Market Power

GholamReza Keshavarz Haddad

Sharif University of Technology
Graduate School of Management and Economics

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Outline

Monoloy pricing

- In this section we study the pricing behavior of a profit-maximizing monopolist, a firm tat is the only producer of a good.

- Which results in q^* and $p(q^*)$, the solution for the optimization problems are not necessarily the same
- We shall focus our analysis on the latter and impose the following restriction on the model

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- The decision variable is production level q

$$Max_{q \geqslant 0} \quad p(q).q - c(q) \tag{1}$$

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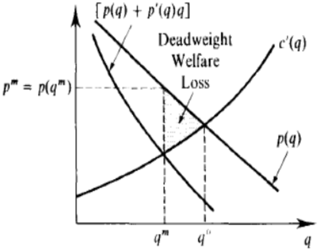
- Since $p'(q) < 0$ at all $q \geqslant 0$ the (4) with equality implies $p(q^m) > c'(q^m)$, so the price under monopoly exceeds marginal cost.
- One consequence of the pricing policy is, $(q^m \leqslant q^0)$

Performance comparison, monopoly and perfect competition

- Welfare loss and quantity distortion



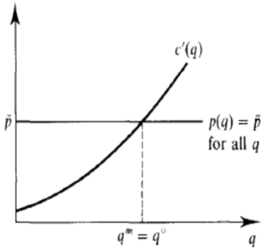


Figure 12.B.1 (left)
The monopoly solution and welfare loss when $p'(\cdot) < 0$.

Figure 12.B.2 (right)
The monopoly solution when $p'(q) = 0$ for all q.

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Example (Monopoly Pricing with a Linear Inverse Demand)

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- Monopolist's optimal output and price level are resulted from F.O.C (4), $a - 2bq = c$, and the corresponding price equilibrium level of q, respectively are $p^m = (a + c)/2$ and the is $q^m = (a - c)/2b$.

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- Bertrand model is a Price competition static game in which the monopoly is the firms simultaneously choose price levels to maximize their profits.

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The Bertrand model of price competition

- Sales for firm j are given by

$$x_j(p_j, p_k) = \begin{cases} x(p_j) & \text{if } \; p_j < p_k \\ \frac{1}{2}x(p_j) & \text{if } \; p_j = p_k \\ 0 & \text{if } \; p_j > p_k \end{cases}$$

Theorem (Bertrand duopoly model, Homogeneous products)

There is a unique Nash equilibrium (p_1^*, p_2^*) in the Bertrand duopoly model. In this equilibrium, both firms set their prices equal to cost:

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- Given prices p_j and p_k, firm j's profits are equal to $(p_j - c)x_j(p_j, p_k)$.
- The Nash equilibrium outcome of this model is presented and proved in the following Proposition.

Theorem (Bertrand duopoly model, Homogeneous products)

There is a unique Nash equilibrium (p_1^*, p_2^*) *in the Bertrand duopoly model. In this equilibrium, both firms set their prices equal to cost:*

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Proof.

- By the C.R.S, in the Nash equilibrium of $p_1^* = p_2^* = c$ the profits are equal to **ZERO**, because $\pi_j = (p_j - c)x_j(p_j, p_k)$.

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  - Both of the firms are incurring loss, is a Nash Equilibrium.
  - Then, one of the firms can increase their price and by losing its market it will get Zero profit, which is greater than negative profit, **NOT a Nash Equilibrium**

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  - With this deviation, the firm j is selling to the entire market and is getting strictly positive profit, **NOT a Nash Equilibrium**

$\square$

Proof.

- **As the final case, suppose that $p_j > c$ and $p_k > c$ and assume that $p_j \leqslant p_k$.**

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Theorem

In any Nash equilibrium of the Bertrand model with $J > 2$ firms, all sales take place at a price equal to cost.

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- **As the final case, suppose that $p_j > c$ and $p_k > c$ and assume that $p_j \leqslant p_k$.**
    - In this case firm k can earn at most $\frac{1}{2}(p_j - c)x(p_j)$, then by undercutting j's price $p_k = p_j - \epsilon$ he can occupy entire market and get the $(p_j - \epsilon - c)x(p_j - \epsilon)$

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  - Since $(p_j - \epsilon - c)x(p_j - \epsilon) > \frac{1}{2}(p_j - c)x(p_j)$ for small $\epsilon > 0$. Therefore, firm k will have profitable price deviation and **the setting is not a Nash Equilibrium as well**.

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Quantitative Competition for Homogeneous products, The Cournot Model

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In any Nash equilibrium of the Cournot duopoly model with cost $c > 0$ per unit for the two firms and an inverse demand function $p(.)$ satisfying $p'(q) < 0$ for all $q \geqslant 0$ and $p(0) > c$, the market price is greater than c (the competitive price) and smaller than the monopoly price.

Example

- Consider a market with the following structure: The market demand $p(q_1 + q_2) = a - b(q_1 + q_2)$, firms' cost functions $c(q_j) = cq_j$.

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- Joint monopoly Equilibrium $q_1^{Jm} = q_2^{Jm} = (a - c)/4b$
- Single monopoly Equilibrium, either $q_1^m = (a - c)/2b, \;\; q_2^m = 0$, or $q_1^m = 0, \;\; q_2^m = (a - c)/2b$

Quantitative Competition, F.O.Cs

- Competitive equilibrium: For firm 1: $p = a - b(q_1 + q_2) = c$, For firm 2 $p = a - b(q_1 + q_2) = c$, equilibrium output and market price respectively are: $q^0 = (a - c)/b$ and $p^0 = c$

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- Single Monopoly equilibrium: For a firm 1 or 2: $Max_q \quad \pi = (a - b(q))(q) - c(q), \quad q = q_1 + q_2$, and $p^m = (a + c)/2$, $q^m = (a - c)/2b$.

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  - Price comparison $p^c = (a + 2c)/3 < (a + c)/2 = p^m$.

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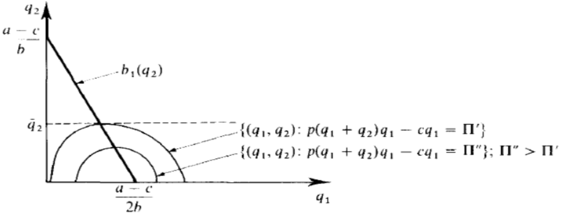
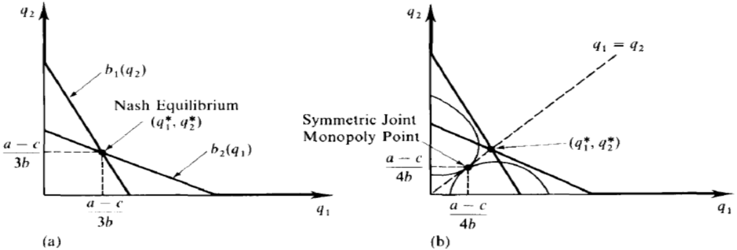
  - Price comparison $p^c = (a + 2c)/3 < (a + c)/2 = p^m$.
- For our example at hand,the last inequality is always true, $2(a + 2c) < 3(a + c)$ gives $a > c$, which is one of our underlying assumptions.

Quantitative Competition, Graphical representation

- Out-put and profit comparison





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$$Max_{p_j} \ (p_j - c)x_j(p_j, \bar{p}_{-j})$$

- Equilibrium price for firm j for $x_j(c, \bar{p}_{-j})$ is $(p_j > c)$, which gives strictly positive profit.

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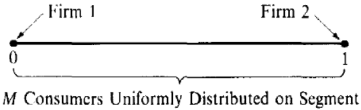

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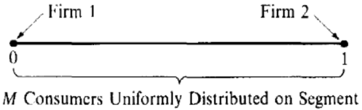

- A consumer's location is indexed by $z \in [0,1]$, the distance from left end of the city, and that from right end of the city is $(1-z)$

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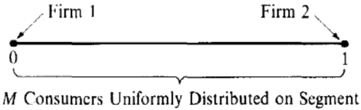

- A consumer's location is indexed by $z \in [0, 1]$, the distance from left end of the city, and that from right end of the city is $(1 - z)$
- The commodities are produced with a C.R.S technology.

Price Competition, Product differentiation: Bertrand model

Example (The linear city)

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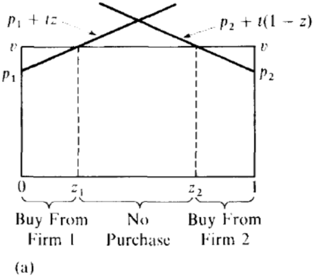

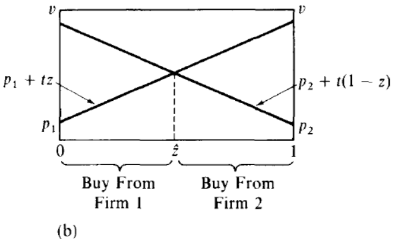

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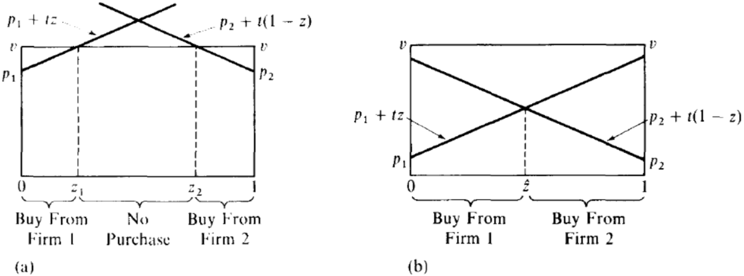

- In what follows, we assume that all consumers buy

Price Competition, Product differentiation: Bertrand model

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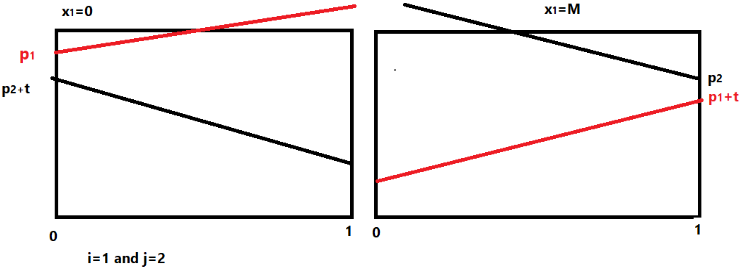
- Assuming $\hat{z} \in [0, 1]$, if i charges p_i and p_j then i's demand is given by:

$$x_i(p_i, p_j) = \begin{cases} 0 & \text{if } \; p_i > p_j + t \\ \frac{t+p_j-p_i}{2t}M & \text{if } \; p_i \in [p_j - t, p_j + t] \\ M & \text{if } \; p_i < p_j - t \end{cases}$$

Example (The linear city)

- Where, M is number of consumers in the linear city
- Two extreme cases, $x_i = 0, \hat{z} = 0$ and $x_i = M, \hat{z} = 1$



Example (The linear city)

- Assuming a symmetric market structure for the firms, demand for firm j is:

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  3. Otherwise ($p_i \in [p_j - t, p_j + t]$), and $\pi_i = (p_i - c)\frac{t+p_j-p_j}{2t}M$, (because $\hat{z} \in [0, 1]$)

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$$p_i = \begin{cases} p_j - t & \text{if } p_j \geqslant c + 3t \\ \frac{(t+p_j+c)}{2} & \text{if } p_j \in (c - t, c + 3t) \\ p_j + t & \text{if } p_j \leqslant c - t \end{cases}$$

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- Finally, for the interior equilibrium price are $p_i^* = p_j^* = c + t$, why? verify it!

Repeated interactions and price competition in heterogeneous product markets

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- The rival will react by cutting its own price to retaliate the policy.
- In this section we turn to a kind of dynamic games in which some firms may undercut its rival's price.

Repeated interactions: Formal setting and Assumptions

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- There is a discount factor $\delta < 1$
- Each firm attempt to maximize the discount value of profits $\Sigma_{t=1}^{\infty} \delta^{t-1} \pi_{jt}$

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- These are special kind of Dynamic Games in which the players play same static simultaneous-move games repeatedly.

$$p_{jt}(H_{t-1}) = \begin{cases} p^m & \text{if all elements of} H_{t-1} \text{equal} (p^m, p^m) \text{ or } t = 1 \\ c & \text{otherwise} \end{cases} \quad (1)$$

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- In the last period T we must be at the Bertrand solution with zero profit, and for all penultimate periods $(T-1)$.
- Now we extend the horizon to infinite number of periods.

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- However, if both firms follow the strategy defined in (1), then, they will end up with monopoly price in every period.

Repeated interactions: Proposition

Theorem

*The strategies described in (1) constitute a subgame perfect Nash equilibrium (**SPNE**) of the infinitely repeated Bertrand duopoly game if and only if* $\delta \geqslant \frac{1}{2}$.

Proof.

the theorem implies that, the firms never deviate the cooperation as long as $\delta \geqslant \frac{1}{2}$.
We have already proved it chapter 9 $\square$

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- We assume that the potential firms that are considering the entry are identical.

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2. All firms that have entry play some oligopolistic game, Bertrand or Cournot and so on.

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  1. No firm must want to change its entry decision given the entry decisions of the other firms.
  2. With this assumption, there is an equilibrium with J^* firms choosing to enter the market if and only if

$$\pi_{J^*} \geqslant K \tag{3}$$

and

$$\pi_{J^*+1} < K \tag{4}$$

Entry to the market as a two step process(game)

- The pure strategy subgame perfect Nash equilibria (**SPNE**s) of the model.
  1. No firm must want to change its entry decision given the entry decisions of the other firms.
  2. With this assumption, there is an equilibrium with J^* firms choosing to enter the market if and only if

$$\pi_{J^*} \geqslant K \tag{3}$$

and

$$\pi_{J^*+1} < K \tag{4}$$

- Condition 3 says that a firm that has chosen to enter does at least as well by doing so as it would do if it were to change its decision to "out", given the J^*

Entry to the market as a two step process(game): Optimal J

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- Condition 4 says that a firm that has decided to stay "out" of the market does strictly worse by Changing its decision to "in".
- It is reasonable to expect that π_J is decreasing in J and that $\pi_J \to 0$ and $J \to \infty$.
- One can assume a unique integer $\hat{J}$ such that $\pi_J \geqslant K$ for all $J \leqslant \hat{J}$ and $\pi_J < K$ for all $J > \hat{J}$, and so $J^* = \hat{J}$.

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Example (Optimal J with Cournot Competition)

- Consider an industry with the aggregate demand $p(q) = a - bq$, the firms cost function $c(q) = cq$ with $a > c \geqslant 0$ and $b > 0$.

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- The stage 2 output per firm, q_J, and profit per firm, π_J, are given by:

$$\pi_j(q) = [a - b(q_1 + q_2 + ... + q_j + ... + q_J)]q_j - cq_j$$

$$\frac{\partial \pi_j}{\partial q_j} = [a - b(q_1 + q_2 + ... + 2q_j + ... + q_J)] - c = 0$$

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- We have J symmetric linear equation with the following solution for firm J

$$q_J = \left(\frac{a - c}{b}\right)\left(\frac{1}{J + 1}\right) \tag{5}$$

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Example (Optimal J with Cournot Competition)

- The profit per firm

$$\pi_J = \left(\frac{a-c}{J+1}\right)^2 \left(\frac{1}{b}\right) \tag{6}$$

1. $\lim_{\to\infty} Jq_J \to (a-c)/b$
2. So, the aggregate $q = q^0$ approaches the competitive level.

- What determines the optimal level of J^*?
- the minimum profit level for entering to the market is $\pi_J = K$, then:

$$(\tilde{J}+1)^2 = \frac{(a-c)^2}{bK}$$

or $\tilde{J} = \frac{(a-c)}{\sqrt{bK}} - 1$

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Theorem (Excess entery in the presence of market power)

- *Assumptions:*
  1. *(A1)* $Jq_J \geqslant J'q_J$, *whenever* $J > J'$;
  2. *(A2)* $q_J \leqslant q_{J'}$, *whenever* $J > J'$;
  3. *(A3)* $p(Jq_J) - c'(q_J) \geqslant 0$ *for all* J.

Suppose that conditions (A1) to (A3) are satisfied by the post-entry oligapoly game, that $p'(.) < 0$, *and that* $c"(.) \geqslant 0$. *Then, the equilibrium number of entrants,* J^*, *is at least* $J^0 - 1$, *where* J^0 *is the socially optimal number of entrants.*