Microeconomics I for Ph.D.
Chapter three: General Equilibrium Under Uncertainty

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Outline

State dependent utility function

Market Economy with Contingent Commodities: Description

Arrow-Debreu Equilibrium

Sequential Trading

Asset Market

State dependent utility function

- We begin by discussing a convenient framework for modeling uncertain alternatives that, in contrast to the lottery apparatus, recognizes underlying states of nature.
- State of Nature representation of Uncertainty
  - we show a state by $s \in S$ and its corresponding probability by $\pi_s > 0$
  - where $\sum_s \pi_s$
- Every uncertain alternative (which usually is a monetary return) is realized with a probability

Definition

Random variable: A random variable is a function $g : S \longrightarrow \mathbb{R}_+$ that maps states into monetary outcomes

- Contingent commodity, if state s occurs, then you will receive 1 \$.
- Example: If a bookmaker offers you odds of 10 to 1 against a certain horse winning, he is saying he will give you £10 if it wins and you will pay him £1 if it loses.

Definition

Extended expected utility representation: the preference relation $\succsim$ has an *extended expected utility representation* if for every $s \in S$, there is a function $u_s : \mathbb{R}^1_+ \longrightarrow \mathbb{R}$ such that for any $(x_1, ..., x_S) \in \mathbb{R}^S_+$ and $(x'_1, ..., x'_S) \in \mathbb{R}^S_+$,

$$(x_1, ..., x_S) \succsim (x'_1, ..., x'_S) \text{ if and only if } \sum_s \pi_s u_s(x_s) \geq \sum_s \pi_s u_s(x'_s). \blacksquare$$

State dependent utility function

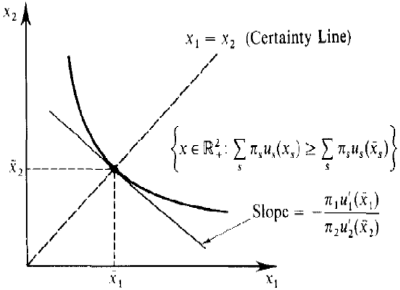

Figure: state dependent preferences

- The marginal rate of substitution at a point $(\bar{x}, \bar{x})$ is $\pi_1 u_1'(\bar{x})/u_2'(\bar{x})$.

State dependent utility function: Demand for insurance

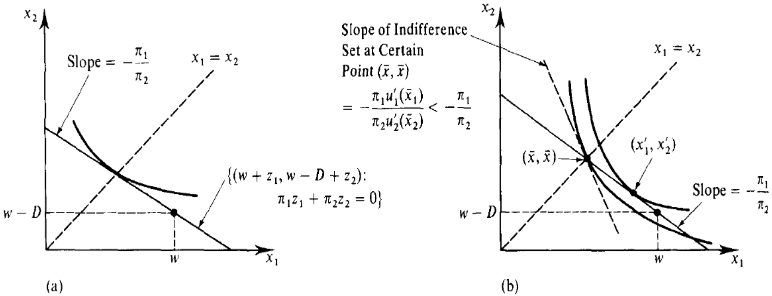

Figure: state dependent preferences

- The marginal rate of substitution at a point $(\bar{x}, \bar{x})$ for a state-dependent utility with non-uniform utility in each state is $\pi_1 u_1'(\bar{x})/\pi_2 u_2'(\bar{x}) < \pi_1/\pi_2$.

- We consider an economy with I consumers, J firms and L physical commodities.
- The new feature of in this section is that **TECHNOLOGY**, **ENDOWMENT**, and **PREFERENCES** are now ***uncertain***.
- The new elements are described by *state of the world.*
- let assume that there are S states $s = 1, ..., S$, like bad weather and good weather, $S = 2$.

Definition

Contingent Commodity: For every physical commodity $l = 1, ..., L$ and state $s = 1, ..., S$, a unit of (state)-contingent commodity ls is a title to receive a unit of the physical good l if and only if s occurs. Accordingly, a (state)-contingent commodity vector is specified by

$$\mathbf{x} = (x_{11}, ..., x_{L1}, ..., x_{1S}, ..., x_{LS}) \in \mathbb{R}^{LS} \tag{1}$$

and is understood as an entitlement to receive the commodity vector $(x_{1s}, ..., x_{Ls})$ if sate s occurs.

- A contingent commodity vector is a collection of L *random variables*, the l^{th} random variable being $(x_{l1}, ..., x_{lS})$.
- Endowments of consumer i is:

$$\omega_i = (\omega_{11i}, ..., \omega_{L1i}, ..., \omega_{1Si}, ..., \omega_{LSi}) \in \mathbb{R}^{LS} \tag{2}$$

- If state s occurs then consumer i has endowment vector $(\omega_{1si}, ..., \omega_{Lsi}) \in \mathbb{R}^{L}$.
- The preference of consumer i may also depend on the state of the world.
  - For example the consumer's enjoyment of some kind of food may well depend on the state of her health.
- the preference relation of consumer i is defined on the $X_i \subset \mathbb{R}^{LS}$, where $x_{lsi} \in x_i$.

Example

State dependent preference: Let the Bernouli state-dependent utility function is $u_{si}(x_{isi}, ..., x_{Lsi})$ we assign probability π_{si} to state s, then the preference of consumer i over two contingent commodity vectors x_i, $x_i' \in X_i \subset \mathbb{R}^{LS}$ satisfy

$$x_i \succsim_i x_i' \text{ if and only if}$$
$$\sum_s \pi_{si} u_{si}(x_{1si}, ..., x_{Lsi}) \geq \sum_s \pi_{si} u_{si}(x'_{1si}, ..., x'_{Lsi}).$$

- The technological possibilities of firm j are represented by a production set $Y_j \subset \mathbb{R}^{LS}$. Where the (state)-contingent production plan $y_j \in \mathbb{R}^{LS}$ is a member of Y_j if for every s the input-output vector $(y_{1sj}, ..., y_{Lsj})$ of physical commodities is feasible for firm j when state s occurs.

- Suppose there are two states, s_1 and s_2, representing good and bad weather. There are two physical commodities: seeds $(l = 1)$ and crops $(l = 2)$. In this case there are 4 elements for Y_j. Assume that seeds must be planted before the resolution of the uncertainty about the weather and that a unit of seeds produces a unit of crops if and only if the weather is good.

- Then
$$y_j = (y_{11j}, y_{21j}, y_{12j}, y_{22j}) = (-1, 1, -1, 0)$$
but the plan $(-1, 1, 0, 0)$ is not feasible.The seeds are planted in both states.

- to keep the model as simple as possible let assume that the ownership shares are constant across the states, however these could also be state dependent, θ_{ji} satisfies $\sum_j \theta_{ji} = 1$ for every i.

Information and the Resolution of Uncertainty

- In reality, states of the world unfold over time.

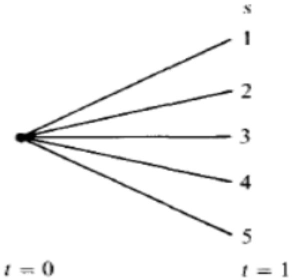

Figure: Two periods, perfect information at $t = 1$.

- We have a period 0 in which there is no information on the true state of the world and a period 1 in which this information has been completely revealed.

Arrow-Debreu Equilibrium: Introduction

- In the previous section we saw that how an economy where uncertainty matters can be described by means of:
  - a set states of the world S,
  - a Consumption set $X_i \subset \mathbb{R}^{LS}$,
  - an endowment vector $\omega_i \in \mathbb{R}^{LS}$,
  - and a preference relation $\succsim_i$ on X_i for every consumer i,
  - together with a production set $Y_j \subset \mathbb{Y}^{LS}$
  - and profit shares $(\theta_{j1}, ..., \theta_{jI})$ for every firm j.
- We now go a step further and make a stronger assumption.
  - We postulate the existence of a market for every contingent commodity ls.
  - These **markets open before the resolution** of uncertainty, at date 0.
  - The price of the commodity is denoted by p_{ls}.

Arrow-Debreu Equilibrium: Introduction

- We now go a step further and make a stronger assumption, **(cont.)**
  - What is being sold (or purchased) in the market at the date 0 is commitments to receive or to deliver amounts of the physical commodity l, if the state s occurs.
  - The information is symmetric across economic agents.

Arrow-Debreu Equilibrium

Definition

Arrow-Debreu Equilibrium: An allocation

$$(x_1^*, ..., x_I^*, y_1^*, ..., y_J^*) \in X_1 \times ... \times X_I \times Y_1 \times ... \times Y_J \subset \mathbb{R}^{LS(I+J)}$$

and a system of prices for the contingent commodities $p = (p_{11}, ..., p_{LS}) \in \mathbb{R}^{LS}$ constitute an arrow Debreu equilibrium if:

1. For every j, y_j^* satisfies $p.y_j^* \geq p.y_j$ for all $y_j \in Y_j$.
2. For every i, x_i^* is maximal for $\succsim_i$ in the budget set
$$\{x_i \in X_i : p.x_i \leq p.\omega_i + \sum_j \theta_{ij} p.y_j^*\}.$$
3. $\sum_i x_i^* = \sum_j y_j^* + \sum_i \omega_i$

Arrow-Debreu Equilibrium: Example

- $I = 2$ and $L = 1$ and $S = 2$

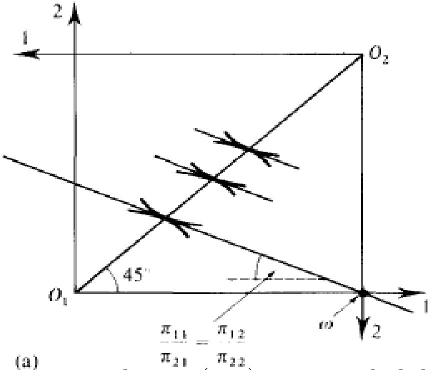

Figure: No Aggregate risk $\omega = (1, 1)$:same probability assessments$\pi_{11} = \pi_{12}$.

- we have $\omega_1 = (1, 0)$ and $\omega_1 = (0, 1)$, the endowments of individual i in the states 1 and 2,respectively.

Arrow-Debreu Equilibrium: Example

- In 4 aggregate uncertainty is zero, $\omega_1 + \omega_1 = (1, 1)$
- the utility functions are $\pi_{1i} u_i(x_{1i}) + \pi_{2i} u_i(x_{2i})$
- π_{1i} and $\pi_{2i}$are the subjective probabilities of states 1 and 2 for individual i.
- Marginal rate of substitution $(x_{1i}$ for $(x_{2i}$ equals π_{1i}/π_{2i}, because marginal utility of xi in two states are the same.
- If the subjective probabilities for each state is the same, then $MRS^1_{x_{11}\ for\ x_{21}} = MRS^2_{x_{12}\ for\ x_{22}}$.
- therefore at equilibrium, the two consumers consume the same quantity of x across the states.

Arrow-Debreu Equilibrium: Example

- $I = 2$ and $L = 1$ and $S = 2$

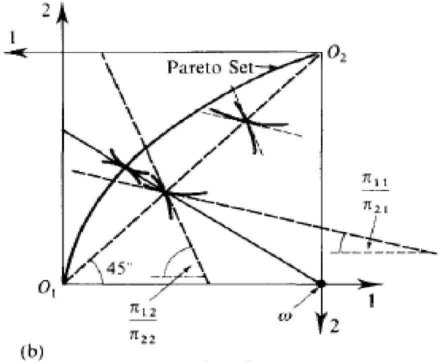

Figure: No Aggregate risk $\omega = (1, 1)$:different probability assessmentsπ_{11}, π_{12}.

Arrow-Debreu Equilibrium: Example

- in figure 5 aggregate uncertainty is zero, $\omega_1 + \omega_1 = (1, 1)$
- the utility functions are $\pi_{1i} u_i(x_{1i}) + \pi_{2i} u_i(x_{2i})$
- π_{1i} and $\pi_{2i}$are the subjective probabilities of states 1 and 2 for individual i.
- Marginal rate of substitution (x_{1i} for (x_{2i} equals π_{1i}/π_{2i}, because marginal utility of xi in two states are the same.
- If the subjective probabilities for each state is not the same($\pi_{11} < \pi_{12}$), then $MRS^1_{x_{11}\ for\ x_{21}} < MRS^2_{x_{12}\ for\ x_{22}}$.
- therefore at equilibrium, each consumer's equilibrium consumption is higher in the state he thinks comparatively more likely the same quantity of x across the states.

Arrow-Debreu Equilibrium: Example

- $I = 2$ and $L = 1$ and $S = 2$

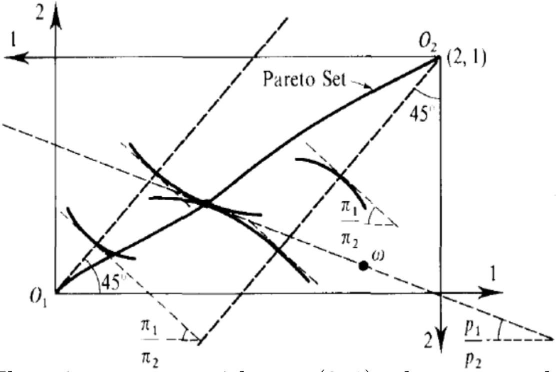

Figure: There is aggregate risk $\omega = (2, 1)$: the same probability assessmentsπ_{11}, π_{12}.

- The utilities are state independent and the probability assessments are the same for the two traders: (π_1, π_2).
- At any point of the Pareto set the common marginal rate of substitution is smaller than the ratio ratio of probabilities.

Proof.

- let (x_1, x_2) be a Pareto optimal allocation. Then $\pi_1 u_1'(x_{11})/\pi_2 u_1'(x_{21}) = \pi_1 u_2'(x_{12})/\pi_2 u_2'(x_{22})$.
- Hence $u_1'(x_{11})/u_1'(x_{21}) = u_2'(x_{12})/u_2'(x_{22})$. Denote this by $k > 0$.
- Then $k \geq 1$ **IFF** $u_1'(x_{11}) \geq u_1'(x_{21})$ and $u_2'(x_{12}) \geq u_2'(x_{22})$.
- But, according to the law of decreasing marginal utilities we must have; $x_{11} \leq x_{21}$ and $x_{12} \leq x_{22}$.

□

Proof.

Cont.

- This implies that
  $2 = \omega_{11} + \omega_{12} = x_{11} + x_{12} \leq x_{21} + x_{22} = \omega_{21} + \omega_{22} = 1.$
- This is a contradiction! So we must have $k < 1$.
- Hence $p_1/p_2 = \pi_1 u_1'(x_{11})/\pi_2 u_1'(x_{21}) = (\pi_1/\pi_2)k < (\pi_1/\pi_2)$.
- If we let $\pi_1 = \pi_2 = 0.5$, then $p_1 < p_2$.
- the price of one contingent unit of consumption is larger for the state for which the consumption good is scarcer.
- its implication in financial markets!

□

Sequential Trading

- **Spot Market**:What is a Spot Market? The spot market is where financial instruments, such as commodities, currencies and securities, are traded for immediate delivery.

- **Spot Price** The current price of a financial instrument is called the spot price. It is the price at which an instrument can be sold or bought at immediately.

- **Forward Market**:A forward market is an over-the-counter marketplace that sets the price of a financial instrument or asset for future delivery. Forward markets are used for trading a range of instruments, but the term is primarily used with reference to the foreign exchange market.

Sequential Trading

- **What Is Over-The-Counter?** Over-the-counter **(OTC)** refers to the process of how securities are traded for companies that are not listed on a formal exchange such as the New York Stock Exchange (NYSE).
- Securities that are traded over-the-counter are traded via a broker-dealer network as opposed to on a centralized exchange. These securities do not meet the requirements to have a listing on a standard market exchange.

Sequential Trading

- **What Is a Futures Market?** A futures market is an auction market in which participants buy and sell commodity and futures contracts for delivery on a specified future date.

- **Examples of futures markets are** the New York Mercantile Exchange, the Kansas City Board of Trade, the Chicago Mercantile Exchange, ***the Chicago Board Options Exchange*** and the Minneapolis Grain Exchange.

Sequential Trading

- The Arrow-Debreu framework provides a remarkable illustration of the power of **GE** theory, but it is hardly realistic.
- Indeed, at an Arrow-Debreu equilibrium all trade take place at time 0 an is a one shot decision.
- What does happen in reality?
- In reality trade take place sequentially over time, as a consequence of information disclosure.
- Consider a pure exchange economy in which we consider $X_i = \mathbb{R}_+^{LS}$.
- Let assume that there are two dates, $t = 0$ and $t = 1$.

Sequential Trading

- The uncertainty is resolved completely at $t = 1$, recall the date-event tree.
- We assume that there is no consumption at time $t = 0$.
- Suppose that markets for LS contingent commodities are set up at $t = 0$, and that $(x_1^*, ..., x_I^*) \in \mathbb{R}^{LSI}$ is an Arrow-Debreu Equilibrium allocation with $(p_{11}, ..., p_{LS})$.
- Recall that the market is a ***Forward Market***. In such a market contracts are executed, at $t = 1$, when the state s is revealed and every consumer i receives $x_{si}^* = (x_{1si}^*, ..., x_{Lsi}^*) \in \mathbb{R}^L$.
- Imagine that there is a ***spot market*** before the actual consumption of x_{si}^*, where the market is opened at $t = 1$.

Sequential Trading

- Would there be any incentive among consumers to trade in these markets? NO!.
- let assume there is a spot market in which consumers may trade in the state s, to increase their utility.
- Then $(x^*_{1i}, ..., x_{si}, ..., x^*_{Si}) \succsim_i (x^*_{1i}, ..., x^*_{si}, ..., x^*_{Si})$, but this contradicts with Pareto optimality of the Arrow-Debreu allocation $(x^*_1, ..., x^*_I) \in \mathbb{R}^{LSI}$.
- Matter will change if there was no contingent commodity market for some of LS commodities, at $t = 0$.
- Then, there would be incentives to reopen the markets and retrade, after revelation of the state s.

Sequential Trading

- Even if not all the contingent commodities are available at $t = 0$, it may still be the case under some conditions that the retarding possibilities at $t = 1$ guarantee that Pareto optimality is reached.

- **That is**, the presence of ***ex post*** trade can make up for an absence of ***ex ante*** markets.

- **Intuition**
  If spot trades can occur within each state, then the only task remaining at $t = 0$ is to transfer the consumer's overall purchasing power efficiently across the states.

- Example: At $t = 0$ consumers have *Expectations* regarding the spot prices prevailing at $t = 1$ for each possible state $s \in S$, the price vector for s is $p_s \in \mathbb{R}^L$ and for over expectation vector is denoted by $p = (p_1, ..., p_S) \in \mathbb{R}^{LS}$.

Sequential Trading

- Suppose that at date $t = 0$ there is trade in the S contingent commodities denoted by $(x_{11}, ..., x_{1S})$ which is contingently traded at $t = 0$ by prices $q = (q_1, ..., q_S) \in \mathbb{R}^S$.
- Faced with prices $q \in \mathbb{R}^S$ at $t = 0$ and expected SPOT prices $(p_1, ..., p_S) \in \mathbb{R}^{LS}$ at $t = 1$, every consumer i formulates a consumption, or trading plan $(z_{1i}, ..., z_{Si}) \in \mathbb{R}^S$ for contingent commodities at $t = 0$, as well as a set of spot market consumption plans $(x_{1i}, ..., x_{Si}) \in \mathbb{R}^{LS}$ for different states that may occur at $t = 1$.
- Let the $U_i(x_{1i}, ..., x_{Si}) = \sum_s \pi_{is} u_{is}(x_{si})$ be a Von-Newman Morgenstern utility function for $\succsim_i$.

Sequential Trading: consumer's problem

- the problem of consumer i is expressed as:

$$\max_{\substack{\{x_{1i},...,x_{Si}\}\in\mathbb{R}^{LS}_{+} \\ \{z_{1i},...,z_{Si}\}\in\mathbb{R}^{S}}} U_i(x_{1i},...,x_{Si}) = \sum_s \pi_{is} u_{is}(x_{si}). \qquad (3)$$

$$\text{s.t.}$$
$$\text{(i) } \sum_s q_s z_{si} \leq 0,$$
$$\text{(ii) } p_s.xsi \leq p_s.\omega_{si} + p_{1s} z_{si} \text{ for every } s.$$

- Restriction (i) is the budget constraint corresponding to $t = 0$.

- The family restrictions (ii) are budget constraints for different spot markets.

- The value of wealth at a state s is composed of two parts:
  - The market value of her initial endowments, $p_s.\omega_{si}$.
  - The market value of the amounts z_{si} of good 1 bought or sold at $t = 0$.
  - if z_{si} is negative and less that $-\omega_{1si}$, then i is making **short** sale at $t = 0$, namely $z_{si} + \omega_{si} < 0$, suggesting that her net trade amount from good 1 is negative and she sales *short*.
    - If s occurs she will actually have to buy in the spot market the extra amount of the first good required for the fulfillment of her commitment
  - if $z_{si} < 0$ but $z_{si} > -\omega$ then $z_{si} + \omega > 0$ and she is a seller in the good 1's contingent market at $t = 0$.
  - if $z_{si} > 0$, then she is buyer in the contingent market of good 1 at $t = 0$.

Self-fulfilled or Rational Expectations

- It is required that consumers' expectations of contingent prices will clear the spot markets for different states s do actually clear them once date $t = 1$ has arrived and a state s is revealed.

Definition

Radner equilibrium: A collection formed by a vector $q = (q_1, ..., q_S) \in \mathbb{R}^S$ for contingent first good commodities at $t = 0$, a spot price vector

$$p_s = (p_{1s}, ..., p_{Ls}) \in \mathbb{R}^L$$

for every s, and, for every consumer i, consumption plan $z_i^* = (z_{1i}^*, ..., z_{Si}^*) \in \mathbb{R}^S$ at $t = 0$ and $x_i^* = (x_{1i}^*, ..., x_{Si}^*) \in \mathbb{R}^{LS}$ at $t = 1$ constitutes a *Radner Equilibrium* if:
(i) For every i, the consumption plans z_i^*, x_i^* solve problem 3.
(ii) $\sum_i z_{si}^* \leq 0$ and $\sum_i x_{si}^* \leq \sum_i \omega_{si}$ for every s. ■

- In the following proposition it is shown that for this model the set of Arrow-Debreu equilibrium allocations and the set of Radner equilibrium allocations are identical.

Theorem

(i) If the allocation $x^* \in \mathbb{R}^{LSI}_{++}$ *and the contingent commodities price vector* $(p_1, ..., p_S) \in \mathbb{R}^{LS}_{++}$ *constitute an Arrow-Debreu equilibrium, then there are prices* $q \in \mathbb{R}^{S}_{++}$ *for contingent first good commodities and consumption plans for these commodities* $z^* = (z_1^*, ..., z_I^*) \in \mathbb{R}^{SI}$ *such that the consumptions plan* x^*, z^*, *the prices* q, *and the spot prices* $(p_1, ..., p_S)$*constitute a Radner equilibrium.*

(ii) Conversely, if the consumption plans $x_* \in \mathbb{R}^{LSI}, z^* \in \mathbb{R}^{SI}$ *and prices* $q \in \mathbb{R}^{S}_{++}, (p_1, ..., p_S) \in \mathbb{R}^{S}_{++}$ *constitute s Radner equilibrium, then there are multipliers* $(\mu_1, ..., \mu_S) \in \mathbb{R}^{S}_{++}$ *such that the allocation* x^* *and the contingent commodities price vector* $(\mu_1 p_1, ..., \mu_S p_S) \in \mathbb{R}^{LS}_{++}$ *constitute an Arrow-Debreu equilibrium.*

Arrow-Debreu Equilibrium, a pure exchange economy

Definition

Arrow-Debreu Equilibrium, a pure exchange economy:
An allocation

$$(x_1^*, ..., x_I^*) \in X_1 \times ... \times X_I \subset \mathbb{R}^{LSI}$$

and a system of prices for the contingent commodities $p = (p_{11}, ..., p_{LS}) \in \mathbb{R}^{LS}$ constitute an arrow Debreu equilibrium if:

1. For every i, x_i^* is maximal for $\succsim_i$ in the budget set
$$\{x_i \in X_i : p.x_i \leq p.\omega_i\}.$$
2. $\sum_i x_i^* = \sum_i \omega_i$

Proof.

- **the necessary condition:** If the allocation $x^* \in \mathbb{R}^{LSI}_{++}$constitute an Arrow-Debreu equilibrium, then there are prices $q \in \mathbb{R}^{S}_{++}$ for contingent first good commodities and consumption plans for these commodities $z^* = (z_1^*, ..., z_I^*) \in \mathbb{R}^{SI}$ such that the consumptions plan x^*, z^*, the prices q, and the spot prices $(p_1, ..., p_S)$constitute a Radner equilibrium.

- $\sum_s p_s.x_{si} \leq \sum_s p_s.\omega_{si}$
  $\sum_s p_s.(x_{si} - \omega_{si}) \leq 0$, which is identical to the budget set of Radner problem, because $\sum_s p_{1s}.z_{si} \leq 0$

- **the sufficient condition:**

$\square$

- $I = 2$ and $L = 2$ and $S = 2$

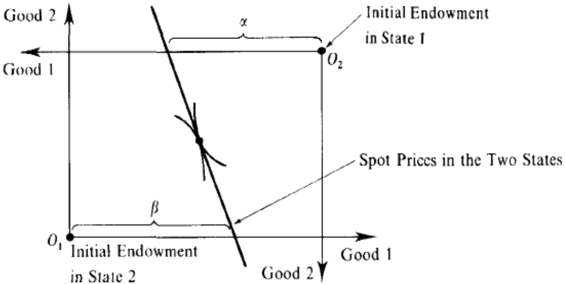

Figure: Reading the Arrow-Debreu equilibrium by means of contingent trade in the first good only, $\pi_1 = \pi_2$

Asset Market: Generalization of Radner Equilibrium

- There is a Given set of K assets, known as an asset structure at date $t = 0$
- Each asset k is characterized by a vector of returns $r_s = [r_{1k}, ..., r_{Sk}]' \in \mathbb{R}^S$.
- There is no initial endowments of assets, but **short** sales are possible.
- $q = (q_1, ..., q_K)$ denotes the assets price vector at $t = 0$.
- A vector of trades in these assets is denoted by$z = (z_1, ..., z_K) \in \mathbb{R}^K$ and we call it a ***portfolio***.
- The VNM utility function of consumer i is denoted by $U_i(x_{1i}, ..., x_{Si}) \in \mathbb{R}_+^{LS}$.

Definition

A collection formed by a price vector $q = (q_1, ..., q_K) \in \mathbb{R}^K$ for assets tradees at $t = 0$, a spot price vector $p_s = (p_{1s}, ..., p_{Ls}) \in \mathbb{R}^L$ for every s, and, for every consumer i, portfolio plans $z_i^* = (z_{1i}^*, ..., z_{Ki}^*) \in \mathbb{R}^K$ at $t = 0$ and consumption plans $x_i^* = (x_{1i}^*, ..., x_{Si}^*) \in \mathbb{R}^{LS}$ at $t = 1$ constitutes a ***Radner equilibrium*** if:

1. for every i, the consumption plans z_i^*, x_i^* solve the problem

$$\max_{\substack{\{x_{1i},...,x_{Si}\} \in \mathbb{R}_+^{LS} \\ \{z_{1i},...,z_{Ki}\} \in \mathbb{R}^K}} U_i(x_{1i}, ..., x_{Si}) = \sum_s \pi_{is} u_{is}(x_{si}).$$

   s.t. (a) $\sum_k q_k . z_k \leq 0$
   (b) $p_s . x_{si} \leq p_s . \omega_{si} + \sum_k p_{1s} z_{ki} r_{sk}$ for every s.

2. $\sum_i z_{ki}^* \leq 0$ and $\sum_i x_{si}^* \leq \sum_i \omega_{si}$ for every k and s.

Theorem

Assume that every return vector is nonnegative and nonzero; that is, $r_k \geq 0$ and $r_k \neq 0$ for all k. Then, for every (column) vector $q \in \mathbb{R}^K$ of asset prices arising in a Radner equilibrium, we can find multipliers $\mu = (\mu_1, ..., \mu_S) \geq 0$, such that $q_k = \sum_s \mu_s r_{sk}$ for all k.

- The proposition says tat we can assign values $(\mu_1, ..., \mu_S)$ to units of wealth in the different states so that the price of a unit of asset k is simply equal to the sum of the returns across states, or ***marginal utility of wealth*** across the sates.

- it is assumed that $U_i(x_{1i}, ..., x_{Si}) = \sum_s \pi_{is} u_{is}(x_{si})$..
- the Bernoulli utilities $u_{si}(.)$ are concave, strictly icreasing, and differentiable.
- Indirect utility function of consumer i in state s is denoted by $v_{si}(p_s, w_{si})$.
- consumer i in sate s has the wealth level $w^*_{si} = p_s.\omega_{si} + \sum_k r_{sk} z^*_{ki}$, remember that $p_{1s} = 1$.

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V

Independence of irrelevant alternative

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The general case: Arrow?s impossibility theorem

Independence of irrelevant alternative