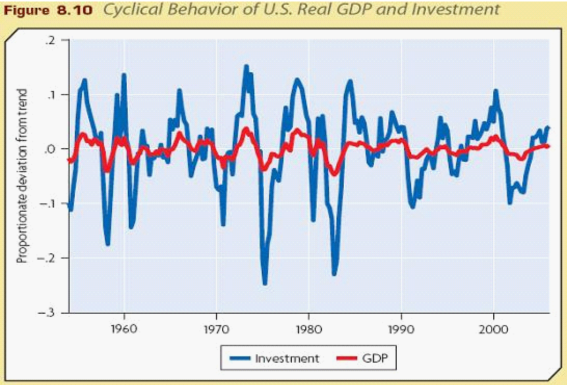
Business Cycles



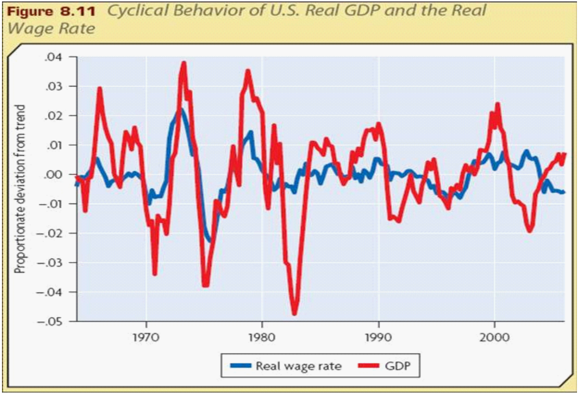
Business Cycles



Business Cycles

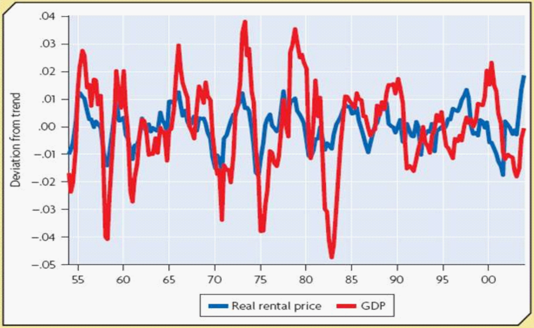


Business Cycles

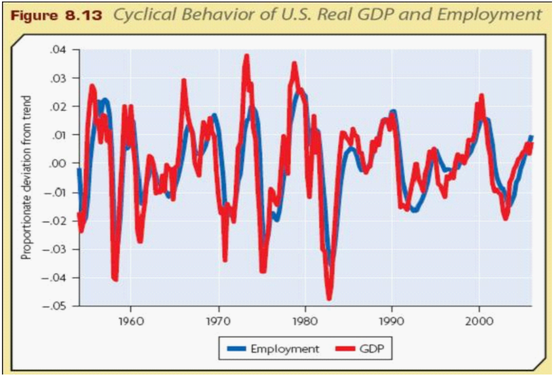


Business Cycles

Figure 8.12 *Cyclical Behavior of U.S. Real GDP and the Real Rental Price of Capital*



Business Cycles



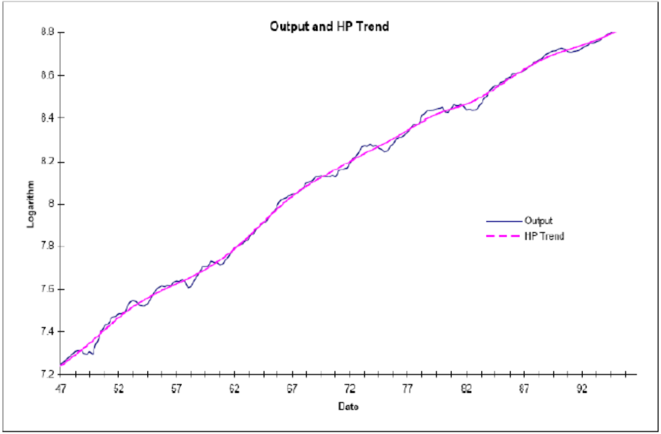
Measuring the Business Cycles

- Hodrick-Prescott (H-P) Filter

$$\min_{\{y_t^g\}_0^\infty} \sum_{t=0}^{\infty} \left\{ (y_t - y_t^g)^2 + \lambda \left[(y_{t+1} - y_t^g) - \left(y_t - y_{t-1}^g \right) \right]^2 \right\}$$

- H-P filter suppresses the really low frequency fluctuations $\simeq$ 8 years
- quarterly data $\lambda = 1600$

Measuring the Business Cycles



	US data		Baseline RBC model	
	Std. Dev (%)	$\rho_{x,GNP}$	Std. Dev (%)	$\rho_{x,GNP}$
GNP	1.72	1.0	1.35	1.0
Consumption (NDS)	0.86	0.77	0.329	0.843
Investment (GPDI)	8.24	0.91	5.954	0.992
Hours	1.59	0.86	0.769	0.986
Productivity	-	-	0.606	0.978

Introduction

- Lucas 1980: “One of the functions of theoretical economics is to provide fully articulated, artificial economic systems that can serve as laboratories in which policies that would be prohibitively expensive to experiment with in actual economies can be tested out at much lower cost. [...] Our task as I see it [...] is to write a FORTRAN program that will accept specific economic policy rules as ‘input’ and will generate as ‘output’ statistics describing the operating characteristics of time series we care about, which are predicted to result from these policies.”

Introduction

- Economists have long been puzzled by the observations that during peacetime industrial market economies display recurrent, large fluctuations in output and employment over relatively short time periods.
- These observations are considered puzzling because the associated movements in labor's marginal product are small.

Introduction

- Prescott:
  - For the United States, in fact, given people's ability and willingness to intertemporally and intratemporally substitute consumption and leisure and given the nature of the changing production possibility set, it would be puzzling if the economy did not display these large fluctuations in output and employment with little associated fluctuations in the marginal product of labor.
  - Moreover, standard theory also correctly predicts the amplitude of these fluctuations, their serial correlation properties, and the fact that the investment component of output is about six times as volatile as the consumption component.
  - This perhaps surprising conclusion is the principal finding of a research program initiated by Kydland and me (1982) and extended by Kydland and me (1984), Hansen (1985a), and Bain (1985).

Introduction

- Economic theory implies that, given the nature of the shocks to technology and people's willingness and ability to intertemporally and intratemporally substitute, the economy will display fluctuations like those the U.S. economy displays.

RBC Model

- A Microfounded general equilibrium macroeconomic model (Proposed by Kydland and Prescott (1982))
  - Explains the short run fluctuations of macroeconomic variables (Business cycle phenomena)
  - Consistent with long run facts: a unique model of growth and business cycles
  - A Dynamic Stochastic General Equilibrium (DSGE) model with rational expectations

- A benchmark RBC model
  - Households:

$$\max_{c_t, k_{t+1}, i_t, h_t} E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, 1-h_t\right)$$

subject to

$$\begin{aligned} c_t + i_t &\leq w_t h_t + v_t k_t + \pi_t \\ k_{t+1} &\leq \left(1-\delta\right) k_t + i_t \\ k_t &\geq 0 \\ k_0 &: \quad \text{Given} \end{aligned}$$

- We assume that the consumer is making all time-t choices (i_t, c_t, k_{t+1}, h_t) conditional on time t information (all variables subscripted t and below).

RBC Model

- Firms

$$\max_{K_t, H_t} A_t F(K_t, H_t) - w_t H_t - v_t K_t$$

- Define:

$$z_t = \log A_t$$

and it follows an AR(1) process:

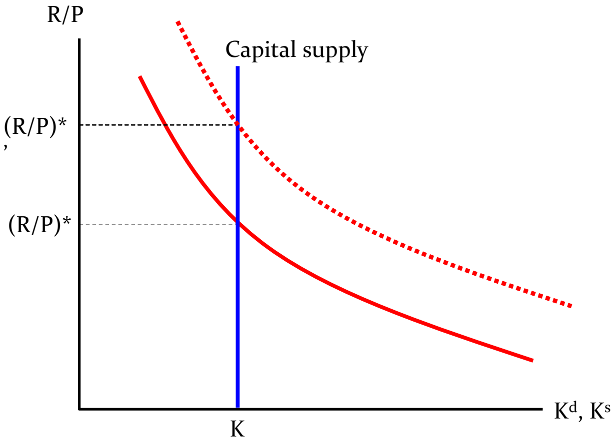
$$z_t = \theta z_{t-1} + \varepsilon_t$$

where ε_t is a mean zero i.i.d random process with variance σ^2

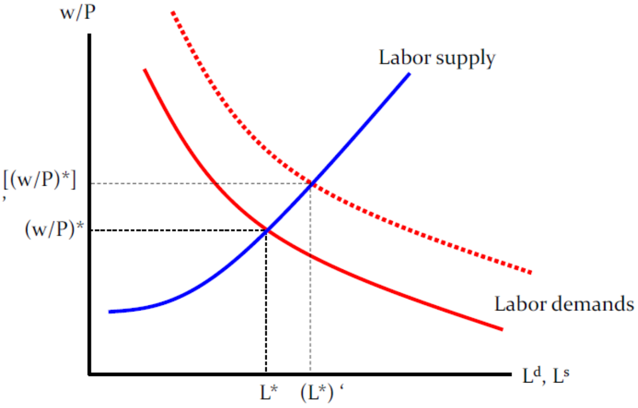
- Equilibrium:
  - An equilibrium in this economy is a joint distribution of prices and allocations where:

$$Y_t = C_t + I_t$$

Effect of an Increase in Technology: Capital Market

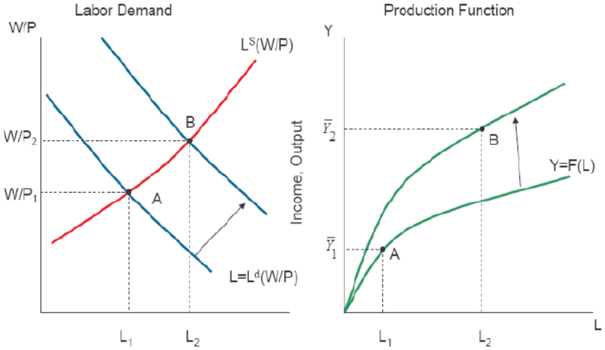


Effect of an Increase in Technology: Labor Market



Transmission Mechanism

- How does a shock transmit over time?



Transmission Mechanism

- Compare
  - A permanent shock
  - One time shock
  - A persistent shock
- Effects
  - Spot
  - Persistency

Steps to Solve Dyanmic Models

1. FOCs
2. Solve for Steady States
3. (Log-Linearize)
4. Solve for the recursive law of motion
5. Calibration and Estimation
    1. Calculate the moments: ratios, correlations, and standard deviations for the different variables both for the artificial economy and for the actual economy
    2. Calibrate or Estimate
6. Evaluatation
    1. Compare how well the model economy matches the actual economy's characteristics
    2. Calculate the IRFs in response to different shocks

Solving the Full Model

- FOC: HH

$$
\begin{aligned}
[c_t] &: \beta^t u_c(c_t, 1-h_t) = \lambda_t \\
[h_t] &: \beta^t u_l(c_t, 1-h_t) = e^{z_t} F_{h,t} \lambda_t \\
[k_{t+1}] &: \lambda_t = E_t \left[\lambda_{t+1} \left(1-\delta + e^{z_{t+1}} F_{k,t+1}\right)\right]
\end{aligned}
$$

- FOC: Firm

$$
\begin{aligned}
w_t &= e^{z_t} F_{h,t} \\
v_t &= e^{z_t} F_{k,t}
\end{aligned}
$$

- Equilibrium Condition: Labor and capital markets clear.

Solving the Full Model

- Consumption Leisure decision (Interpretation!)

$$
\begin{aligned}
u_l(c_t, 1-h_t) &= u_c(c_t, 1-h_t) w_t \\
\text{where } w_t &= e^{z_t} F_{h,t}
\end{aligned}
$$

- Euler Equation

$$
\begin{aligned}
u_{c,t} &= \beta E_t \left[u_{c,t+1} (1-\delta+v_{t+1}) \right] \\
\text{where } v_t &= e^{z_t} F_{k,t}
\end{aligned}
$$

- Resource constraint:

$$
c_t + k_{t+1} \leq e^{z_t} F(k_t, h_t) + (1-\delta) k_t
$$

- RBC model does not have any distortion or market imperfection, therefore the welfare theorems apply to these models:
  1) the competitive equilibrium is pareto-optimal
  2) a pareto-optimal allocation can be decentralized as a competitive equilibrium
- The social planner equilibrium and the competitive equilibrium are identical and admit a unique solution
- So we can instead solve the planner problem instead.

Solving the RBC Model: The planner problem

- RBC model in the planner form:

$$\max_{C_t, K_{t+1}, H_t} E_0 \sum_{t=0}^{\infty} u\left(C_t, 1 - H_t\right)$$

subject to

$$\begin{aligned} C_t + K_{t+1} &\leq e^{z_t} F\left(K_t, H_t\right) + \left(1 - \delta\right) K_t \\ z_t &= \theta z_{t-1} + \varepsilon_t \end{aligned}$$

Solving the Full Model

- FOC

$$
\begin{aligned}
[c_t] &: \quad \beta^t u_c(c_t, 1-h_t) = \lambda_t \\
[h_t] &: \quad \beta^t u_l(c_t, 1-h_t) = e^{z_t} F_{h,t} \lambda_t \\
[k_{t+1}] &: \quad \lambda_t = E_t \left[\lambda_{t+1} \left(1 - \delta + e^{z_{t+1}} F_{k,t+1} \right) \right]
\end{aligned}
$$

Solving the Full Model

- Consumption Leisure decision (Interpretation!)

$$u_l(c_t, 1-h_t) = u_c(c_t, 1-h_t)e^{z_t}F_{h,t}$$

(remember: $w_t = e^{z_t}F_{h,t}$)

- Euler Equation

$$u_{c,t} = \beta E_t\left[u_{c,t+1}(e^{z_{t+1}}F_{k,t+1} + 1 - \delta)\right]$$

(remember: $v_t = e^{z_t}F_{k,t}$)

- Resource constraint:

$$c_t + k_{t+1} \leq e^{z_t}F(k_t, h_t) + (1-\delta)\,k_t$$

Example

- Sample Utility Functions

$$
\begin{aligned}
U(c_t, h_t) &= \frac{\left(c_t^{1-\xi}(1-h_t)^{\xi}\right)^{1-\gamma} - 1}{1-\chi} \\
U(c_t, h_t) &= (1-\xi)\log(c_t) + \xi \log(1-h_t) \\
U(c_t, h_t) &= \frac{c_t^{1-\gamma} - 1}{1-\gamma} - \xi \frac{h_t^{1+\phi}}{1+\phi}
\end{aligned}
$$

- Sample Production functions

$$
\begin{aligned}
F(K, H) &= K^{\alpha} H^{1-\alpha} \\
F(K, H) &= \left(\alpha K^{1-1/\eta} + (1-\alpha) H^{1-1/\eta}\right)^{\frac{\eta}{\eta-1}}
\end{aligned}
$$

Numerical Solution

- Numerical Methods to solve the model:
  - Bellman's equation, and apply numerical dynamic programming methods.
  - Linear-quadratic approximation around the steady states
  - Log-linearize the model around the steady state

Log Linearization

- For $x \sim 0$:
$$e^x \approx 1 + x$$
- For x_t , let $\hat{x}_t = \log\left(\frac{x_t}{\bar{x}}\right)$ be the log-deviation of x_t from its steady state. Thus, $100 * \hat{x}_t$ is (approximately) the percent deviation of x_t from $\bar{x}$. Then,
$$x_t = \bar{x} e^{\hat{x}_t} \approx \bar{x}\left(1 + \hat{x}_t\right)$$
- Formally: first order Taylor expansion,

$$\begin{aligned}
g\left(x_t\right) &= g\left(\bar{x} e^{\hat{x}_t}\right) \\
g_t &= g\left(\bar{x} + \bar{x}\hat{x}_t\right) \\
g\left(\bar{x}\right)\left(1 + \hat{g}_t\right) &\approx g_t \approx g\left(\bar{x}\right)\left(1 + \frac{g'\left(\bar{x}\right)\bar{x}}{g\left(\bar{x}\right)}\hat{x}_t\right) \\
\hat{g}_t &\approx \frac{g'\left(\bar{x}\right)\bar{x}}{g\left(\bar{x}\right)}\hat{x}_t = \frac{g'\bar{x}}{g}\hat{x}_t
\end{aligned}$$

Solving the Full Model: An example

- Take: $U\left(c_t, h_t\right) = \frac{c_t^{1-\gamma}-1}{1-\gamma} - \xi \frac{h_t^{1+\phi}}{1+\phi}$ and $F\left(K, H\right) = K^{\alpha} H^{1-\alpha}$
- Consumption Leisure decision

$$\xi H_t^{\phi} = C_t^{-\gamma} e^{z_t} K_t^{\alpha} H_t^{-\alpha}$$

(remember: $w_t = e^{z_t} F_{h,t}$)

- Euler Equation

$$C_t^{-\gamma} = \beta E_t \left[C_{t+1}^{-\gamma} \left(e^{z_{t+1}} K_{t+1}^{\alpha-1} H_{t+1}^{1-\alpha} + 1 - \delta \right) \right]$$

(remember: $v_t = e^{z_t} F_{k,t}$)

- Resource constraint:

$$C_t + K_{t+1} \leq e^{z_t} F\left(K_t, H_t\right) + \left(1-\delta\right) K_t$$

Solution: Recursive Law of Motion

- We guess a decision rule

$$\begin{aligned}\hat{k}_{t+1} &= \gamma_1 \hat{k}_t + \gamma_2 z_t \\ \hat{c}_t &= \eta_1 \hat{k}_t + \eta_2 z_t\end{aligned}$$

- Then verify by substituting into the FOCs.

Example (1)

- HH problem with no capital and no labor supply decision

$$\max \sum_{t=0}^{\infty} \beta^t \ln(c_t)$$

$$\text{s.t. } c_t + b_{t+1} = y_t + (1 + R_t) b_t$$

- Intuition of how the EE is working
- How Permanent Income Hypothesis (PIH) is in place

1. $R_t = \bar{R}, \ y_t = \theta y_{t-1} + \varepsilon_t$
2. $R_t = \theta R_{t-1} + \varepsilon_t, \ y_t = \bar{y}$

Example (2)

- RBC with no capital and no labor supply decision

$$\max \sum_{t=0}^{\infty} \beta^t \ln(c_t)$$

$$\text{s.t. } c_t + b_{t+1} = w_t l_t + \pi_t + (1 + R_t) b_t$$

$$\begin{aligned} y_t &= e^{z_t} l_t^{1-\alpha} \\ z_t &= \theta z_{t-1} + \varepsilon_t \end{aligned}$$

$$\pi_t = y_t - w_t l_t$$

- Solve the GE problem: R_t becomes endogenous versus an exogenous variable for the HH.

Example (3)

- RBC with labor supply decision but no capital

$$\max \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma} - 1}{1-\sigma} - \xi \frac{l^{1+\phi}}{1+\phi} \right)$$

$$\text{s.t. } c_t + b_{t+1} = w_t l_t + \pi_t + (1 + R_t)\, b_t$$

$$\begin{aligned} y_t &= e^{z_t} l_t^{1-\alpha} \\ z_t &= \theta z_{t-1} + \varepsilon_t \end{aligned}$$

$$\pi_t = y_t - w_t l_t$$

- Intuition of how the EE is working

Example (4)

- A dynamic Household problem with capital but no labor supply decision

$$\max \sum_{t=0}^{\infty} \beta^t \ln (c_t)$$

$$\text{s.t. } c_t + k_{t+1} = w_t l_t + v_t k_t + (1-\delta) k_t$$

- Assume $w_t = w$, $l_t = l$
- Take $v_t = \varepsilon_t$, pure iid white noise shock
- Take $v_t = \theta v_{t-1} + \varepsilon_t$, an AR(1) shock

Example (5)

- RBC with capital but no labor supply decision

$$\max \sum_{t=0}^{\infty} \beta^t \ln\left(c_t\right)$$

$$\begin{aligned} \text{s.t. } c_t + k_{t+1} &= e^{z_t} k_t^{\alpha} + (1-\delta) k_t \\ z_t &= \theta z_{t-1} + \varepsilon_t \end{aligned}$$

- FOC:

$$\frac{1}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} R_{t+1} \right]$$

where $R_t = \alpha e^{z_t} k_t^{\alpha-1} + (1-\delta)$

- How the Golden Rule comes into place
- Discuss the Spot and Persistency Effects

Example (5)

- Show how persistence of a shock can affect R_t and then consumer's decision
- Show graphically how a shock affect the capital market and rate of return.
  - No persistence
  - Full persistence
  - Mild persistence

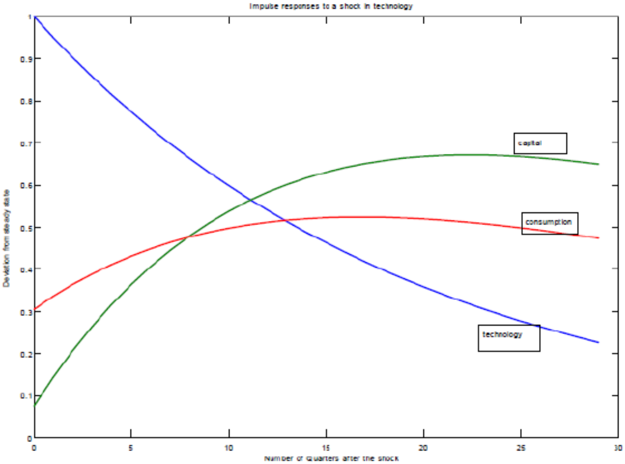
Example (5)

- Take $\delta = 1$
- Solve using the Golden rule of the log-linearized equations

$$\begin{aligned} \hat{k}_{t+1} &= \alpha \hat{k}_t + \left(\frac{1}{\theta \alpha \beta} - 1 \right) z_t \\ \hat{c}_t &= \alpha \hat{k}_t + \frac{1 + \alpha\beta - \frac{1}{\theta}}{1 - \alpha\beta} z_t \end{aligned}$$

- Intuition for the role of θ
- Propagation Mechanism
- Spot and Persistency Effects
- k_t is an AR(2) process! Economic Structural effects on the Propagation Mechanism
- Find unconditional variances

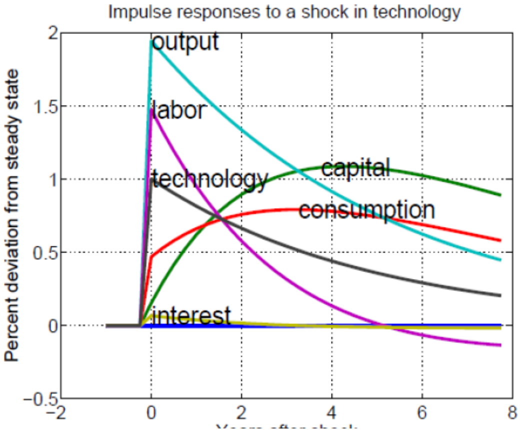
Example (5)



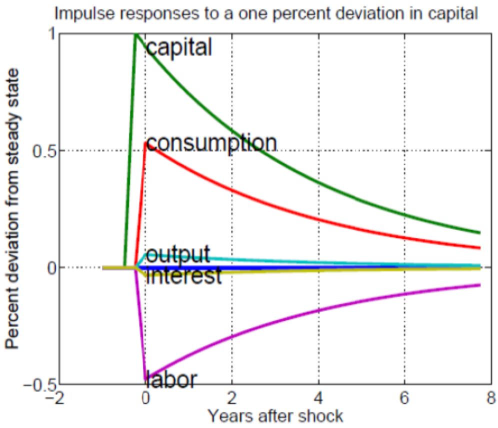
Impulse responses to a shock in technology
capital
consumption
technology
Deviation from steady state
Number of quarters after the shock
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
5
10
15
20
25
30

Hansen RBC model

- Technology Shock!
  - Hansen RBC model



- Capital stock Shock!



More Examples

- See Sargent paper
- DSGE user guide
- Uhlig's lectures
- Dr Tavakkolian's Book on DSGE and Dynare

Calibration

- β :At the non-stochastic steady state, we have $R = \frac{1}{\beta}$. The average real interest rate in the U.S. is usually around 4% annually which is about 1% quarterly
  - $\beta = 0.99$
- α : $1-\alpha$ will be labor's share of output, a quantity that can be estimated from the national income accounts
  - $\alpha = 0.4$
- χ :Estimates from micro studies of the typical worker's intertemporal elasticity of substitution are in the range of $\chi \simeq 1$
  - $\chi = 1$

$$u(c, 1-h) = (1-\xi)\ln c + \xi \ln(1-h)$$

Calibration

- α : By solving for the steady states we find that:

$$\frac{\alpha}{1-h} = (1-\alpha)\,(1-\alpha)\,\frac{y}{ch}$$

    - From long run data, 31% of available time is spent working$\Rightarrow$ $\bar{h} = 0.31$
    - The steady state output to consumption ratio is about 1.33 $\left(\frac{y}{c}\right)$
    - $\Rightarrow \alpha = 0.64$

- Cooley and Prescott estimate that depreciation is 4.8% annually, so 1.2% quarterly ($\delta = 0.012$).
- ν : Use quarterly population growth rate
    - $\nu = 0.012$

Calibration

- θ and σ_ε :This model has perfect competition and constant returns to scale.
  - So $z_t - z_{t-1}$ is the Solow residual.
  - The average value of the Solow residual gives us our estimate for γ. Cooley and Prescott set $\gamma = 0.0156$, giving about 1.6% annual TFP growth.
  - Once we subtract out this average, we can estimate an AR(1) model
  - $\theta = 0.95$ and $\sigma_\varepsilon = 0.007$

Estimation

- Byesian Estimation is an alternative method to calibration.
- We assume a prior distribution for our parameters.
- We simulate the model and try to match the models outcome with the actual outcome from data
- We find a posterior distribution for the parameters

Simulation and the Test

- Simulation:
  - Now we can simulate the model on a computer and we get time series for output, employment, productivity, investment, consumption, and capital.
- Test
  - We look at the moments of real and simulated data

Revisiting Calibration and Estimation

- Parameters estimation
  - Matching with the moments of data
  - Matching with the data

	US data		Baseline RBC model	
	Std. Dev (%)	$\rho_{x,GNP}$	Std. Dev (%)	$\rho_{x,GNP}$
GNP	1.72	1.0	1.35	1.0
Consumption (NDS)	0.86	0.77	0.329	0.843
Investment (GPDI)	8.24	0.91	5.954	0.992
Hours	1.59	0.86	0.769	0.986
Productivity	-	-	0.606	0.978

Evaluation

- To an RBC theorist, these numbers represent success.
- We've managed to write down a very simple model that duplicates many of the properties (moments) of the actual data.
- There are few failures though.
- The RBC approach to this failing is to investigate why the model doesn't match, and adjust the model so that it does match.

Issues

- Understate the variability of both consumption and hours
  - The consumption variability is simple. Even with careful measurement, a lot of "consumption" is actually purchase of consumer durables, which really belongs in investment
- In order to generate higher variation in hours worked for each individual worker, we need to make them more willing to substitute intertemporally - work less when wages are low and more when they are high.
  - micro studies show a low IES, so we can't justify simply lowering χ
  - Introduce Unemployment (Gary Hansen 1985)

Issues

- Persistence of fluctuations
  - However, their persistence really isn't much more than that of the Solow residual, which is the exogenous source of shocks.
  - The problem is that new investment is very small relative to the capital stock, so the capital stock itself varies little.
  - So new mechanisms for propagation:
    - Financial markets frictions
    - Labor market search

Critics

- Why matching moments is a desire property? There could be many other alternative
- If solow residual are the sources of shocks, so recessions are results of technical regress.
- It is not clear what particular technological advances or disasters can be associated with any of the major short-term swings in the Solow residual.

RBC Model Implications: a Revolution

- Main policy conclusion: Fluctuations of all variable (output, consumption, employment, investment...) are the optimal responses to technology shocks exogenous changes in the economic environment.
- Shocks are not always desirable. But once they occur, this is the best possible outcome: business cycle fluctuations are the optimal response to technology shocks => no need for government interventions: it can be only deleterious
- Financial sector has no role in determining the bysiness cycles (Money has no role)

RBC Model Implications: a Revolution

- The policy implication of this research is that costly efforts at stabilization are likely to be counterproductive.
- Economic fluctuations are optimal responses to uncertainty in the rate of technological change.
- However, this does not imply that the amount of technological change is optimal or invariant to policy.
- If policies adopted to stabilize the economy reduce the average rate of technological change, then stabilization policy is costly.
- To summarize, attention should be focused not on fluctuations in output but rather on determinants of the average rate of technological advance.

RBC Revolution

- Furious response from "people from the Oceans"
- From mid'80s to mid'90s: ten years lost in useless ideological debates between the Oceans and the Lakes
- From mid'90s: convergence on methodology: "the RBC approach as the new orthodoxy in macroeconomics"

Some Intuitions

- Relative labour supply responds to relative wages between two different periods $=>$ households substitute labour intertemporally
- Also the interest rate matters for labour supply $=> \uparrow$ r $=> \uparrow h^s$ today, because MPK is high $=>$ crucial channel for employment fluctuations
- What is the effect of $\uparrow$ w or $\uparrow$ r?

Some Intuitions

- temporary $\uparrow w \Rightarrow$ substitution effect prevails $\uparrow h^s \Rightarrow \downarrow \left(\frac{c_t}{w_t}\right)$ (given the intratemporal trade-off between consumption and labour: $\frac{u_l(c_t,1-h_t)}{u_c(c_t,1-h_t)} = w_t$
- permanent $\uparrow w =>$ income and substitution effects cancel out, no change in h_t^s and $\left(\frac{c_t}{w_t}\right)$
- Temporary increase in both w and r $=>$ intertemporal substitution both in labour and consumption $=> \uparrow\uparrow h_t^s$

Some Intuitions

- The standard neoclassical intratemporal trade-off between consumption and labour

$$\frac{u_l(c_t, 1 - h_t)}{u_c(c_t, 1 - h_t)} = w_t$$

  hence, for a given wage, C and H tend to move in the opposite direction
- How one can get both C and H highly pro-cyclical?
- Highly procyclical real wage (=> productivity shocks!!)

Key Readings:

- Kydland and Prescott, Econometrica 1982, "Time to build and aggregate fluctuations" .
- Prescott 1986 "Theory ahead of business cycle measurement"

- A simple 2-period example to show the transmission mechanism ($\delta = 1$)

$$\max U(c_0) + \beta U(c_1)$$

$$\begin{aligned} \text{s.t.} \quad c_0 + k_1 &= A_0 k_0^{\alpha} \\ c_1 &= A_1 k_1^{\alpha} \end{aligned}$$

A few simple examples (7)

- A simple 2-period example to show how does uncertainty work
  - Without Capital:

$$\max U(c_0) + \beta U(c_1)$$

$$\begin{aligned} \text{s.t.} \quad c_0 + b_1 &= y_1 \\ c_1 &= y_2 + (1+r)\, b_1 \end{aligned}$$

$y_2 = \bar{y}_2 + \Delta y$ with prob. $\frac{1}{2}$ and $y_2 = \bar{y}_2 - \Delta y$ with prob $\frac{1}{2}$

  - With Capital and $\delta = 1$:

$$\max U(c_0) + \beta U(c_1)$$

$$\begin{aligned} \text{s.t.} \quad c_0 + k_1 &= A_1 k_0^{\alpha} \\ c_1 &= A_2 k_1^{a} \end{aligned}$$

A few simple examples (8)

- A 2-period model with labor supply:
  - $$U = \log c_t - \frac{h_t^{1+\phi}}{1+\phi}$$

  becomes

  $$h_t = \left(\frac{w_t}{c_t}\right)^{\frac{1}{\phi}}$$

  - So the elasticity of labor supply w.r.t. real wages $= \frac{1}{\phi}$:Frisch elsticity
  - Show intertemporal labor substitutions

A few simple examples (5)

- RBC with no labor, log utility, $\delta = 1$:

$$\max_{c_t, k_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \log c_t$$

subject to

$$\begin{aligned} c_t + k_{t+1} &\leq A_t F(k_t) \\ \log A_t &= \theta \log A_{t-1} + \varepsilon_t \end{aligned}$$

- Analytical solution
- Log-linearized solution

A few simple examples (5)

- RBC model with no labor supply:

$$\max_{c_t, k_{t+1}} E_0 \sum_{t=0}^{\infty} u\left(c_t\right)$$

subject to

$$\begin{aligned} c_t + k_{t+1} &\leq A_t k_t^{\alpha} + (1-\delta)\, k_t \\ \log A_t &= \theta \log A_{t-1} + \varepsilon_t \end{aligned}$$

Example (5)

- If $\delta = 1$: Guess:

$$\begin{aligned} k_{t+1} &= \Pi e^{z_t} k_t^{\alpha} \\ c_t &= \Gamma e^{z_t} k_t^{\alpha} \end{aligned}$$

- Then:

$$\begin{aligned} \Pi &= \alpha\beta \\ \Gamma &= 1 - \alpha\beta \end{aligned}$$

- Intuitions!

Example (1)

- HH problem with no capital and no labor supply decision

$$\max \sum_{t=0}^{\infty} \beta^t \ln (c_t)$$

$$\text{s.t. } c_t + b_{t+1} = y_t + (1 + R_t)\, b_t$$

- Intuition of how the EE is working
- How Permanent Income Hypothesis (PIH) is in place:

$$R_t = \theta R_{t-1} + \varepsilon_t, y_t = \bar{y}$$

Example (1)

- HH problem with no capital and no labor supply decision

$$\max \sum_{t=0}^{\infty} \beta^t \ln(c_t)$$

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- Intuition of how the EE is working
- How Permanent Income Hypothesis (PIH) is in place:

$$R_t = \bar{R}, y_t = \theta y_{t-1} + \varepsilon_t$$

Example (1)

$$\begin{aligned}
\hat{c}_t &= E_t\left[\hat{c}_{t+1}\right] - \frac{1}{\sigma} E\left[R_{t+1}\right] = E_t\left[\hat{c}_{t+1}\right] \\
\bar{c}\hat{c}_t + \bar{b}\hat{b}_{t+1} &= \bar{y}y_t + \left(1+\bar{R}\right)\bar{b}\hat{b}_t \\
\bar{c} &= \bar{y} + \bar{R}\bar{b} \\
\bar{c}_t &= \hat{y}_t + \left(1+\bar{R}\right)\gamma\hat{b}_t - \gamma\hat{b}_{t+1} \\
\hat{y}_t + \left(1+\bar{R}\right)\gamma\hat{b}_t - \gamma\hat{b}_{t+1} &= E_t\left[\hat{y}_{t+1} + \left(1+\bar{R}\right)\gamma\hat{b}_{t+1} - \gamma\hat{b}_{t+2}\right]
\end{aligned}$$

$$\left(1+\bar{R}\right)\gamma\hat{b}_t - \left(2+\bar{R}\right)\gamma\hat{b}_{t+1} + \gamma E_t\left[\hat{b}_{t+2}\right] + \left(1-\theta\right)\hat{y}_t = 0$$

Example (1)

$$\left(1+\bar{R}\right)\gamma\hat{b}_t-\left(2+\bar{R}\right)\gamma\hat{b}_{t+1}+\gamma E_t\left[\hat{b}_{t+2}\right]+\left(1-\theta\right)\hat{y}_t=0$$

Assume:

$$\hat{b}_{t+1}=\xi\hat{b}_t+\eta\hat{y}_t$$

$$\left(1+\bar{R}\right)\gamma\hat{b}_t-\left(2+\bar{R}\right)\gamma\left(\xi\hat{b}_t+\eta\hat{y}_t\right)+\gamma E_t\left[\left(\xi\hat{b}_{t+1}+\eta\hat{y}_{t+1}\right)\right]+\left(1-\theta\right)\hat{y}_t=$$

$$\left(1+\bar{R}\right)\gamma\hat{b}_t-\left(2+\bar{R}\right)\gamma\left(\xi\hat{b}_t+\eta\hat{y}_t\right)+\gamma E_t\left[\left(\xi\left(\xi\hat{b}_t+\eta\hat{y}_t\right)+\eta\hat{y}_{t+1}\right)\right]+\left(1-\right.$$

$$\left(1+\bar{R}\right)\gamma\hat{b}_t-\left(2+\bar{R}\right)\gamma\left(\xi\hat{b}_t+\eta\hat{y}_t\right)+\gamma\left(\xi^2\hat{b}_t+\xi\eta\hat{y}_t+\theta\eta\hat{y}_t\right)+\left(1-\theta\right)\hat{y}_t=$$

$$\left(\left(1+\bar{R}\right)\gamma-\left(2+\bar{R}\right)\gamma\xi+\gamma\xi^2\right)\hat{b}_t+\left(\gamma\eta\left(\xi+\theta\right)-\left(2+\bar{R}\right)\gamma\eta+\left(1-\theta\right)\right)\hat{y}_t$$

Example (1)

$$\left(\left(1+\bar{R}\right)+\left(2+\bar{R}\right)\xi+\xi^2\right)\gamma\hat{b}_t+\left(\gamma\eta\left(\xi+\theta\right)-\left(2+\bar{R}\right)\gamma\eta+\left(1-\theta\right)\right)\hat{y}_t=$$

$$\begin{aligned}\left(1+\bar{R}\right)+\left(2+\bar{R}\right)\xi+\xi^2 &= 0\\ \left(\gamma\left(\xi+\theta\right)-\left(2+\bar{R}\right)\gamma\right)\eta+\left(1-\theta\right) &= 0\end{aligned}$$