Money in Utility

- Introduction
Money in Utility

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Money in Utility

- To use the general equilibrium framework to analyze monetary issues, a role for money should be specified so that agents are willing to hold a positive quantity of money.
- Here, we assume that money yields direct utility by incorporating real money balances into the utility function (Sidrauski 1967).
  - We can think of it as saving the individual's time from barter.
- Money does not earn interest.
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- A basic MIU model:

\[
\max_{\{c_t,l_t,m_t,b_t\}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t, m_t)
\]

subject to

\[P_t c_t + B_t + P_t k_{t+1} + M_t = W_t l_t + V_t k_t + (1 - \delta) P_t k_t + (1 + i_t) B_t\]

where \(m_t = \frac{M_t}{P_t}\) is the real money holding.
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- In real terms
  \[ c_t + \frac{B_t}{P_t} + k_{t+1} + \frac{M_t}{P_t} = \frac{W_t}{P_t} l_t + \frac{V_t}{P_t} k_t + (1 - \delta) k_t + (1 + i_t) \frac{B_{t-1}}{P_{t-1}} \frac{P_t}{P} \]

- Redefining variables
  \[ c_t + b_t + k_{t+1} + m_t = w_t l_t + (v_t + 1 - \delta) k_t + \frac{1 + i_t}{1 + \pi_t} b_{t-1} + \frac{1}{1 + \pi_t} \]
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- FOCs:

  
  
  \[ \begin{align*}
  [c_t] & : \quad \beta^t U_{ct} = \lambda_t \\
  [l_t] & : \quad \beta^t U_{lt} = w_t \lambda_t \\
  [b_t] & : \quad \lambda_t = E_t \left[ \frac{1 + i_{t+1}}{1 + \pi_{t+1}} \lambda_{t+1} \right] \\
  [k_{t+1}] & : \quad \lambda_t = E_t \left[ \lambda_{t+1} (v_{t+1} + 1 - \delta) \right] \\
  [m_t] & : \quad U_{mt} + \beta E_t \left[ \frac{1}{1 + \pi_{t+1}} \lambda_{t+1} \right] = \lambda_t
  \end{align*} \]

  

  - mC and mB analysis.
  - In Equilibrium, return on bond, money (and capital) should be equalized to prevent arbitrage.
  - Why people holds money?
  - Neither \( M \) nor \( P \) matter but their ratio \( m \). \( \Rightarrow \) Money Neutrality
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- Euler Equation

\[ U_{ct} = \beta E_t \left[ \frac{1 + i_{t+1}}{1 + \pi_{t+1}} U_{ct+1} \right] \]

- Note that these terms depend on \( m \) (Real money balances)
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\[ U_{ct} = U_{mt} + \beta E_t \left[ \frac{U_{ct+1}}{1 + \pi_{t+1}} \right] \]

**Interpretation**
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- Asset pricing Equation of money

\[ \frac{\lambda_t}{P_t} \]

Unit value of C I could purchase with my 1$

\[ = \frac{U_{mt}}{P_t} + \beta E_t \left[ \frac{1}{P_{t+1}} \lambda_{t+1} \right] \]

Utility value of money balances I buy instead

Discounted utility value of consumption if I spend the 1$ next period

- Value of Stock today = It’s dividend payment today + It’s expected future value.
Using stock price analogy, we get (one out of many stationary solution)

\[ \frac{1}{P_t} = \frac{1}{\lambda_t} E_t \left[ \sum_{i=0}^{\infty} \beta^i \left( \frac{U_{m(t+i)}}{P_{t+i}} \right) \right] \]

It moves like a stock price. But in reality we don’t see such movements in the aggregate price index. Thus we need some price rigidities!
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- Perfect foresight:

\[
1 + r_{t+1} \equiv \frac{\nu_{t+1} + 1 - \delta}{1 + i_{t+1}} = \frac{1 + \pi_{t+1}}{1 + \pi_{t+1}}
\]
Using Euler equation, FOCs for money and bond, we find that:

\[
\frac{U_{mt}}{U_{ct}} = \frac{i_{t+1}}{1 + i_{t+1}}
\]

Interpretation: MRS = relative price, since

\[1 + i_{t+1} = (1 + \pi_{t+1})(1 + r_{t+1})\]

We get money demand equation: As \( i \uparrow, m \downarrow \)

Consumer surplus! and welfare cost of inflation
lost CS due to $i^* > 0$.
How does the money demand curve look like?

\[
\ln(m) = \ln(A) - \eta i
\]

\[
i \rightarrow 0 \Rightarrow \ln(m) \rightarrow \ln(A)
\]

\[
\ln(m) = \ln(B) - \eta \ln(i)
\]

\[
i \rightarrow 0 \Rightarrow \ln(m) \rightarrow \infty
\]
Lucas 1994 used 1900 to 1985 Us data to estimate money demand equation

Ireland 2009 used more recent data and showed semi-log money demand curve has better fit.

Estimates of the welfare cost of inflation varies from 0.85% to 3% of GDP (Gillman 1995)
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- **Equilibrium:**
  - HHs maximizes utility
  - Firms only hire labor optimally.
  - Central Bank supplies money (constant growth rate $\mu$, for example)

\[
M_t^s = (1 + \mu) M_{t-1}^s \\
V_t = \mu M_{t-1}^s
\]

- Goods, labor, bond and money markets clear

\[
B_t = 0 \\
M_t^d = M_t^s
\]
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- Neutrality
  - Changes in level of money doesn’t affect the real terms.

- Super-neutrality: Changes in the growth rate of money supply
  - $\pi \uparrow \Rightarrow i \uparrow \Rightarrow m \downarrow$
  - If separable utilites: No change in real terms $\Rightarrow$ Super Neutral.
    Inflation only induces a welfare cost.
  - Non-Seperable: Not super Neutral
Steady States: Separable case

- $m$ constant $\Rightarrow$ growth of $P_t = \text{growth of } M_t \Rightarrow \pi = \mu$
- We had (Fisher Equation):
  
  $$1 + r_t = \frac{1 + i_t}{1 + \pi_t}$$

- Euler Equation (EE) $\Rightarrow$

  $$1 = \beta \frac{1 + i}{1 + \pi}$$

  $$1 + \rho = \frac{1}{\beta} = \frac{1 + i}{1 + \pi} = 1 + r$$

  $$r = \rho$$

  $\Rightarrow$

  $$i = \pi + r = \mu + \rho$$

- given $\beta$ (or $\rho$), $i$ tracks $\pi$. 
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- Inflation results in welfare loss due to lower money demand.
- Is there an optimal rate of $\pi$ that maximizes the welfare?
  - The private opportunity cost of the private market depends on the nominal interest rate.
  - The social marginal cost of producing money is essentially zero.
  - If $i > 0$ then there is a wedge between private and social cost $\Rightarrow$ inefficiency.
  - So the optimal rate is $i = 0$. 
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- Optimal inflation

\[ i = 0 \Rightarrow \]
\[ \mu = \pi = -\rho < 0 \]

Called "The Friedman Rule" (Friedman 1969, Bailey 1956)
- Some economists do not like this rule!
- Phelps 1973 argues that Friedman rule holds only in environments where gov can raise lump sum taxes.
A Simple Example

\[ U = \log (c) + \alpha \log (1 - l) + \gamma \log (M) \]
Short Run analysis

- Log-linearization
- Dynare
More References

- Coerria, Teles (1996, 1999)