In the name of GOD

Fiscal Policy

c. Public debt

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- Optimal taxation

Ricardian equivalence

- Government: Two-period model:

$$G_1 = T_1 + B_1$$

$$G_2 = T_2 - (1+r)B_1$$

$$G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r}$$

Ricardian equivalence

- Government:

$$G_t = T_t + B_t - (1 + r)B_{t-1}$$

Transversality condition → $\lim_{s\to\infty} \left(\frac{1}{1+r}\right)^{s-1} B_s = 0$

$$\sum_{t=1}^{\infty} \frac{G_t}{(1+r)^{t-1}} = \sum_{t=1}^{\infty} \frac{T_t}{(1+r)^{t-1}}$$

Ricardian equivalence

- Households: Two-period model

$$\max_{c} u(c_1) + \beta u(c_2)$$

$$s.t. \qquad c_1 + B_1 = Y_1 - T_1$$

$$c_2 = Y_2 - T_2 + (1+r)B_1$$

$$c_1 + \frac{c_2}{1+r} = Y_1 + \frac{Y_2}{1+r} - \left(T_1 + \frac{T_2}{1+r}\right)$$

Ricardian equivalence

- Households:

$$\max_{c} \sum_{t=1}^{\infty} \beta^{t-1} U(C_t)$$

$$s.t. \qquad Y_t + (1+r)B_{t-1} = C_t + T_t + B_t$$

$$\sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^{t-1} (Y_t - T_t) = \sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^{t-1} C_t$$

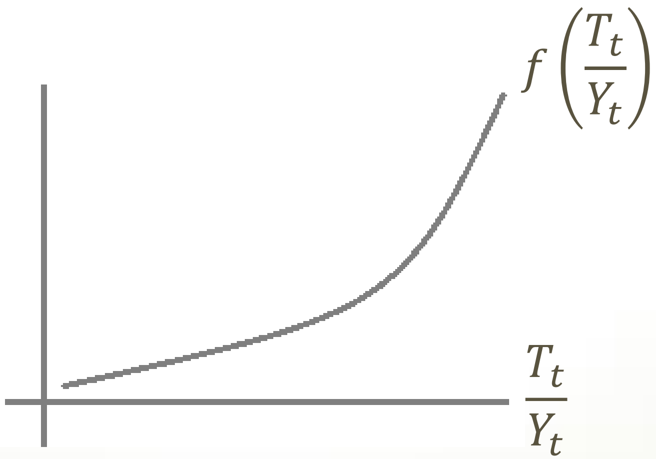
Ricardian equivalence

- Government: $\sum_{t=1}^{\infty} \frac{G_t}{(1+r)^{t-1}} = \sum_{t=1}^{\infty} \frac{T_t}{(1+r)^{t-1}}$
- Households: $\sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^{t-1} (Y_t - T_t) = \sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^{t-1} C_T$

$$\sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^{t-1} C_T = \sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^{t-1} (Y_t - G_t)$$

Optimal taxation

- $f\left(\frac{T_t}{Y_t}\right) = the\ fraction\ of\ income\ that\ the\ economy\ loses\ because\ of\ distortions$
- $f' > 0 \quad , \quad f'' > 0$
- $total\ loss = Y_t . f\left(\frac{T_t}{Y_t}\right)$



Optimal taxation

- Government:

$$\min_{\{T_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^{t-1} Y_t . f\left(\frac{T_t}{Y_t}\right)$$

$$s.t. \qquad \sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^{t-1} G_t \leq \sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^{t-1} T_t$$

$f'\left(\frac{T_t}{Y_t}\right) = \lambda$ ➡ tax smoothing

Optimal taxation

- Result:

$$\sum \frac{G_t}{(1+r)^t} = \left(\sum \frac{Y_t}{(1+r)^t} \right) \tau^*$$

$$\tau^* = \frac{NPV\{G_t\}}{NPV\{Y_t\}}$$

Optimal taxation

- Microeconomics where price elasticity of goods are the same:
  - $number\ of\ goods = L\ , l = 1, \ldots, L\ \rightarrow \tau_l = \tau^*$
- Macroeconomics where tax is a source of deadweight loss & r is constant over time:
  - $number\ of\ periods = T\ , t = 1, \ldots, T \rightarrow \tau_t = \tau^*$
- Macroeconomics where tax rate is different for consumption and investment:
  - $\tau_{I_t} = \bar{\tau}_I\ \&\ \tau_{C_t} = \bar{\tau}_C$

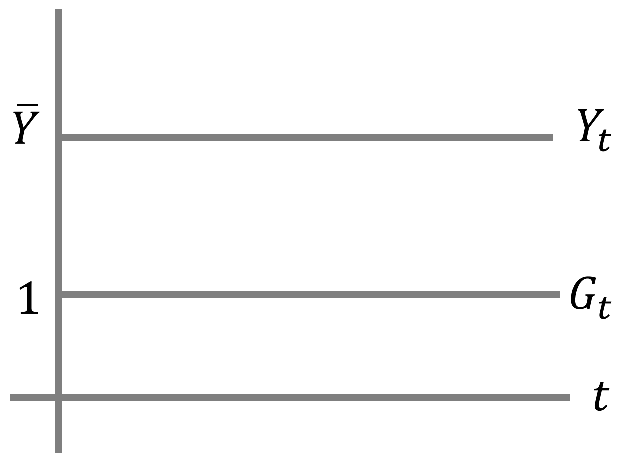
Optimal taxation: example 1

- $Y_t = \bar{Y}, \quad \forall t$
- $G_t = 1 < \bar{Y}, \quad \forall t$
- $\left(\frac{T_t}{Y_t}\right) = constant \quad \& \quad Y_t = \bar{Y} \quad \rightarrow \quad T_t = \bar{T}$
- $\sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^{t-1} G_t = \sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^{t-1} T_t$

$$1.\sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^{t-1} = \bar{T}.\sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^{t-1}$$

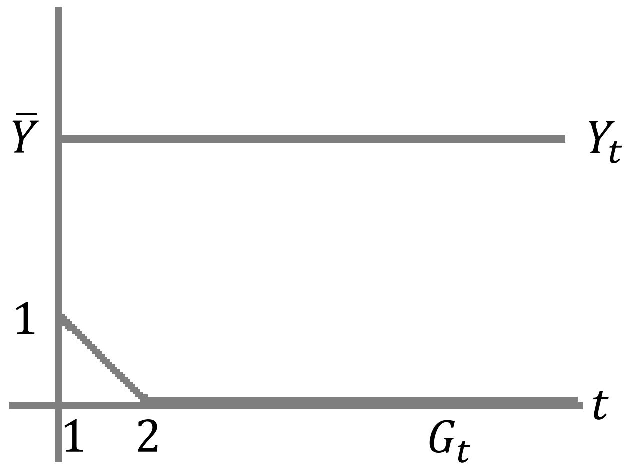
$$\bar{T}=1 \qquad \& \qquad B_t = 0$$

Optimal taxation: example 1



Optimal taxation: example 2

- $Y_t = \bar{Y}, \qquad \forall t$
- $G_1 = 1, G_2 = G_3 = \cdots = 0$



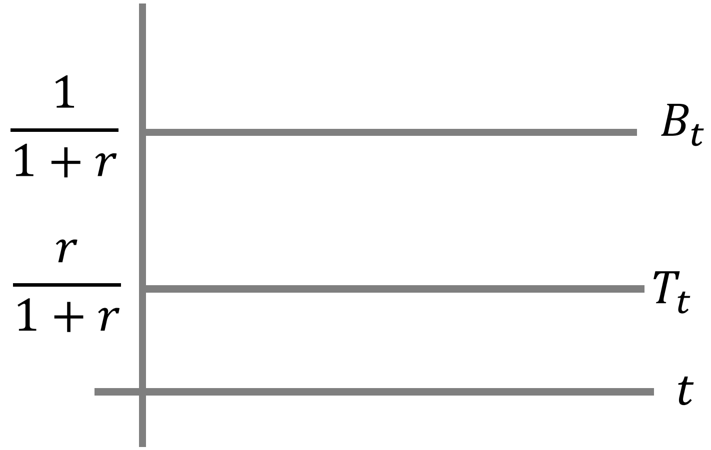
Optimal taxation: example 2

- $\left(\frac{T_t}{Y_t}\right) = constant \quad \& \quad Y_t = \bar{Y} \quad \rightarrow \quad T_t = \bar{T}$
- $\sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^{t-1} G_t = \sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^{t-1} T_t$

$$1 = \bar{T}.\sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^{t-1}$$

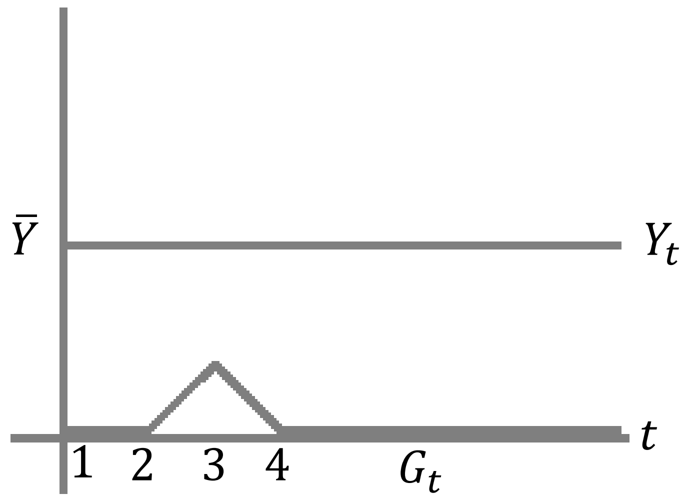
$$\bar{T} = \frac{r}{1+r} \qquad \& \qquad B_t = \frac{1}{1+r}$$

Optimal taxation: example 2



Optimal taxation: example 3

- $Y_t = \bar{Y}, \qquad \forall t$
- $G_1 = G_2 = 0, G_3 = 1, G_4 = G_5 = \cdots = 0$



Optimal taxation: example 3

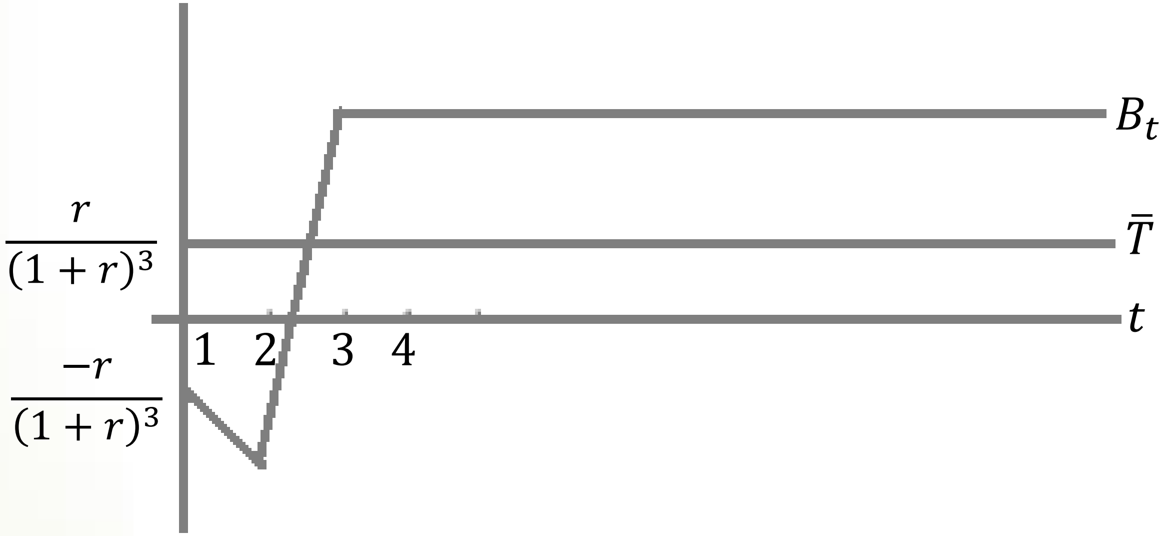
- $\left(\frac{T_t}{Y_t}\right) = constant \quad \& \quad Y_t = \bar{Y} \quad \rightarrow \quad T_t = \bar{T}$
- $\sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^{t-1} G_t = \sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^{t-1} T_t$

$$\frac{1}{(1+r)^2}.G_3 = \bar{T}.\frac{1+r}{r}$$

$$\bar{T} = \frac{r}{(1+r)^3}$$

$$B_1 = \frac{-r}{(1+r)^3} \quad \& \quad B_2 = \frac{-r(r+2)}{(1+r)^3} \quad \& \quad B_3 = B_4 = B_5 = \cdots = \frac{1}{(1+r)^3}$$

Optimal taxation: example 3



Supplemental topics

- Optimal tax scheme: $t_k = 0$, $t_l = constant$
- W(G,T)
- W(beta)

Conclusion

- Proportional taxes create distortions and reduce the source on which they are levied.
- Lump-sum taxes are not distortionary.
- Who you tax also matters: the results are sensitive to the choice of the utility function.
- Increasing taxes is not a popular policy option for an elected government. Debt (and seignorage) are more popular.