# Trade in Intermediate Goods and The Welfare Gain from International Trade 

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#### Abstract

In this paper, we introduce a general equilibrium model of international trade that takes into account the industry linkages between intermediate goods and consumption goods producers to quantitatively measure the countries welfare gains and losses from different trade policies on intermediate and consumption goods. Our model extends the inter-industry trade model of Ricardo (1817), intra-industry model of Krugman (1980), and firm heterogeneous model of Melitz (2003). Reduction of tariffs affect welfare of countries through direct effects on income, cost of export, and cost of input bundles of production (labor and intermediate goods), and indirectly through changes in term of trades, and endogenous entries and exits in many industries due to the general equilibrium adjustment of the input-output linkages. The model have three main implication on estimating the welfare gain from trade. First, since the share of domestic production in expenditure of an industry can correspond to different shares of domestic production in consumption and intermediate goods expenditure on that industry, taking not into account some differences between intermediate and consumption goods (as it common in the literature) can significantly distort the estimated welfare gains from trade. Second, only the elasticities of substitution of intermediate goods sectors affect the welfare gain from trade. Third, the trade elasticities of consumption goods have much lower effect on welfare gain from trade than trade elasticities of intermediate goods.


Keywords: Gain from Trade, Trade Policy, Intermediate Goods, Input-Output Linkage

## 1 Introduction

How does the trade of intermediate goods affect the welfare gain of countries from international trade? Estimating the welfare gain from trade remained one of the major areas of research in international
economics. As more than half of the world's trade of goods is in intermediate goods, the important question is how to appropriately take into account trade of these goods in the quantitative models of international trade.

We propose a multi-country multi-industry general equilibrium model of international trade that takes into account the difference between intermediate and consumption goods and the linkage between industries throughout the input-output loop. Our model features inter-industry trade, intra-industry trade, and firm heterogeneity. There are two sectors in each industry: the sector that produce consumption goods that are demanded by consumers, and the sector that produce intermediate goods that are demanded by producers in other intermediate and consumption sectors for production. Industries are related to each other in an input-output environment.

Our work stands in the literature of quantitative trade models that estimate the welfare gains from trade. Arkolakis et al. (2012) assess that with single industry gravity models, the welfare gain from trade for United States is around 2 percents. However, it is shown that the significant portion of welfare gain from trade comes from trade in intermediate inputs. Ossa (2015) recognizes the input-output loop in a simple Armington model and finds the median welfare gain from trade of countries around 50 percents. Eaton and Kortum (2002), Alvarez and Lucas (2007), and Caliendo and Parro (2014) also consider trade of intermediate goods in Eaton and Kotum model and find a sizable welfare gain from trade. But, since in this environment the measure of producers in each industry is exogenous, the effects of entries in and out of industries resulting from trade are missed. Thus, same as Balistreri et al. (2011), Di Giovanni and Levchenko (2013), and Costinot and Rodriguez-clare (2014), we take into account trade of intermediate goods in Melitz (2003) environment. ${ }^{1}$

However, in Caliendo and Parro (2014) and Costinot and Rodriguez-clare (2014), it is assumed that all goods can be used both as intermediate and consumption goods. Here, we assume that goods are either used solely as inputs of production or as consumption goods. Taking into account the differences between consumption and intermediate goods in trade elasticities, elasticity of substitution, and share of domestic production can significantly change the estimated welfare gain from trade.

We use the sufficient static approach as Arkolakis et al. (2012) to provide a single formula for welfare gain of countries from trade. We make use of simulation of an artificial economy to assess the main contributions of our model in estimating welfare gain form trade.

[^0]First, we demonstrate that shares of domestic production in consumption goods and intermediate goods together determine the welfare gain from trade. However, if the lower shares of domestic production in intermediate goods is the reason behind the lower shares of domestic production, because of the proliferation effects of intermediate goods in economy, its effect on welfare gain will be more.

Second, it is shown that only elasticities of substitution of intermediate goods appear in the welfare gain formula. The elasticity of substitution of consumption goods does not have any effect on the welfare gain of countries from trade. As the demand of firms determine the elasticity of substitution of intermediate goods and demand of households determinate the elasticity of substitution of intermediate goods, these two statistics are not necessary similar in one industry.

Third, we show that different estimations for trade elasticities of intermediate goods significantly affect the estimated welfare gain from trade, while different estimations for trade elasticities of consumption goods have much lower impact on estimation of the welfare gain.

To evaluate different trade policies on intermediate goods, we solve the equilibrium in relative changes using the exact hat algebra as Dekle et al. (2008). This help us to assess the welfare effects of trade policies without information on parameters like fixed costs that are difficult to estimate empirically. Using only information on trade flows available in multi-countries input-output tables, elasticities of substitution, and trade elasticities of sectors, we are able to evaluate the welfare effects of trade policies on intermediate and consumption goods.

The remainder of paper is organized as follows. In Section 2, we develop the theoretical framework and characterize the equilibrium. Section 3 describes the channels that changes in tariffs affect welfare. Section 4 lays out the welfare gain from trade in the model. In section 5 , we simulate an artificial economy and discuss the welfare implication of the model. Section 6 concludes.

## 2 Model

### 2.1 Households

There are N countries and S industries. In each country j there are $L_{j}$ representative households whom their preferences are defined by:

$$
\begin{equation*}
U_{j}=\prod_{s=1}^{S} C_{j s}^{\mu_{j s}} \tag{1}
\end{equation*}
$$

The budget constraint is $\sum_{s=1}^{S} P_{j s} C_{j s}=R_{j}$, where $R_{j}$ is country j's total income. Assuming trade is balanced between countries (no deficit) and free entry condition (no profit), the only source of income is labor income that is equal to $R_{j}=w_{j} L_{j}$.

### 2.2 Market Structure

In each industry, there are two sectors of intermediate goods and consumption goods. In each industry, $\delta \in\{I, C\}$ shows intermediate and consumptions sectors, respectively. The market structure is monopolistic competition same as in Melitz (2003). Firms in each sector enter as long as the expected profit is higher than the entry sunk cost $w_{i} f_{i s}^{e} .{ }^{2}$ After entry, firms pick their random productivity from a distribution $g_{i s}^{\delta}(\varphi)$. After realization of $\varphi$, firms produce for their domestic if their profit considering fixed cost of production $\left(f_{i i s}^{\delta}\right)$ is positive. To export to country j , firms must pay the fixed cost of $f_{i j s}^{\delta}$ by hiring labor in country j beside the variable cost of export. They decide to export to country j if only their profit from serving that market is positive. We define $\varphi_{i j s}^{*, \delta}$ as the cut-off of productivity of export to country j.

Same as Chaney (2008), we assume that productivities of firms are derived from a Pareto distribution $G_{i s}^{\delta}(\varphi)=1-\left(\frac{b_{i s}^{\delta}}{\varphi}\right)^{\theta_{s}^{\delta}}$. Where $b_{i s}^{\delta}$ is the Pareto location parameter (higher $b_{i s}^{\delta}$ means more productive industry) and $\theta_{s}^{\delta}$ is the Pareto shape parameter (higher $\theta_{s}^{\delta}$ means less variety in productivity of firms).

The ex-post distribution of firms would be:

$$
\begin{equation*}
g_{i s}^{\delta}\left(\varphi \mid \varphi>\varphi_{i j s}^{\delta, *}\right)=\frac{g_{i s}^{\delta}(\varphi)}{1-G_{i s}^{\delta}\left(\varphi_{i j s}^{\delta, *}\right)}=\frac{\theta_{s}^{\delta}}{\varphi}\left(\frac{\varphi_{i j s}^{\delta, *}}{\varphi}\right)^{\theta_{s}^{\delta}} \tag{2}
\end{equation*}
$$

In each country and each industry, two final goods are produced. In each intermediate sector, a final intermediate good is produced from a CES aggregate production function over all available intermediate goods. The same is also true for each consumption sector

$$
\begin{equation*}
Q_{j s}^{\delta}=\left(\sum_{i=1}^{N} \int q_{i j s}^{\delta}(\varphi)^{\frac{\sigma_{s}^{\delta}-1}{\sigma_{s}^{\delta}}} M_{i s}^{\delta} d \varphi_{i s}^{\delta}(\varphi)\right)^{\frac{\sigma_{s}^{\delta}}{\sigma_{s}^{\delta}-1}} \tag{3}
\end{equation*}
$$

where $Q_{j s}^{\delta}$ is the final good produced from the available goods $q_{i j s}^{\delta}$ in sector $(s, \delta)$ that are available in country j , and $M_{i s}^{\delta}$ is measure of producers in each country and each sector. Final consumption goods are only demanded by consumers and final intermediate goods are demanded by producers of consumption and intermediate goods for production. ${ }^{3}$

[^1]
### 2.3 Production

For the production, a firm uses labor and the final intermediate goods from all industries as inputs. Its technology is constant return to scale and its production function is given by:

$$
\begin{equation*}
q_{i s}^{\delta}(\varphi)=\varphi\left(l_{i s}^{\delta}(\varphi)\right)^{\beta_{i s}}\left(\prod_{r=1}^{S}\left(\bar{Q}_{i s, r}^{\delta}(\varphi)\right)^{\eta_{s r}^{i}}\right)^{1-\beta_{i s}} \tag{4}
\end{equation*}
$$

Where $\varphi$ shows the productivity of firm, $\beta$ is share of labor and $\eta_{s k}^{i}\left(1-\beta_{i s}\right)$ is share of final intermediate good of $k$ industry in production function of intermediate goods and consumption goods producers of industry s.

The final consumption goods available in each country and each industry are demanded by consumers and final intermediate goods are demanded by intermediate goods and consumption goods producers as inputs of production. The total expenditures on sector $(s, C)$ and $(s, I)$ in country $j$ are equal to:

$$
\begin{align*}
& E_{i s}^{C}=P_{i s}^{C} Q_{i s}^{C}  \tag{5}\\
& E_{i s}^{I}=\sum_{r=1}^{S} \sum_{\delta \in\{I, C\}}\left[M_{i r}^{e, \delta} \sum_{j=1}^{N} p r\left(\varphi>\varphi_{i j r}^{*, \delta}\right) \mathbb{E}\left(P_{j s}^{\delta} \bar{Q}_{i j r, s}^{\delta}(\varphi) \mid \varphi>\varphi_{i j r}^{*, \delta}\right)\right] .
\end{align*}
$$

Where $\bar{Q}_{i j r, s}^{\delta}(\varphi)$ is the amount of final intermediate good demanded from sector (s,I) by a firm in sector $(r, \delta)$ of country i with productivity of $\varphi$ for production of its export to country j and $P_{i k}^{\delta}$ is the aggregate price index of sector $(s, \delta)$ in country i and is given by

$$
\begin{equation*}
P_{j s}^{\delta}=\left(\sum_{i=1}^{N} \int_{\varphi_{i j s}^{*, \delta}}^{\infty} M_{i s}^{\delta}\left(d_{i j s}^{\delta} \tau_{i j s}^{\delta} p_{i s}^{\delta}(\varphi)\right)^{1-\sigma_{s}^{\delta}} d \varphi_{i s}\right)^{\frac{1}{1-\sigma_{s}^{\delta}}} \tag{6}
\end{equation*}
$$

### 2.4 Equilibrium for given tariffs

Profit maximization of each producer of intermediate and consumption goods yields the pricing rule:

$$
\begin{equation*}
p_{i j s}^{\delta}(\varphi)=\frac{\sigma_{s}^{\delta}}{\sigma_{s}^{\delta}-1}\left(\frac{d_{i j s}^{\delta} \tau_{i j s}^{\delta} \bar{c}_{i s}}{\varphi}\right) \tag{7}
\end{equation*}
$$

Where $d_{i j s}^{\delta}$ is iceberg cost defined as number of goods that must shipped from country $i$ to $j$ for one unit of intermediate good, $\tau_{i j s}^{\delta}$ is tariff, and $\bar{c}_{i s}$ is defined as

$$
\begin{equation*}
\bar{c}_{i s}=\left(\frac{w_{i}}{\beta_{i s}}\right)^{\beta_{i s}}\left(\frac{\prod_{k=1}^{S}\left(\frac{P_{i k}^{I}}{\eta_{s k}}\right)^{\eta_{s k}^{i}}}{1-\beta_{i s}}\right)^{1-\beta_{i s}} \tag{8}
\end{equation*}
$$

Where $\frac{\bar{c}_{i s}}{\varphi}$ is marginal cost of firm with productivity of $\varphi$. By FOC of a firm, the demand of a firm for labor and intermediate final goods for serving each market are:

$$
\begin{align*}
l_{i j s}^{\delta}(\varphi) & =\frac{\beta_{i s}}{w_{i}} \frac{d_{i j s}^{\delta} \tau_{i j s}^{\delta} \bar{c}_{i s}}{\varphi} q_{i j s}^{\delta}(\varphi)  \tag{9}\\
P_{j s}^{I} \bar{Q}_{i j r, s}^{\delta}(\varphi) & =\left(1-\beta_{i s}\right) \eta_{r s}^{i} \frac{d_{i j s}^{\delta} \tau_{i j s}^{\delta} \bar{c}_{i s}}{\varphi} q_{i j s}^{\delta}(\varphi) \tag{10}
\end{align*}
$$

Utility and profit maximizations yield that the revenue of a firm with productivity of $\varphi$ in sector $(s, \delta)$ of country $i$ from serving market of country $j$ is

$$
\begin{equation*}
q_{i j s}^{\delta}(\varphi) p_{i j s}^{\delta}(\varphi)=\left(\frac{p_{i j s}^{\delta}(\varphi)}{P_{j s}^{\delta}}\right)^{1-\sigma_{s}^{\delta}} E_{j s}^{\delta} \tag{11}
\end{equation*}
$$

The share of total expenditure spent on product of a firm is determined by its price advantage in the market it serves. The more is the elasticity of substitution in a sector, the more a firm loses its revenue because of its higher price.

Firms serving market of a country must gain positive profit by entering to that market. Assuming $\mathbb{E}\left(\pi\left(\varphi_{i s}^{*, \delta}\right)\right)=0$, the productivity cut-offs of exports are:

$$
\begin{equation*}
\varphi_{i j s}^{*, \delta}=\left(\frac{\sigma_{s}^{\delta} w_{j} f_{i j s}^{\delta}}{E_{j s}^{\delta}}\right)^{\frac{1}{\sigma_{s}^{\delta}-1}} \frac{\sigma_{s}^{\delta}}{\sigma_{s}^{\delta}-1} \frac{d_{i j s}^{\delta} \tau_{i j s}^{\delta} \bar{c}_{i s}^{\delta}}{P_{j s}^{\delta}} \tag{12}
\end{equation*}
$$

Free entry causes the expected profit of a firm to be equal to the entry sunk cost.

$$
\begin{equation*}
\mathbb{E}\left(\pi_{i s}^{\delta}(\varphi)\right)=\sum_{j=1}^{N}\left[\operatorname{pr}\left(\varphi>\varphi_{i j s}^{*, \delta}\right) E\left(\pi_{i j s}^{\delta}(\varphi) \mid \varphi>\varphi_{i j s}^{*, \delta}\right)\right]=w_{i} f_{i s}^{e} \tag{13}
\end{equation*}
$$

It can be shown that under Pareto distribution we have

$$
\begin{equation*}
\mathbb{E}\left(r_{i j s}^{\delta}(\varphi) \mid \varphi>\varphi_{i j s}^{* \delta}\right)=\frac{\theta_{s}^{\delta} \sigma_{s}^{\delta}}{\sigma_{s}^{\delta}-1} w_{i} f_{i s}^{e} \tag{14}
\end{equation*}
$$

Hence, the free entry condition becomes

$$
\begin{equation*}
\sum_{j=1}^{N}\left(\frac{b_{i s}^{\delta}}{\varphi_{i j s}^{* \delta}}\right)^{\theta_{s}^{\delta}} \frac{\sigma_{s}^{\delta}-1}{1+\theta_{s}^{\delta}-\sigma_{s}^{\delta}} w_{j} f_{i j s}=w_{i} f_{i s}^{e} \tag{15}
\end{equation*}
$$

The expenditure of consumers in each industry is $\mu_{i s} w_{i} L_{i}$ and the expenditure of firms come from substituting Equation (10) in Equation (5) and using Equations (14) and (15):

$$
\begin{equation*}
E_{i s}^{I}=\sum_{r=1}^{S} \sum_{\delta \in\{I, C\}}\left[\left(1-\beta_{i r}\right) \eta_{r s}^{i} \theta_{r}^{\delta} w_{i} M_{i r}^{e, \delta} f_{i r}^{e}\right] \tag{16}
\end{equation*}
$$

Potential producers hire labors from their country to derive their productivity (sunked cost of production). Active firms must also hire labors from their country for production and from countries they serve to pay fixed cost of export. Therefore, labor market in each country clears as

$$
\begin{equation*}
L_{i}=\sum_{s=1}^{S} \sum_{\delta \in\{I, C\}}\left[M_{i s}^{e, \delta}\left(f_{i s}^{e}+\sum_{j=1}^{N} p r\left(\varphi>\varphi_{i j s}^{*, \delta}\right) E\left(l_{i j s}^{\delta}(\varphi) \mid \varphi>\varphi_{i j s}^{*, \delta}\right)\right)+\sum_{j=1}^{N} M_{j i s}^{\delta} f_{j i s}\right] \tag{17}
\end{equation*}
$$

Substituting Equation (9) and using Equations (14) to (16), the labor market clearing condition yields

$$
\begin{equation*}
\alpha_{i} L_{i}=\sum_{s=1}^{S}\left[\left(\frac{1}{\sigma_{s}^{I}}\left(1+\theta_{s}^{I}\right)\left(1+\beta_{i s}^{I}\left(\sigma_{s}^{I}-1\right)\right) M_{i s}^{e, I} f_{i s}^{e}\right)+\left(1+\beta_{i s}^{C} \theta_{i s}^{C}\right) M_{i s}^{e, C} f_{i s}^{e}\right] \tag{18}
\end{equation*}
$$

Where $\alpha_{i}=\sum_{s=1}^{S} \frac{\left(\theta_{s}^{C}+1\right)\left(\sigma_{s}^{C}-1\right)}{\sigma_{s}^{C} \theta_{s}^{C}} \mu_{i s}$ is constant. Imposing Pareto distribution in Equation (6), the price indeces are determined by:

$$
\begin{equation*}
P_{i s}^{\delta}=\gamma_{s}^{\delta}\left(\frac{\sigma_{s}^{\delta} w_{i}}{E_{i s}^{\delta}}\right)^{\frac{\theta_{s}^{\delta}-\left(\sigma_{s}^{\delta}-1\right)}{\theta_{s}^{\delta}\left(\sigma_{s}^{\delta}-1\right)}}\left[\sum_{v=1}^{N}\left(\frac{b_{v s}^{\delta}}{d_{v i s}^{\delta} \tau_{v i s}^{\delta} \bar{c}_{v s}^{\delta}}\right)^{\theta_{s}^{\delta}} M_{v s}^{e, \delta} f_{v i s} \frac{\left(\sigma_{s}^{\delta}-1\right)-\theta_{s}^{\delta}}{\sigma_{s}^{\delta}-1}\right]^{\frac{-1}{\theta_{s}^{\delta}}} \tag{19}
\end{equation*}
$$

Where $\gamma_{s}^{\delta}=\left(\frac{\sigma_{s}^{\delta}}{\sigma_{s}^{\delta}-1}\right)\left(\frac{\theta_{s}^{\delta}-\left(\sigma_{s}^{\delta}-1\right)}{\theta_{s}^{\delta}}\right)^{\frac{1}{\theta_{s}^{\delta}}}$ is constant. Using Equations (14) and (19), we can derive the final free entry condition:

$$
\begin{equation*}
\frac{\sigma_{s}^{\delta} \theta_{s}^{\delta}}{\sigma_{s}^{\delta}-1} f_{i s}^{e} w_{i}=\sum_{j=1}^{N} \frac{f_{i j s}^{\frac{\left(\sigma_{s}^{\delta}-1\right)-\theta_{s}^{\delta}}{\sigma_{s}^{\delta}-1}}\left(\frac{b_{i s}^{\delta}}{d_{i j s}^{\delta} \tau_{i j s}^{\delta} \bar{c}_{i s}}\right)^{\theta_{s}^{\delta}}}{\sum_{m=1}^{N} M_{m s}^{e, \delta} f_{m j s} \frac{\frac{\left(\sigma_{s}^{\delta}-1\right)-\theta_{s}^{\delta}}{\sigma_{s}^{\delta}-1}}{\left(\frac{b_{m s}^{\delta}}{d_{m j s}^{\delta} \tau_{m j s}^{\delta} \bar{c}_{m s}}\right)^{\theta_{s}^{\delta}}}} E_{j s}^{\delta} \tag{20}
\end{equation*}
$$

The equilibrium for given tariffs is $\mathrm{N} w_{i}$ and $\mathrm{NS} M_{i s}^{e, C}, M_{i s}^{e, I}, R_{i s}^{I}, P_{i s}^{I}$, and $\bar{c}_{i s}$ that satisfy system of N equations of Equation (18), 2NS equations of Equation (20), and NS equations of Equations (8), (16) and (19).

### 2.5 Equilibrium for given tariffs changes

Estimating sunk costs and fixed costs used in previous equilibrium are empirically difficult. However, for examining the effects of changes in tariffs we don't need to know all the parameters. We solved the model in relative changes with the method inspired by Dekle et al. (2008). Defining $\hat{X}=\frac{X^{\prime}}{X}$ for all variables, Equations (8), (16) and (18) to (20) become:

$$
\begin{align*}
1 & =\sum_{s=1}^{S}\left(\Psi_{i s}^{I} \widehat{M_{i s}^{e, I}}+\Psi_{i s}^{C} \widehat{M_{i s}^{e, C}}\right)  \tag{21}\\
\widehat{\bar{c}_{i s}} & =\widehat{w}_{i} \beta_{i s}\left(\prod_{k=1}^{S} \widehat{P_{i k}^{I}} \eta_{s k}^{i}\right)^{1-\beta_{i s}}  \tag{22}\\
\widehat{w_{i}} & =\sum_{j=1}^{N} \frac{v_{i j s}^{I}\left(\widehat{\tau_{i j s}^{I}} \widehat{\bar{c}_{i s}}\right)^{-\theta_{s}^{I}} \widehat{w_{j}} \sum_{r=1}^{S} \kappa_{j s}^{r}\left(\kappa_{j r}^{I} \widehat{M_{j r}^{e, I}}+\kappa_{j r}^{C} \widehat{M_{j r}^{e, C}}\right)}{\sum_{m=1}^{N} a_{m j s}^{I} \widehat{M_{m s}^{e, I}\left(\widehat{\tau_{m j s}^{I}} \widehat{\bar{c}_{m s}}\right)^{-\theta_{s}^{I}}}}  \tag{23}\\
\widehat{w_{i}} & =\sum_{j=1}^{N} \frac{v_{i j s}^{C}\left(\widehat{\tau_{i j}^{C}} \widehat{\widehat{c_{i s}}}\right)^{-\theta_{s}^{C}} \widehat{w_{j}}}{\sum_{m=1}^{N} a_{m j s}^{C} \widehat{M_{m s}^{e, C}}\left(\widehat{\tau_{m j s}^{C}} \widehat{\bar{c}_{m s}}\right)^{-\theta_{s}^{C}}}  \tag{24}\\
\widehat{P_{i s}^{I}} & \left.=\left(\frac{1}{\widehat{w_{i}}} \sum_{r=1}^{S} \kappa_{i s}^{r}\left(\kappa_{i s}^{r, I} \widehat{M_{i r}^{e, I}}+\kappa_{i s}^{r, C} \widehat{M_{i r}^{e, C}}\right)\right)^{\frac{\left(\sigma_{s}^{I}-1\right)-\theta_{s}^{I}}{\theta_{s}^{I}\left(\sigma_{s}^{I}-1\right)}}\left(\sum_{j=1}^{N} a_{j i s}^{I} \widehat{M_{j s}^{e, I}} \widehat{\left(\tau_{j i s}^{I}\right.} \widehat{\bar{c}_{j s}}\right)^{-\theta_{s}^{I}}\right)^{-\frac{1}{\theta_{s}^{I}}} \tag{25}
\end{align*}
$$

Where $\Psi_{i s}^{I}, \Psi_{i s}^{C}, \kappa_{i s}^{r, I}$, and $\kappa_{i s}^{r, C}$ are determined by trade flows, $\beta_{i s}, \eta_{s k}^{i}, \theta_{s}^{\delta}, \sigma_{s}^{\delta 4}$ and $a_{i j s}^{\delta}=$ $T_{i j s}^{\delta} / \sum_{m} T_{m j s}^{\delta}, v_{i j s}^{\delta}=T_{i j s}^{\delta} / \sum_{m} T_{i m s}^{\delta}$. Equation (21) for each country, Equations (22) to (24) for each intermediate and consumption sectors of each country, and Equation (25) for each intermediate sector of each country represent a system of $\mathrm{N}+\mathrm{NS}+\mathrm{NS}+\mathrm{NS}+\mathrm{NS}$ equations in the $\mathrm{N}+\mathrm{NS}+\mathrm{NS}+\mathrm{NS}+\mathrm{NS}$ unknowns variables $\left\{\hat{w}_{i}, \hat{M}_{i s}^{e, C}, \hat{M}_{i s}^{e, I}, \hat{P}_{i s}^{I}, \hat{c}_{i s}\right\}$. These equations can be solved only with the information of $\beta_{i s}, \eta_{s k}^{i}, \theta_{s}^{\delta}, \sigma_{s}^{\delta}$, and observable trade flows in intermediate and consumption goods that are available in Input-output tables. $T_{i j s}^{r}$ shows the flow of intermediate goods from sector s in country j to sector r in country and $T_{i j s}^{C}$ and $T_{i j s}^{I}$ show the flow of consumption and intermediate goods from country i to $j$, respectively.

$$
\begin{aligned}
& 4 \\
& \Psi_{i s}^{I}=\frac{\sum_{j} \frac{\left(\sigma_{s}^{I}-1\right)\left(1+\theta_{s}^{I}\right)}{\theta_{s}^{I} \sigma_{s}^{I 2}}\left(1+\beta_{i s}\left(\sigma_{s}^{I}-1\right)\right) T_{i j s}^{I}}{\sum_{r} \sum_{j}\left(\frac{\left(\sigma_{r}^{I}-1\right)\left(1+\theta_{r}^{I}\right)}{\theta_{r}^{I} \sigma_{r}^{I}{ }^{2}}\left(1+\beta_{i r}\left(\sigma_{r}^{I}-1\right)\right) T_{i j r}^{I}+\frac{\sigma_{r}^{C}-1}{\sigma_{r}^{C} \theta_{r}^{C}}\left(1+\beta_{i r} \theta_{r}^{C}\right) T_{i j r}^{C}\right)}, \\
& \Psi_{i s}^{C}=\frac{\sum_{j} \frac{\sigma_{s}^{C}-1}{\sigma_{s}^{C} \theta_{s}^{C}}\left(1+\beta_{i s} \theta_{s}^{C}\right) T_{i j s}^{C}}{\sum_{r} \sum_{j}\left(\frac{\left(\sigma_{r}^{I}-1\right)\left(1+\theta_{r}^{I}\right)}{\theta_{r}^{I} \sigma_{r}^{I}}\left(1+\beta_{i r}\left(\sigma_{r}^{I}-1\right)\right) T_{i j r}^{I}+\frac{\sigma_{r}^{C}-1}{\sigma_{r}^{C} \theta_{r}^{C}}\left(1+\beta_{i r} \theta_{r}^{C}\right) T_{i j r}^{C}\right)}, \\
& \kappa_{i s}^{r}=\frac{\sum_{j} T_{j i s}^{r}}{\sum_{j} T_{j i s}^{I}}, \kappa_{i s}^{r, I}=\frac{1}{1+\frac{\sigma_{s}^{I}}{\sigma_{s}^{C}} \frac{\sigma_{s}^{C}-1}{\sigma_{s}^{I}-1} \frac{\sum_{j} T_{i j s}^{C}}{\sum_{j} T_{i j s}^{I}}} \text {, and } \kappa_{i s}^{r, C}=1-\kappa_{i s}^{r, I}
\end{aligned}
$$

## 3 Welfare effects of changes in tariffs

We are interested in evaluating welfare change caused by general equilibrium adjustments of changes in tariffs. Change in welfare equals to change in nominal wage deflated by change in the ideal aggregate price index of consumption goods: $\hat{W}_{j}=\hat{w}_{j} / \hat{P}_{j}^{C}$. Since preferences of consumers are Cobb-Douglas across sectors, change in welfare can be written as $\hat{W}_{j}=\hat{w}_{j} / \prod_{s}\left(\hat{P}_{j s}^{C}\right)^{\mu_{j s}}$. To better understand the general equilibrium effects of change in tariffs, we use log-linear approximation around factual.

Change in price index of consumption and intermediate goods sector in industry $s$ in country $j$ can be written as:

$$
\begin{align*}
& \frac{\Delta P_{j s}^{I}}{P_{j s}^{I}}=\sum_{i=1}^{N} \frac{T_{i j s}^{I}}{\sum_{m=1}^{N} T_{m j s}^{I}}[\underbrace{\frac{\Delta \tau_{i j s}^{I}}{\tau_{i j s}^{I}}}_{1}+\underbrace{\frac{\Delta \bar{c}_{i s}}{\bar{c}_{i s}}}_{2}-\frac{1}{\theta_{s}^{I}} \underbrace{\frac{\Delta M_{i, I}^{e, I}}{M_{i s}^{e, I}}}_{3}  \tag{26}\\
& \quad+\left(\frac{1}{\theta_{s}^{I}}-\frac{1}{\sigma_{s}^{I}-1}\right)(\sum_{r=1}^{S} \frac{T_{i j s}^{I}}{\sum_{m=1}^{N} T_{m j s}^{I}} \underbrace{\left.\frac{T_{i j s}^{r, I}}{T_{i j s}^{I}} \frac{\Delta M_{i r}^{e, I}}{M_{i r}^{e, I}}+\frac{T_{i j s}^{r, C}}{T_{i j s}^{I}} \frac{\Delta M_{i r}^{e, C}}{M_{i r}^{e, C}}\right)})] \\
& \frac{\Delta P_{j s}^{C}}{P_{j s}^{C}}=\sum_{i=1}^{N} \frac{T_{i j s}^{C}}{\sum_{m=1}^{N} T_{m j s}^{C}}[\underbrace{\frac{\Delta \tau_{i j s}^{C}}{\tau_{i j s}^{C}}}_{1}+\underbrace{\frac{\Delta \bar{c}_{i s}}{\bar{c}_{i s}}}_{2}-\frac{1}{\theta_{s}^{C}} \underbrace{\frac{\Delta M_{i s}^{e, C}}{M_{i s}^{e, C}}}_{3}] \tag{27}
\end{align*}
$$

Change in price index of each sector is weighted average of changes in price indeces of imported product from each country proportioned to share of import from that country to the total final good available in that sector. In general, change in imported products price index from each country obtains from changes in tariffs, changes in marginal cost of bundles of inputs of production

$$
\begin{equation*}
\frac{\Delta \bar{c}_{j s}}{\bar{c}_{j s}}=\beta_{j s} \Delta \frac{w_{j}}{w_{j}}+\left(1-\beta_{j s}\right) \sum_{k=1}^{S} \eta_{s k} \Delta \frac{P_{j k}^{I}}{P_{j k}^{I}} \tag{28}
\end{equation*}
$$

entries to that sector, changes in demand, and changes in fixed cost of production $w_{i} f_{i s}^{e}$.

To understand how changes in tariffs affect price of a sector throughout these five channels, imagine a $1 \%$ reduction in import tariff of a sector. First, it affect wages and marginal costs of production and induces entries in and out of industries. Taking out these changes, a $1 \%$ reduction in tariffs changes the price index of imported products by directly decreasing it $1 \%$. Further, as the price of exporting goods has became cheaper, it reduces the cut-off productivity of export to the country $1 \%$. As it shown in Figure 1, this reduction affect the price index of imported products from two different
channels: change in entry to export market and change in average productivity of exporting firms. Since the cut-off productivity of export is now $1 \%$ lower, the entrants to export market increases by $\theta^{\delta} \%\left(\right.$ as $\left.M_{i j s}^{\delta}=\left(b_{i s}^{\delta} / \varphi_{i j s}^{*, \delta}\right)_{s}^{\delta} M_{i s}^{e, \delta}\right)$. Also, $1 \%$ decrease in the cut-off productivity to export reduces the average productivity of exporting firms by $1 \%$.


Figure 1: Change in Price Index of exporting products in response to change in tariffs

By definition of aggregate price index in Equation (6), any change in entry to a sector affect the price index of exported products by elasticity of $-1 /\left(\sigma_{s}^{\delta}-1\right)$ and any change in average productivity changes the price index by elasticity of -1 . Thus, the price index will be reduced by $1+\theta_{s}^{\delta} /\left(\sigma_{s}^{\delta}-1\right)$ $\%$. But, this reduction in price index make competition in export market easier. So, it further reduces the cut-off productivity to export. The overall elasticity that can be calculated from a geometric series is $\left(\sigma_{s}^{\delta}-1\right) / \theta_{s}^{\delta 5}$. Since it is assumed that $\theta_{s}^{\delta}>\sigma_{s}^{\delta}-1$, this elasticity is always positive and lower than one. It means that if the cut-off productivity to export decreases, the price index of exporting products will be lower.

Therefore, the overall elasticity of change in tariff (beside it effect on cost of production, wage, and entries) which is sum of the initial effect on price index and the effect on price index by changing the cut-off productivity is $1^{6}$.

Second, since a change in marginal cost of production makes the exporting products cheaper, its effect is same as a change in tariff and changes cut-off productivity to export by $1 \%$.

[^2]Third, the induced entry to a sector from changes in tariffs changes entry to export market. Therefore, it changes the price index of target market by increasing the measure of exporters with elasticity of $-1 /\left(\sigma_{s}^{\delta}-1\right)$ by Equation (6). Hence, as any initial change in price index of export market magnifies and changes the price index by elasticity of $\left(\sigma_{s}^{\delta}-1\right) / \theta_{s}^{\delta}$, changes in entries change the price index of exporting products by $-1 /\left(\sigma_{s}^{\delta}-1\right) \cdot\left(\sigma_{s}^{\delta}-1\right) / \theta_{s}^{\delta}=-1 / \theta_{s}^{\delta}$. Lower the trade elasticity of a sector, the entry to that sector reduces the price index of exporting products more.

Changes in wages have two opposing effect on the cut-off productivity of export that would cancel out each other. Since the only fixed cost of export is hiring labor from target country, if changes in tariffs increase wage in target country, it would raises the fixed cost of export and consequently the cut-off productivity of export by $1 /\left(\sigma_{s}^{\delta}-1\right) \hat{w}_{j} / w_{j}$. On the other hand, increase in wage also means increase in demand for both consumption and intermediate goods. For consumption goods, this effect is easier to grasp. Since the demand of a consumption sector is proportional to income $\left(\mu_{j s} L_{j} w_{j}\right)$, a raise in wage of target country would increases the demand for consumption goods in same amounts. For intermediate goods, the demand is determined by both intensive and extensive margins. How much each firm wants to buy inputs of production is a portion of revenue of the firm (by CobbDouglas production function). Because of free entry condition, the expected profit of a firm should be equal to the fixed sunk of entry $\left(w_{j} f_{j s}^{e}\right)$. The expected revenue is proportional to the expected profit (by CES aggregate functions), so change in wage solely determine change in the expected revenue and demand for intermediate goods of a firm. The change in demand changes the cut-off productivity to export with the elasticity of $-1 /\left(\sigma_{s}^{\delta}-1\right)$ from Equation (12). Therefore, the two opposing forces of changes in wage in the destination country on the cut-off productivity of export (change in cost of export and change in demand for goods) are canceling out each other for both consumption and intermediate good, so it does not affect the price index of exporting products.

The demand of an intermediate goods sector also changes by extensive margin: entries in and out of other industries that are using the produced goods of that sector as inputs of production (forth effect for price of intermediate goods). The change in demand that an intermediate goods sector faces affect prices throughout the change in cut-off productivity. As it is stated before, a reduction in cut-off productivity reduces the price index by changing the entry and the average productivity of firms that export. Thus, the overall elasticity of exporting intermediate goods price index to both these channels is $1 / \theta_{s}^{I}-1 /\left(\sigma_{s}^{I}-1\right)^{7}$.

$$
\text { As } \hat{W}_{j}=\hat{w}_{j} / \prod_{s=1}^{S}\left(\hat{P_{j s}^{C}}\right)^{\mu_{j s}}, \text { the log-linear approximation around factual yields: }
$$

$$
7 \frac{-1}{\sigma_{s}^{I}-1} \cdot\left(-1+\frac{\theta_{s}^{I}}{\sigma_{s}^{I}-1}\right) \cdot \frac{\sigma_{s}^{I}-1}{\theta_{s}^{I}}
$$

$$
\begin{align*}
\frac{\Delta W_{j}}{W_{j}} & =\sum_{i=1}^{N} \sum_{s=1}^{S} \mu_{j s} \frac{T_{i j s}^{C}}{\sum_{m=1}^{N} T_{m j s}^{C}}[\underbrace{\frac{\Delta w_{j}}{w_{j}}}_{\text {Income effect }} \underbrace{-\frac{\Delta \tau_{j i s}^{C}}{\tau_{j i s}^{C}}}_{\begin{array}{c}
\text { Cost of export of } \\
\text { consumption goods effect }
\end{array}}
\end{align*} \underbrace{-\frac{\Delta \bar{c}_{j s}}{\bar{c}_{j s}}}_{\text {Cost of production effect }}
$$

The first term is the direct effect of change in income of labor on welfare. The second term is changes in tariffs of consumption goods that must be paid by firms country $j$ on its welfare. The third term is direct effect of changes in cost of production. Note that the first and third effects are direct consequences of considering intermediate goods in production function. Since in the models without intermediate goods for production the only cost of production is wage, these two effects cancel out each other.

The forth term is the traditional terms of trade effect. It captures the direct effect of changes in cost of inputs of production (wages and price indeces of intermediate sectors) and tariffs have on the prices of the consumption goods exported by country $j$ relative to the direct effect changes in tariffs and cost of inputs of production have on the prices of the consumption goods imported by country $j$. Higher term of trade would be beneficial for a country since it consumes cheaper imports and sales more expensive exports.

The fifth term is the home market effect emphasized by Venables (1987). It captures the indirect effect of adjustments in entry and exit have on the aggregate consumption price index in country $j$. Changes in entry to a consumption sector affect price index by changing the range of products available in country $j$ and by changing the average productivity of exporters to country $j$. Note, the home market effect along the other effects of intermediate sectors are captured by changes in costs of production.

In Table 1, we estimated the effects of multilateral $10 \%$ reduction in tariffs on intermediate and consumption goods between two similar countries using equations in Section 2.5. We considered three scenarios when the reduction in tariffs happen for both intermediate and consumption (row 1), and when only tariffs of intermediate or consumption goods decreases without any change in tariffs of other products (row 2 and 3 , respectively). In all scenarios, the income effect and home market effects of consumption goods are subtle. From $8.9 \%$ welfare gain of reduction in tariffs, $4.9 \%$ is due to reduction

Table 1: Effect of 10 \% reduction in tariffs for two similar countries


Note: The two countries are similar. Share of labor in production is 0.3 for all industries and each industry uses equal share of inputs from other industries. There are 4 industries that are similar in all features except $\theta$ and $\sigma . \theta_{s}=4,5,6,7$ and $\sigma_{s}=1.5,2.5,3.5,4.5$ for these four industries. Intermediate and consumption sectors are similar in each industry.
in cost of production that is missed when only tariffs of consumption goods decreases. These gain comes from changes in tariffs of intermediate goods, changes in entries, changes in demands (due to entries in and out of other industrious), and changes in fixed costs of export (since changes in wages are almost zero, this effect does not present). On the other hand, $3.6 \%$ of welfare gain comes from change in cost of export of consumption goods. Note, since the two countries are similar, the term of trade of countries does not change, and its effect is always $0 \%$.

## 4 Welfare gain of international trade

In the same way as Arkolakis et al. (2012), we can evaluate changes in welfare in response to trade shocks using some sufficient statistics without need to solve the equilibrium in Section 2.5.

From Equation (20), the share of expenditure on domestic product in each sector $(s, \delta)$ of country $j$ equals to:

$$
\begin{equation*}
\lambda_{j j s}^{\delta}=M_{j s}^{e, \delta} \frac{f_{j j s}^{\frac{\left(\sigma_{s}^{\delta}-1\right)-\theta_{s}^{\delta}}{\sigma_{s}^{\delta}-1}}\left(\frac{b_{i s}^{\delta}}{d_{i j s}^{\delta} \tau_{i j s}^{\delta} \bar{c}_{i s}}\right)^{\theta_{s}^{\delta}}}{\sum_{m=1}^{N} M_{m s}^{e, \delta} f_{m j s} \frac{\left(\sigma_{s}^{\delta}-1\right)-\theta_{s}^{\delta}}{\sigma_{s}-1}}\left(\frac{b_{m s}^{\delta}}{d_{m j s}^{\delta} \tau_{m j s}^{\delta} \bar{c}_{m s}}\right)^{\theta_{s}^{\delta}} . \tag{30}
\end{equation*}
$$

Substituting Equation (30) in Equation (19), the price index of consumption goods equals to

$$
\begin{equation*}
P_{j s}^{C}=\gamma_{s}^{C}\left(\frac{\sigma_{s}^{C} f_{j j s}^{C}}{\mu_{j s} L_{j}}\right)^{\frac{\theta_{s}^{C}-\left(\sigma_{s}^{C}-1\right)}{\theta_{s}^{C}\left(\sigma_{s}^{C}-1\right)}}\left[\lambda_{j j s}^{C} \frac{L_{j} w_{j}}{\mathbb{E}\left(r_{j s}^{C}(\varphi)\right)} \pi_{j s}^{R, C}\right]^{\frac{-1}{\theta_{s}^{C}}} b_{j s}^{C-1} w_{j} \beta_{j s} \prod_{r=1}^{S} P_{j k}^{I} \eta_{s r}^{j}\left(1-\beta_{j s}\right) . \tag{31}
\end{equation*}
$$

Where $\gamma_{j s}^{\prime}$ is constant. ${ }^{8}$ After some matrix algebra it can be shown that:

$$
\begin{equation*}
W_{j}=\prod_{s=1}^{S}\left[\frac{b_{j s}^{C}}{\gamma_{s}^{C}}\left(\frac{\sigma_{s}^{C} f_{j j s}^{C}}{\mu_{j s} L_{j}}\right)^{\frac{\left(\sigma_{s}^{C}-1\right)-\theta_{s}^{C}}{\theta_{s}^{C}\left(\sigma_{s}^{C}-1\right)}}\left(\frac{L_{j} w_{j}}{\mathbb{E}\left(r_{j s}^{C}(\varphi)\right)} \pi_{j s}^{R, C} \lambda_{j j s}^{C}\right)^{\frac{1}{\theta_{s}^{C}}} \prod_{r=1}^{S}\left[\frac{b_{j r}^{I}}{\gamma_{r}^{I}}\left(\frac{\sigma_{r}^{I} f_{j j r}^{I}}{\pi_{j r}^{E, I} L_{j}}\right)^{\frac{\left(\sigma_{r}^{I}-1\right)-\theta_{r}^{I}}{\theta_{r}^{I}\left(\sigma_{r}^{I}-1\right)}}\left(\frac{L_{j} w_{j}}{\mathbb{E}\left(r_{j r}^{I}(\varphi)\right)} \pi_{j r}^{R, I} \lambda_{j j r}^{I}\right)^{\frac{1}{\theta_{r}^{I}}}\right]^{\tilde{a}_{s r}^{j}}\right]^{\mu_{j s}} \tag{32}
\end{equation*}
$$

$$
{ }^{8} \gamma_{j s}^{\prime}=\gamma_{s} \beta_{j s}^{-\beta_{j s}}\left(1-\beta_{j s}\right)^{-\left(1-\beta_{j s}\right)} \prod_{r=1}^{S} \eta_{s r} \eta_{s r}\left(1-\beta_{j r}\right) .
$$

Where $\pi_{j s}^{E, \delta}$ and $\pi_{j s}^{R, \delta}$ are the share of expenditure on a sector to total income $w_{j} L j$ and the share of revenue of a sector in to to total income, respectively. $\tilde{a}_{s r}^{j}$ is the $(s, r)$ element of the adjusted Leontief inverse matrix $A_{j}\left(I-A_{j}\right)^{-1}$, where $A_{j}$ is the matrix whose its $(s, r)$ element is $\eta_{s r}\left(1-\beta_{j r}\right)$. $\tilde{a}_{s r}^{j}$ is the elasticity of price index of consumption goods in industry $s$ with respect to price index of intermediate goods in industry $r$ in country $j$.

The welfare change in response to trade shocks is:

$$
\begin{equation*}
\left.\widehat{W_{j}}=\prod_{s=1}^{S}\left[\widehat{\left(\lambda_{j j s}^{C}\right.} \widehat{\pi_{j s}^{R, C}}\right)^{\frac{1}{\theta_{s}^{C}}} \prod_{r=1}^{S}\left[\left(\widehat{\pi_{j r}^{E, I}}\right)^{\frac{\theta_{r}^{I}-\left(\sigma_{r}^{I}-1\right)}{\theta_{r}^{I}\left(\sigma_{r}^{I}-1\right)}}\left(\widehat{\pi_{j r}^{R, I}} \widehat{\lambda_{j j r}^{I}}\right)^{\frac{1}{\theta_{r}^{I}}}\right]^{\tilde{a}_{s r}^{j}}\right]^{\mu_{j s}} \tag{33}
\end{equation*}
$$

In autarky, the share of domestic production is 1 for in all sectors and for each sector revenue and expenditure are the same. Therefore, it can be shown that the welfare of gain of moving from observed equilibrium to autarky is:

$$
\begin{equation*}
\widehat{W}_{j}^{O E \rightarrow A}=1-\prod_{s=1}^{S}\left[\left(\frac{\pi_{j s}^{R, C}}{\mu_{j s}} \lambda_{j j s}^{C}\right)^{\frac{-1}{\theta_{s}^{C}}} \prod_{r=1}^{S}\left[\left(\frac{\pi_{j r}^{E, I}}{\Delta_{j r}}\right)^{\frac{\left(\sigma_{r}^{I}-1\right)-\theta_{r}^{I}}{\theta_{r}^{I}\left(\sigma_{r}^{I}-1\right)}}\left(\frac{\pi_{j r}^{R, I}}{\Delta_{j r}} \lambda_{j j r}^{I}\right)^{\frac{-1}{\theta_{r}^{I}}}\right]^{\tilde{a}_{s r}^{j}}\right]^{\mu_{j s}} \tag{34}
\end{equation*}
$$

Where $\Delta_{j s}$ is $\pi_{j s}^{R, I}=\pi_{j s}^{E, C}$ in autarky and only depends on structural parameters.
In the same model but without different sectors for intermediate and consumption goods (the same feature used in Caliendo and Parro (2014) and Costinot and Rodriguez-clare (2014)), the welfare of gain of moving from observed equilibrium to autarky is:

$$
\begin{equation*}
\widehat{W}_{j}^{O E \rightarrow A}=1-\prod_{s=1}^{S} \prod_{r=1}^{S}\left[\left(\frac{\pi_{j r}^{E}}{\Delta_{j r}^{\prime}}\right)^{\frac{\left(\sigma_{r}-1\right)-\theta_{r}}{\theta_{r}\left(\sigma_{r}-1\right)}}\left(\frac{\pi_{j r}^{R}}{\Delta_{j r}^{\prime}} \lambda_{j j r}\right)^{\frac{-1}{\theta_{r}}}\right]^{a_{s r}^{j} \mu_{j s}} \tag{35}
\end{equation*}
$$

Where $a_{s r}^{j}$ is the $(s, r)$ element of the Leontief inverse matrix $\left(I-A_{j}\right)^{-1}$, where $A_{j}$ is the matrix whose its $(s, r)$ element is $\eta_{s r}\left(1-\beta_{j s}\right)$ and $\pi_{j s}^{E}$ and $\pi_{j s}^{R}$ are the share of expenditure on an industry to total income and revenue of an industry to total income, respectively. $\Delta_{j s}^{\prime}$ is $\pi_{j s}^{E}=\pi_{j s}^{R}$ in autarky and only depends on structural parameters.

## 5 Simulation

There are three main differences between Equation (35) and Equation (34). In order to shed light on these differences, we estimate the welfare gain from trade in these two models for an artificial economy. The values of parameters used to estimate equations Equation (35) and Equation (34) are shown in Table 2. It is assumed that all parameters are similar across industries, so we drop subscript
of industries from parameters.

Table 2: Parameters used in simulation

|  |  |
| :---: | :---: |
| Parameter | Value |
| $S$ | 50 |
| $\beta$ | 0.3 |
| $\sigma, \sigma^{I}$ | 3.8 |
| $\theta, \theta^{I}, \theta^{C}$ | 5 |
| $\lambda \lambda^{I}, \lambda^{C}$ | 0.75 |
| $\pi^{C}, \pi^{I}$ | 0.5 |

### 5.1 Share of domestic production

First, in Equation (35), it is the share of expenditure on domestic production in each industry that determines the welfare gain from trade. However, the share of intermediate goods and consumption goods can vary in each sector. In Equation (34) both shares of domestic production in consumption goods and intermediate goods determine the welfare gain from trade together. However, these statistics affect the welfare gain from trade differently.

To see how with the same $\lambda_{j j s}$ different $\lambda_{j j s}^{I}$ and $\lambda_{j j s}^{C}$ affect the welfare gain from trade, look at Section 5.1. In the model that does not consider the different usages of goods for consumption and production (shown with asterisk), no matter whether the share of domestic production comes from intermediate goods or consumption goods, the welfare gain from trade is the same. Contrary, as it is shown in Section 5.1, with the same $\lambda_{j j s}$ different $\lambda_{j j s}^{I}$ and $\lambda_{j j s}^{C}$ alter the welfare gain from trade. As with the same $\lambda_{j j s}$, higher $\lambda_{j j s}^{I}$ only happens if $\lambda_{j j s}^{C}$ is lower, there are two opposing effects. As it can be seen from Section 5.1 the maximun welfare gain does not happen in extremes. If lower $\lambda_{j j s}^{I}$ is the reason behind the lower $\lambda_{j j s}$, because of the proliferation effects of intermediate goods in economy, its effect on welfare gain will be more severe.

Moreover, as it can be seen in Section 5.1, as a country becomes more and more dependent on foreign intermediate goods, it's loss from going to autarky will significantly become higher. This effect is much smaller if a country is more dependent on foreign consumption goods, they can still produce consumption goods from domestic intermediate goods with less increase in cost of production.


Figure 2: Effect of change in $\lambda_{\text {iis }}^{C}$ and $\lambda_{\text {iis }}^{I}$ on welfare gain from trade

### 5.2 Elasticity of substitution

Second, here, the effect of elasticity of substitution of intermediate goods is stronger than elasticity of substitution of consumption goods. As it can be seen in Figure 3, different $\sigma^{I}$ s lead to highly different welfare implications.


Figure 3: Effect of change in $\sigma_{I}$ on welfare gain from trade

### 5.3 Trade elasticity

Third, in the model that does not take into account the difference between consumption and intermediate goods, it is implicitly assumed that consumption and intermediate goods in each sector have the same trade elasticity. However, the trade elasticity of consumption and intermediate goods could widely differ. Some intermediate goods, like raw materials, will be easily replaced by firms in response to changes in prices, so they have high trade elasticity. In contrast, some intermediate goods embodies technology with them. It is very costly for firms to switch to another technology in response to a change in trade barrier. Thus, these sectors have lower trade elasticity. In Figure 4, we compute the welfare gain from trade when for different fixed $\theta^{C}, \theta^{I}$ changes. The difference between curves of welfare gain with $\theta^{C}=3$ and $\theta^{C}=8$ is roughly 15 percents. However, as $\theta^{I}$ vary in the same range, the welfare gain from trade greatly changes. It indicate that the welfare gain from trade is much less influenced by trade elasticity of consumption goods than intermediate goods.


Figure 4: Effect of change in $\theta^{I}$ and $\theta^{C}$ on welfare gain from trade

## 6 Conclusion

This paper provides a general equilibrium model of international trade that takes into account the endogenous entry and exit of intermediate goods and consumption goods producers and the industry linkages to quantitatively measure the countries welfare gains and losses from different trade policies on intermediate and consumption goods. The model features inter-industry trade model Ricardo (1817), intra-industry model of Krugman (1980), and firm heterogeneous model of Melitz (2003) in input-output environment that consider the different usages of intermediate and consumption goods in economy. Taking into account the differences between consumption and intermediate goods in trade elasticities, elasticity of substitution, and share of domestic production can significantly change the estimated welfare gain from trade.

We make use of simulation of an artificial economy to assess the main contributions of our model in estimating welfare gain form trade. First, we demonstrate that shares of domestic production in consumption goods and intermediate goods together determine the welfare gain from trade. However, if the lower shares of domestic production in intermediate goods is the reason behind the lower shares of domestic production, because of the proliferation effects of intermediate goods in economy, its effect on welfare gain will be more. Second, we show that elasticity of substitution of consumption goods does not have any effect on the welfare gain of countries from trade, and elasticity of substitution
of intermediate goods solely determine the welfare gain from trade. Third, different estimations for trade elasticities of intermediate goods significantly affect the estimated welfare gain from trade, while different estimations for trade elasticities of consumption goods have much lower impact on estimates of the welfare gain.

Solving the equilibrium in hat algebra as Dekle et al. (2008) provides us with a general framework to evaluate different trade policies on intermediate goods. The model can be calibrated with only information on trade flows available in multi-countries input-output tables, elasticity of substitution and trade elasticity of sectors.

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## A Trade Imbalances

Before taking the model in Section 2.5 into data, we solved the model with trade imbalances between countries. Taking into account an exogenous trade deficit $D_{i}$ as Eaton and Kortum (2002) into Equations (22) to (25) yield:

$$
\begin{align*}
1 & =\sum_{s=1}^{S}\left(\Psi_{i s}^{I} \widehat{M_{i s}^{e, I}}+\Psi_{i s}^{C} \widehat{M_{i s}^{e, C}}\right)  \tag{36}\\
\widehat{\bar{c}_{j s}} & =\widehat{w}_{i} \beta_{i s}\left(\prod_{k=1}^{S} \widehat{P_{i k}^{I}} \eta_{s k}^{i}\right)^{1-\beta_{i s}}  \tag{37}\\
\widehat{w_{i}} & =\sum_{j=1}^{N} \frac{v_{i j s}^{I}\left(\widehat{\tau_{i j s}^{I}} \widehat{c_{i s}}\right)^{-\theta_{s}^{I}} \widehat{w_{j}}}{\sum_{m=1}^{N} a_{m j s}^{I} \widehat{M_{m s}^{e, I}}\left(\widehat{\tau_{m j s}^{I}} \widehat{\widehat{c_{m s}}}\right)^{-\theta_{s}^{I}}}\left[\sum_{r=1}^{S} \kappa_{j s}^{r}\left(\kappa_{j r}^{I} \widehat{M_{j r}^{e, I}}+\kappa_{j r}^{C} \widehat{M_{j r}^{e, C}}\right)\right]  \tag{38}\\
\widehat{w_{i}} & =\sum_{j=1}^{N} \frac{v_{i j s}^{C}\left(\widehat{\tau_{i j s}^{C}} \widehat{\bar{c}_{i s}}\right)^{-\theta_{s}^{C}}}{\left.\sum_{m=1}^{N} a_{m j s}^{C} \widehat{M_{m s}^{e, C}} \widehat{\left(\widehat{\tau_{m j s}^{C}}\right.} \widehat{\bar{c}_{m s}}\right)^{-\theta_{s}^{C}}}\left[\frac{D_{j}}{E_{j}^{c}}+\left(1-\frac{D_{j}}{E_{j}^{c}}\right) \widehat{w_{j}}\right]  \tag{39}\\
\widehat{P_{i s}^{I}} & \left.=\left(\frac{1}{\widehat{w}_{i}} \sum_{r=1}^{S} \kappa_{i s}^{r}\left(\kappa_{i s}^{r, I} \widehat{M_{i r}^{e, I}}+\kappa_{i s}^{r, C} \widehat{M_{i r}^{e, C}}\right)\right)^{\frac{\left(\sigma_{s}^{I}-1\right)-\theta_{s}^{I}}{\theta_{s}^{I}\left(\sigma_{s}^{I}-1\right)}}\left(\sum_{j=1}^{N} a_{j i s}^{I} \widehat{M_{j s}^{e, I}} \widehat{\left(\tau_{j i s}^{I}\right.} \widehat{c_{j s}}\right)^{-\theta_{s}^{I}}\right)^{-\frac{1}{\theta_{s}^{I}}} \tag{40}
\end{align*}
$$


[^0]:    ${ }^{1}$ Positive effects of use of foreign intermediate goods on productivity of domestic producers have been also extensively documented empirically. Amiti and Konings (2007) estimates the impact of the reduction of tariffs of intermediate goods on Indonesian firms. They find that the elasticity of productivity of firms with respect to tariffs on inputs is 1.2 in Indonesia, which is twice the effect of a reduction in the tariffs of final goods on productivity. Kasahara and Rodrigue (2008) assess the effect of imported inputs on the productivity of Chilean firms. Their estimate varies from 12.9 to 22 percent depending on the method of estimation. Goldberg et al. (2010) finds that a major determinant of producing new products in India was lower input tariffs. On the contrary, Van Biesebroeck (2003) and Muendler (2004) show that the positive effects of the reduction of input tariffs did not happen in Columbia and Brazil, respectively. Schor (2004) finds that the effect of decreasing tariffs is the same for intermediate and consumption goods.

[^1]:    ${ }^{2}$ We assume that I and C sectors in each industry have the same entry cost and production function. These assumptions do not alter the main results and only simplify solution and calibration of the model.
    ${ }^{3}$ The alternative model (as in Caliendo and Parro (2014)) is that in each industry, all firms produce intermediate goods. Then, these intermediate goods are aggregated to final goods. Part of a final good is demanded by households for consumption and part of it is demand by intermediate goods producers for production.

[^2]:    ${ }^{5} 1-\left(-1+\frac{\theta_{s}^{\delta}}{\sigma_{s}^{\delta}-1}\right)-\left(-1+\frac{\theta_{s}^{\delta}}{\sigma_{s}^{\delta}-1}\right)^{2}-\cdots=\frac{\sigma_{s}^{\delta}-1}{\theta_{s}^{\delta}}$
    ${ }^{6}\left(1-1+\frac{\theta_{s}^{\delta}}{\sigma_{s}^{\delta}-1}\right) \frac{\sigma_{s}^{\delta}-1}{\theta_{s}^{\delta}}=1$

