Microeconomics I
44715 (1396-97 1st Term) - Group 1
Chapter Three
Classical Demand Theory

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Introduction

Definitions (Rationality, desirability and convexity): The preference relation $\succsim$ on $X$ is

- **Rational**: if it is complete and transitive.
- **Monotone**: if $x, y \in X$ and $y \gg x$ then $y \succ x$.
- **Strongly Monotone**: if $x, y \in X$ and $y \geq x$ and $y \neq x$ then $y \succ x$.
- **Locally non-satiated**: 
  $\forall x \in X$ and $\forall \epsilon > 0$, $\exists y \in X$ such that $\|y - x\| \leq \epsilon$ and $y \succ x$.

It can be shown that strong monotonicity is a stronger assumption than monotonicity; and monotonicity is also a stronger assumption than local non-satiation.
Introduction

- **Convex:** if \( \forall x \in X, \) the upper contour set \( \{ y \in X : y \succeq x \} \) is convex.
  or \( \forall x \in X, \) if \( y \succeq x \) and \( z \succeq x \), then \( \alpha y + (1 - \alpha)z \succeq x \) for \( \forall \alpha \in [0, 1] \).

- **Strictly Convex:**
  if \( \forall x \in X, y \succ x, z \succ x, \) and \( y \neq z \) then \( \alpha y + (1 - \alpha)z \succ x \) for \( \forall \alpha \in [0, 1] \).

  *Convexity can be interpreted in terms of diminishing marginal rates of substitution.*

Can we deduce the consumer entire preference relation from a single indifference set?

Sometimes.
**Introduction**

**Definition (homotheticity):** A monotone preference relation $\succeq$ on $X = \mathcal{R}^L_+$ is homothetic if all indifference sets are related by proportional expansion along rays: this is if $x \sim y$ then $\alpha x \sim \alpha y$ for $\forall \alpha \geq 0$.

**Definition (quasilinearity):** A preference relation $\succeq$ on $X = (-\infty, \infty) \times \mathcal{R}^{L-1}_+$ is quasilinear with respect to commodity 1 (numeraire commodity) if:

i) All the indifference sets are parallel displacements of each other along the axis of commodity 1; this is if $x \sim y$ then $x + \alpha e_1 \sim y + \alpha e_1$ for $e_1 = (1, 0, \ldots, 0)$ and $\forall \alpha \in \mathcal{R}$.

ii) Commodity 1 is desirable; $x + \alpha e_1 \succ x$ for $\forall x$ and $\forall \alpha > 0$. 
For analytical purposes it is useful to interpret consumers’ preferences by a utility function.

However, our assumptions so far do not guarantee a rational preference relation is not necessarily representable with a utility function.

The assumption that is needed to ensure the existence of a utility function for a rational preference relation is continuity.
Preference and Utility

Definition (continuity): The preference relation \( \succsim \) on \( X \) is continuous if it is preserved under limits; this is

for any sequence of pairs \( \{(x^n, y^n)\}_{n=1}^{\infty} \)

with \( x^n \sim y^n \) for all \( n \), \( x = \lim_{n \to \infty} x^n \)

and \( y = \lim_{n \to \infty} y^n \),

we have \( x \sim y \).

OR

A preference relation \( \succsim \) on \( X \) is continuous if whenever \( a \succ b \), there are neighborhoods \( B_a \) and \( B_b \) around \( a \) and \( b \), respectively, such that for all \( x \in B_a \) and \( y \in B_b \), \( x \succ y \).
Preference and Utility

Example for non-continuity: lexicographic preferences.

**Proposition (MWG 3.C.1):** Suppose that the rational preferences relation \( \succeq \) on \( X \) is continuous, then there is a continuous utility function \( u(x) \) that represents \( \succeq \).
Note:

- The utility function $u(.)$ that represents a preference relation is not unique.
- Any strictly increasing transformation of $u(.)$ is also a utility function.
- Not all utility representation of a preference relation is continuous (one can consider a strictly increasing transformation which is not continuous).
- For analytical purposes, usually we consider the utility functions to be differentiable (not always e.g. Leontief preferences).
- We usually assume that the utility functions to be twice continuously differentiable, but we do not formally discuss the necessary assumptions on preferences that implies this property.
- How do restrictions on preferences imply restrictions on Utility functions?
Preference and Utility

- **Monotonicity** of preference relation ⇒ Utility function is increasing.
  \[ y \succ x \Rightarrow u(y) > u(x) \]

- **Convexity** of preference relation ⇒ Utility function is quasiconcave.
  \[ u(\alpha x + (1 - \alpha)y) \geq \min(u(x), u(y)) \quad \forall x, y \quad \& \quad \alpha \in [0, 1] \]

- **Strict Convexity** of preference relation ⇒ Utility function is strictly quasiconcave.
  \[ u(\alpha x + (1 - \alpha)y) > \min(u(x), u(y)) \quad \forall x, y \quad \& \quad x \neq y \quad \& \quad \alpha \in (0, 1) \]

Note: Quasiconcavity is a weaker property than concavity

\[ u(\alpha x + (1 - \alpha)y) \geq \alpha u(x) + (1 - \alpha)u(y) \]
Preference and Utility

- **Homotheticity** of a continuous preference relation \( \Leftrightarrow \) Utility function is homogenous of degree one
  
  \[ y(\alpha x) = \alpha u(x) \quad \forall \alpha > 0 \]

- **Quasilinearty** of a preference relation with respect to \( x_1 \) \( \Leftrightarrow \) Utility function is quasilinear function
  
  \[ u(x) = x_1 + f(x_2, \ldots x_L) \]

3D demonstrations of utility function:

- http://www2.hawaii.edu/~fuleky/anatomy/anatomy.html
- http://www2.hawaii.edu/~fuleky/anatomy/anatomy2.html
The consumer problem can be written as:

\[
\begin{align*}
\max_{x \geq 0} & \quad u(x) \\
\text{s.t} & \quad p \cdot x \leq w
\end{align*}
\]

where \( p \gg 0 \) and \( w > 0 \)

**Proposition (MWG 3.D.1):** If \( p \gg 0 \) and \( u(.) \) is continuous, then the utility maximization problem has a solution.

**Proof:** A continuous function always has a maximum value on any compact set.

**The Walrasian Demand Correspondence/Function**
The rule that assigns the set of optimal consumption vectors in the utility maximization problem to the price-wealth pairs.
Proposition (MWG 3.D.2): Suppose that \( u(\cdot) \) is a continuous utility function representing a locally nonsatiated preference relation \( \succeq \) defined on the consumption set \( X = \mathbb{R}^L_+ \). Then the Walrasian demand correspondence \( x(p, w) \) possesses the following properties

i) Homogeneity of degree zero in \((p, w)\)

\[ x(\alpha p, \alpha w) = x(p, w) \quad \text{for} \quad \forall p, w \quad \& \quad \alpha > 0 \]

ii) Walras’ law

\[ p \cdot x = w \quad \forall x \in x(p, w) \]

iii) Convexity/Uniqueness

If \( \succeq \) is convex (strictly convex) then \( x(p, w) \) is a convex set (singleton)

Proof:

i) The feasible set remains the same.

ii) Simple; using the fact that \( \succeq \) is nonsatiated.

iii) Can be proved using the fact that \( u(\cdot) \) is quasiconcave (strictly quasiconcave) and any linear combination of two optimal bundles is feasible.
If \( u(.) \) is continuously differentiable and \( x^* \) is a solution to the utility maximization problem, then the Kuhn-Tucker conditions say:

\[
\frac{\partial u(x^*)}{\partial x_l} \leq \lambda p_l \quad \text{if} \quad x^*_l = 0 \quad \text{and} \quad \frac{\partial u(x^*)}{\partial x_l} = \lambda p_l \quad \text{if} \quad x^*_l > 0
\]

or

\[
x^*_l \left( \frac{\partial u(x^*)}{\partial x_l} - \lambda p_l \right) = 0
\]

in matrix notation

\[
x^*. (\nabla u(x^*) - \lambda p) = 0
\]

where \( \lambda \geq 0 \) is the Lagrange multiplier.
The Utility Maximization Problem

Then for an interior solution \( x^* \gg 0 \) we must have \( \nabla u(x^*) = \lambda p \)

And for any two goods \( l \) and \( k \):

\[
\frac{\partial u(x^*)}{\partial x_l} - \lambda p_l = \frac{\partial u(x^*)}{\partial x_k} - \lambda p_k = 0
\]

So

\[
\frac{\partial u(x^*)}{\partial x_l} = p_l
\]

\[
\frac{\partial u(x^*)}{\partial x_k} = p_k
\]

\[
MRS_{lk}(x^*) = \frac{\partial u(x^*)}{\partial x_l} \frac{\partial x_k}{\partial u(x^*)}
\]
The Utility Maximization Problem

In contrast for a boundary solution $x^* \geq 0$ and $x_l^* = 0$ for some $l$ we might have:

$$MRS_{lk}(x^*) \neq \frac{p_l}{p_k}$$

$\lambda$ is the marginal (shadow) value of relaxing the budget constraint.

**Example:** The demand function for a Cobb-Douglas utility function.

It can be shown that if preferences are continuous, strictly convex, and locally nonsatiated then the Walrasian demand function is always continuous at all $(p, w) \gg 0$. Furthermore, if the determinant of the bordered Hessian of $u(.)$ is nonzero at $x^*$, the Walrasian demand function is differentiable.
The Utility Maximization Problem
The Indirect Utility Function

The Indirect Utility Function
The utility value of the utility maximization problem.

Proposition (MWG 3.D.3): Suppose that \( u(.) \) is a continuous utility function representing a locally non-satiated preference relation \( \succsim \) defined on the consumption set \( X = \mathcal{R}^L_+ \). Then the indirect utility function \( \nu(p, w) \) possesses the following properties:

i. Homogeneity of degree zero in \((p, w)\)

\[
\nu(\alpha p, \alpha w) = \nu(p, w) \quad \forall p, w \quad \& \quad \alpha > 0
\]

ii. Strictly increasing in \( w \) and non-increasing in \( p \)

\[
p.x = w \quad \forall x \in x(p, w)
\]

iii. Quasiconvex

\[
\{(p, w) : \nu(p, w) \leq \bar{\nu}\} \text{ is a convex set} \quad \forall \bar{\nu}
\]

iv. Continuous in \( p \) and \( w \)
The consumer’s problem can also be written as (which is the dual problem to the utility maximization problem):

$$\min_{x \geq 0} p \cdot x$$

$$s.t \quad u(x) \geq u$$

where $p \gg 0$ and $u > u(0)$. The EMP reverses the roles of objective function and constraint.
Proposition (MWG 3.E.1): Suppose that $u(.)$ is a continuous utility function representing a locally non-satiated preference relation $\succsim$ defined on $X = \mathbb{R}_+^L$ and that the price vector is $p \gg 0$. We have:

i) If $x^*$ is optimal in the UMP when wealth is $w > 0$, then $x^*$ is optimal in the EMP when the required utility level is $u(x^*)$. Moreover, the minimized expenditure level in this EMP is exactly $w$.

ii) If $x^*$ is optimal in the EMP when the required utility level is $u > u(0)$, then $x^*$ is optimal in the UMP when wealth is $p.x^*$. Moreover, the maximized utility level in this UMP is exactly $u$.

*Students need to go through the proof in MWG*

The Expenditure Function

Given the prices and the required utility, the value of the EMP is denoted $e(p, u)$ which is called the expenditure function. Its value is $p.x^*$ where $x^*$ is any solution to the EMP.
The Expenditure Minimization Problem

**Proposition (MWG 3.E.2):** Suppose that \( u(\cdot) \) is a continuous utility function representing a locally non-satiated preference relation \( \succeq \) defined on \( X = \mathcal{R}_+^L \). Then the expenditure function \( e(p, u) \) is

i. Homogeneous of degree one in \( p \)

\[
e(\alpha p, u) = \alpha e(p, u) \quad \forall p, u \quad \& \quad \alpha > 0
\]

ii. Strictly increasing in \( u \) and nondecreasing in \( p_l \); \( \forall l \)

iii. Concave in \( p \)

iv. Continuous in \( p \) and \( u \)

**Proof:**

i. The constraint set remains unchanged and the minimization problem is equivalent to the original one.

ii. Use the continuity of \( u(\cdot) \) for the first part and the optimization condition for the second part.

iii. Can be proved using the definition of expenditure function.
The Expenditure Minimization Problem

Note: For any $p \gg 0$, $w > 0$ and $u > u(0)$

\[ e(p, \nu(p, w)) = w \quad \text{and} \quad \nu(p, e(p, u)) = u \]

are inverse to one another.

The Hicksian (Compensated) Demand Function

The set of optimal commodity vectors in the EMP is denoted by $h(p, u) \subset R^L_+$. 
Proposition (MWG 3.E.3): Suppose that $u(.)$ is a continuous utility function representing a locally non-satiated preference relation $\succsim$ defined on $X = \mathbb{R}^L_+$. Then for any $p \gg 0$, the Hicksian demand $h(p, u)$ is

i. Homogeneous of degree zero in $p$

$$h(\alpha p, u) = h(p, u) \quad \forall p, u \text{ and } \alpha > 0$$

ii. No excess Utility

$$\forall x \in h(p, u) : \quad u(x) = u$$

iii. Convexity/Uniqueness

If $\succsim$ is convex (strictly convex) then $h(p, u)$ is a convex set (singleton)

Note: the Walrasian and Hicksian demand functions are equivalent as:

$$h(p, u) = x(p, e(p, u)) \quad \text{and} \quad x(p, w) = h(p, v(p, w))$$
The Hicksian demand function is also called compensated demand function since it can show the level of wealth compensation required to keep the consumer at the same level of utility after a change in prices.

**The Hicksian Demand and the Compensated Law of Demand**

**Proposition (MWG 3.E.4):** Suppose that \( u(\cdot) \) is a continuous utility function representing a locally nonsatiated preference relation \( \succsim \) and that \( h(p, u) \) consists of a single element for all \( p \gg 0 \). Then \( h(p, u) \) satisfies the compensated law of demand:

\[
\forall p', p''; \quad (p'' - p').[h(p'', u) - h(p', u)] \leq 0
\]
Relationships between Demand, Indirect Utility, and Expenditure Functions

The UMP
\[
\begin{align*}
\max_{x \geq 0} u(x) \\
\text{s.t. } p \cdot x \leq w
\end{align*}
\]

→

Walrasian Demand Function
\[x(p, w)\]

\[u(x(p, w))\]

Indirect Utility Function
\[v(p, w)\]

The EMP
\[
\begin{align*}
\min_{x \geq 0} p \cdot x \\
\text{s.t. } u(x) \geq u
\end{align*}
\]

→

Hicksian Demand Function
\[h(p, u)\]

\[p \cdot h(p, u)\]

Expenditure Function
\[e(p, u)\]
Relationships between Demand, Indirect Utility, and Expenditure Functions

**Proposition (MWG 3.G.1):** *(Shephard’s lemma)* Suppose that \( u(\cdot) \) is a continuous utility function representing a locally non-satiated and strictly convex preference relation \( \succeq \) defined on \( X = \mathbb{R}_+^L \). Then

\[
\forall p, u \quad \therefore \quad h(p, u) = \nabla_p e(p, u)
\]

Or

\[
\forall p, u \quad \therefore \quad h_l(p, u) = \frac{\partial e(p, u)}{\partial p_l} \quad \forall l = 1, \ldots, L
\]

The Economic interpretation of this proposition can be seen if we look more carefully at the following:

\[
\frac{\partial e(p, u)}{\partial p_l} = \frac{\partial}{\partial p_l} \sum_k p_k h_k(p, u) = h_l(p, u) + \sum_k p_k \frac{\partial h_k(p, u)}{\partial p_l}
\]

\[
\partial e(p, u) = h_l(p, u) \partial p_l + \sum_k p_k \partial h_k(p, u)
\]
Walrasian Demand Function **Proposition (MWG 3.G.2):** Suppose that $u(.)$ is a continuous utility function representing a locally non-satiated and strictly convex preference relation $\succsim$ defined on $X = \mathbb{R}_+^L$. Suppose also that $h(., u)$ is continuously differentiable at $(p, u)$. Then

i. $D_p h(p, u) = D_p^2 e(p, u)$

ii. $D_p h(p, u)$ is a negative semidefinite matrix.

iii. $D_p h(p, u)$ is a symmetric matrix.

iv. $D_p h(p, u)p = 0$

Note:

(ii) implies that compensated own price effects are nonpositive.

(iii) implies that for compensated price cross derivatives we must have:

$$\frac{\partial h_k(p, u)}{\partial p_l} = \frac{\partial h_l(p, u)}{\partial p_k}$$

We also define two goods as substitutes and complements based on the sign of $\frac{\partial h_k(p,u)}{\partial p_l}$. And (iv) implies that every good has at least one substitute.
Relationships between Demand, Indirect Utility, and Expenditure Functions

- Walrasian Demand Function: $x(p, w)$
- Indirect Utility Function: $u(x(p, w))$
- Hicksian Demand Function: $h(p, u)$
- Expenditure Function: $e(p, u)$

Shepard’s Lemma:

$p \cdot h(p, u)$
Proposition (MWG 3.G.3): Suppose that \( u(\cdot) \) is a continuous utility function representing a locally non-satiated and strictly convex preference relation \( \succsim \) defined on \( X = \mathbb{R}_+^L \). Then

\[
\forall p, w, u = v(p, w) \quad \therefore \quad D_p h(p, u) = D_p x(p, w) + D_w x(p, w) x(p, w)^T
\]

Or

\[
\forall p, w, u = v(p, w) \quad \therefore \quad \frac{\partial h_i(p, u)}{\partial p_k} = \frac{\partial x_i(p, w)}{\partial p_k} + \frac{\partial x_i(p, w)}{\partial w} x_k(p, w) \quad \forall l, k
\]

Note:
The Slutsky equation describes the relationship between the slope of the ordinary and compensated demand functions.

If the good is normal: \( \frac{\partial h_i(p, u)}{\partial p_i} < \frac{\partial x_i(p, w)}{\partial p_i} \) and

If the good is inferior: \( \frac{\partial h_i(p, u)}{\partial p_i} > \frac{\partial x_i(p, w)}{\partial p_i} \).
Relationships between Demand, Indirect Utility, and Expenditure Functions

\[ D_p h = D_p x + D_w x x^T \]

Walrasian Demand Function
\[ x(p, w) \]

Indirect Utility Function
\[ u(x(p, w)) \]

Hicksian Demand Function
\[ h(p, u) \]

Expenditure Function
\[ e(p, u) \]
Proposition (MWG 3.G.4): (Roy’s Identity) Suppose that \( u(.) \) is a continuous utility function representing a locally non-satiated and strictly convex preference relation \( \succeq \) defined on \( X = \mathbb{R}^L_+ \). Suppose also that \( v(.) \) is differentiable at \((\bar{p}, \bar{w}) \gg 0\). Then

\[
x(\bar{p}, \bar{w}) = -\frac{1}{\nabla_w v(\bar{p}, \bar{w})} \nabla_p v(\bar{p}, \bar{w})
\]

Or

\[
x_l(\bar{p}, \bar{w}) = -\frac{\partial v(\bar{p}, \bar{w})}{\partial p_l} \frac{\partial v(\bar{p}, \bar{w})}{\partial w} \quad \forall l = 1, \ldots, L
\]

Note:
Roy’s Identity and Shepard’s lemma are parallel results for UMP and EMP.
Relationships between Demand, Indirect Utility, and Expenditure Functions

- **Walrasian Demand Function**: \( x(p, w) \)
- **Indirect Utility Function**: \( v(p, w) \)
- **Hicksian Demand Function**: \( h(p, u) \)
- **Expenditure Function**: \( e(p, u) \)

\[
D_p h = D_p x + D_w x x^T
\]

- **The Slutsky Equation**
- **Shepard’s Lemma**
- **Roy’s Identity**
If we observe a demand function which
- is homogeneous of degree zero,
- satisfies the Walras’ law, and
- has a substitution matrix that is symmetric and negative semidefinite

can we find preferences that rationalize that?

The answer to this question is important because:
1) Whether these conditions are sufficient conditions as well?
2) Completes the comparison of preference-based and choice-based theories of demand.
3) When can we recover the customer preferences from consumer behavior observations for welfare analysis?
4) Simple form demand functions may not correspondent to simple form preferences.
Integrability

The problem of integrability can be divided to two questions:

1. Recovering an expenditure function from the demand function.
2. Recovering the preferences from the expenditure function.
Definition: Money metric indirect utility function:

\[ e(\bar{p}, v(p^1, w)) - e(\bar{p}, v(p^0, w)) \quad \forall \bar{p} \]

In particular if \( \bar{p} = p^0 \):

Equivalent variation:

\[ EV = e(p^0, v(p^1, w)) - e(p^0, v(p^0, w)) \]
\[ = e(p^0, u^1) - e(p^0, u^0) \]

And if \( \bar{p} = p^1 \):

Compensating variation:

\[ CV = e(p^1, v(p^1, w)) - e(p^1, v(p^0, w)) \]
\[ = e(p^1, u^1) - e(p^1, u^0) \]
Welfare Evaluation of Economic Changes

The representation of equivalent variation in terms of Hicksian demand (when only $p_1$ changes):

$$EV = e(p^0, u^1) - e(p^0, u^0)$$
$$= e(p^0, u^1) - w$$
$$= e(p^0, u^1) - e(p^1, u^1)$$

$$EV(p^0, p^1, w) = \int_{p_1^0}^{p_1^1} h_1(p_1, \bar{p}_{-1}, u^1)dp_1$$

The representation of compensating variation in terms of Hicksian demand (when only $p_1$ changes):

$$CV(p^0, p^1, w) = \int_{p_1^0}^{p_1^1} h_1(p_1, \bar{p}_{-1}, u^0)dp_1$$
Welfare Evaluation of Economic Changes

Example:

\[ u(x_1, x_2) = 0.6\ln(x_1) + 0.4\ln(x_2) \]
Welfare Evaluation of Economic Changes

\[ x_1(p_1, p_2, w) = 0.6 \frac{w}{p_1} \]

\[ h_1(p_1, p_2, u) = u - 0.4 \ln\left( \frac{2p_1}{3p_2} \right) \]
Welfare Evaluation of Economic Changes

In this case:

\[ p_1^0 = p_2^0 = 1 \quad \& \quad w = 14.484 \Rightarrow x_1^0 = 8.690 \quad \& \quad x_2^0 = 5.793 \quad \& \quad u^0 = 2 \]
\[ p_1^1 = 2, p_2^1 = 1 \quad \& \quad w = 14.484 \Rightarrow x_1^1 = 4.345 \quad \& \quad x_2^1 = 5.793 \quad \& \quad u^1 = 1.584 \]

\[ e(p^0, u^1) = 9.556 \quad \& \quad e(p^1, u^0) = 21.953 \]

\[ EV(p^0, p^1, w) = 9.556 - 14.484 = -4.928 \]
\[ CV(p^0, p^1, w) = 14.484 - 21.953 = -7.469 \]

(recall that \( x_1 \) is normal): \( 0 > EV(p^0, p^1, w) > CV(p^0, p^1, w) \)

Example: The deadweight loss from commodity taxation
Welfare Analysis with Partial Information  So far, we learnt how it is possible to calculate the welfare effect of a price change when we know the consumer’s expenditure function. The latter is not always the case.
Firstly, a test is introduced to evaluate whether a price change improves the welfare or not.

Proposition (MWG 3.I.1): Suppose that the consumer has a locally nonsatiated rational preference relation $\succeq$. If $(p^1 - p^0).x^0 < 0$, then the consumer is strictly better off under price-wealth situation $(p^1, w)$ than under $(p^0, w)$. 
Welfare Evaluation of Economic Changes

Proof: \((p^1 - p^0).x^0 < 0 \Rightarrow p^1.x^0 < p^0.x^0 = w\) which means \(x^0\) is affordable under \((p^1, w)\) and is an interior point in the budget constraint. Then since preferences are nonsatiated then there should be a better option available under \((p^1, w)\).

Proposition (MWG 3.1.2): Suppose that the consumer has a differentiable expenditure function. Then if \((p^1 - p^0).x^0 > 0\), there is a sufficiently small \(\bar{\alpha} \in (0, 1)\) such that for all \(\alpha < \bar{\alpha}\) we have \(e((1 - \alpha)p^0 + \alpha p^1, u^0) > w\), and so the consumer is strictly better under price-wealth situation \((p^0, w)\) than under \(((1 - \alpha)p^0 + \alpha p^1, w)\).
Approximation of the Welfare Effect using the Walrasian Demand Curve

Since Hicksian demand is not directly observable, the Area Variation (AV) has been used extensively in the literature:

$$AV(p^0, p^1, w) = \int_{p_1^1}^{p_1^0} x_1(p_1, \bar{p}_1, w) dp_1$$

It can be seen that for a normal good:

$$EV(p^0, p^1, w) > AV(p^0, p^1, w) > CV(p^0, p^1, w)$$

And for an inferior good:

$$EV(p^0, p^1, w) < AV(p^0, p^1, w) < CV(p^0, p^1, w)$$
Welfare Evaluation of Economic Changes

When \((p^1 - p^0)\) is small, a better approximation is:

\[
\int_{p_1^0}^{p_1^1} \tilde{h}_1(p_1, \bar{p}_{-1}, u^0) dp_1
\]

where \(\tilde{h}_1\) is a Taylor approximation of \(h_1\):

\[
\tilde{h}(p, u^0) = h(p, u^0) + D_p h(p, u^0)(p - p^0)
\]

This measure is also computable from the knowledge of the observable Walrasian demand function:

\[
\tilde{h}(p, u^0) = x(p, w) + S(p, w)(p - p^0)
\]

And since only the price of good 1 is changing:

\[
\tilde{h}_1(p_1, \bar{p}_{-1}, u^0) = x_1(p_1^0, \bar{p}_{-1}, w) + s_{11}(p_1^0, \bar{p}_{-1}, w)(p_1 - p_1^0)
\]

where

\[
s_{11}(p_1^0, \bar{p}_{-1}, w) = \frac{\partial x_1(p_1^0, w)}{\partial p_1} + \frac{\partial x_1(p_1^0, w)}{\partial w} x_1(p_1^0, w)
\]
The Strong Axiom of Revealed Preferences

It represents the conditions on consumer choice in the same fashion as WARP such that consumer behaviour can be rationalized by preferences.

**Definition (MWG 3.J.1):** The market demand function \( x(p, w) \)
Satisfies SARP if for any list

\[
(p^1, w^1), ..., (p^N, w^N)
\]

with \( x(p^{n-1}, w^{n-1}) \neq x(p^n, w^n) \) for all \( n \leq N - 1 \),

we have:

\[
p^N \cdot x(p^1, w^1) > w^N \text{ whenever } p^n \cdot x(p^{n+1}, w^{n+1}) \leq w^n \text{ for } \forall n \leq N - 1.
\]
The Strong Axiom of Revealed Preferences

Proposition (MWG 3.J.1): If the market demand function $x(p, w)$ satisfies SARP then there is a rational preference relation $\succsim$ that rationalizes $x(p, w)$, that is, such that for all $(p, w)$, $x(p, w) \succ y$ for every $y \neq x(p, w)$ with $y \in B_{p, w}$.

Note: For $L = 2$ the SARP and the WARP are equivalent. But for $L > 2$, the SARP is stronger than the WARP.