Sharif University of Technology

Financial Contagion

by Allen and Gale

JPE, 2000

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Outline

1. Motivation
2. Interbank markets
3. Fragility
4. Conclusion

1. Motivation

- Financial crisis are prevalent
- Financial crises are important because they raise the costsof intermediation and restrict credit
- Small shocks spread to the rest of the financial sector
- U.S. subprime mortgage market $\longrightarrow$ effect all over the world

- t = 0,1,2
- Continuum of agents, identical at t = 0
- Deposit one unit of consumption at t = 0 in exchange of (c_1, c_2)
- Agents may store between t = 1 and t = 2 at no cost
- Privately observed idiosyncratic preference shock at t = 1,

$$U(c_1, c_2) = \begin{cases} u(c_1), & \omega \\ u(c_2), & 1 - \omega \end{cases}$$

- Intermediary has access to two assets
  - Short-term: return of 1
  - Long-term: return of $R > 1$ at t = 2 or $r < 1$ at t = 1

2. Interbank Markets

- Multiple equilibria: even identical intermediaries can encounter different demands for liquidity

Allen & Gale (2000)

- Similar set-up
- Contagion: overlapping claims banks have on one another
- Complete information
- Focus on the Pareto-efficient equilibrium (no sunspots)

- Intermediaries are identical at $t = 0$
- At $t = 1$, privately observe proportion of early dyers, $\omega_H > \omega_L$

TABLE 1

REGIONAL LIQUIDITY SHOCKS

	A	B	C	D
S_1	ω_H	ω_L	ω_H	ω_L
S_2	ω_L	ω_H	ω_L	ω_H

Let $\gamma = \frac{\omega_H + \omega_L}{2}$

- Social planner

$$\max_{c_1, c_2, x, y} \gamma u(c_1) + (1-\gamma) u(c_2)$$

$$s.t. \quad x + y \leq 1, \quad \gamma c_1 \leq y, \quad (1-\gamma) c_2 \leq Rx$$

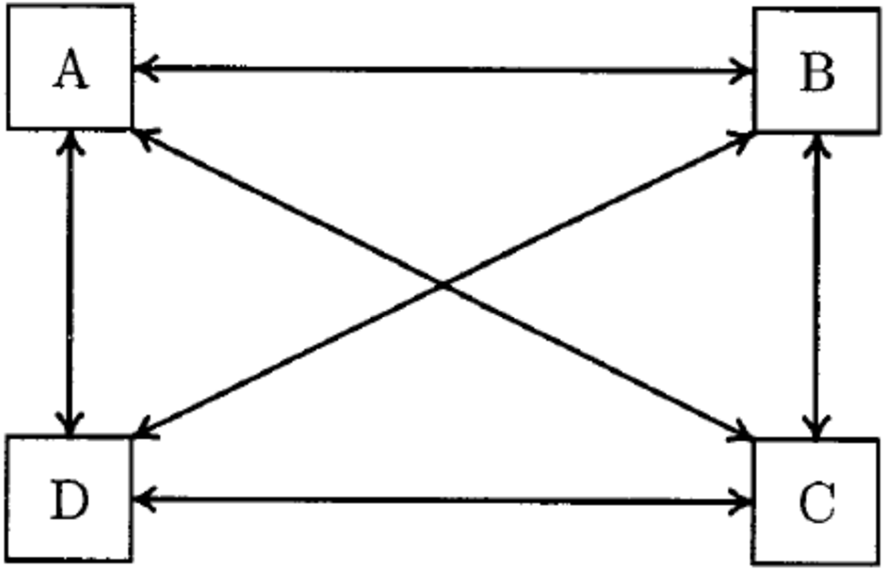

FIG. 1.—Complete market structure

- Each bank holds $z^i = \frac{(\omega_H - \gamma)}{2}$ deposits in each region $j \neq i$

- At $t = 1$, high demand for liquidity banks

$$\left(\omega_H + \frac{\omega_H - \gamma}{2}\right) c_1 = y + \frac{3(\omega_H - \gamma)}{2} \Leftrightarrow \gamma c_1 = y$$

- At $t = 1$, low demand for liquidity banks

$$(\omega_L + \omega_H - \gamma) c_1 = y \Leftrightarrow \gamma c_1 = y$$

- At t = 2, banks that had high demand for liquidity

$$[(1-\omega_H) + (\omega_H - \gamma)]c_2 = Rx \Leftrightarrow (1-\gamma)c_2 = Rx$$

- At t = 2, banks that had low demand for liquidity

$$\left[(1-\omega_L) + \frac{\omega_H - \gamma}{2}\right]c_2 = Rx + \frac{3(\omega_H - \gamma)}{2} \Leftrightarrow (1-\gamma)c_2 = Rx$$

- All constraints are satisfied ⇒ First best is achieved

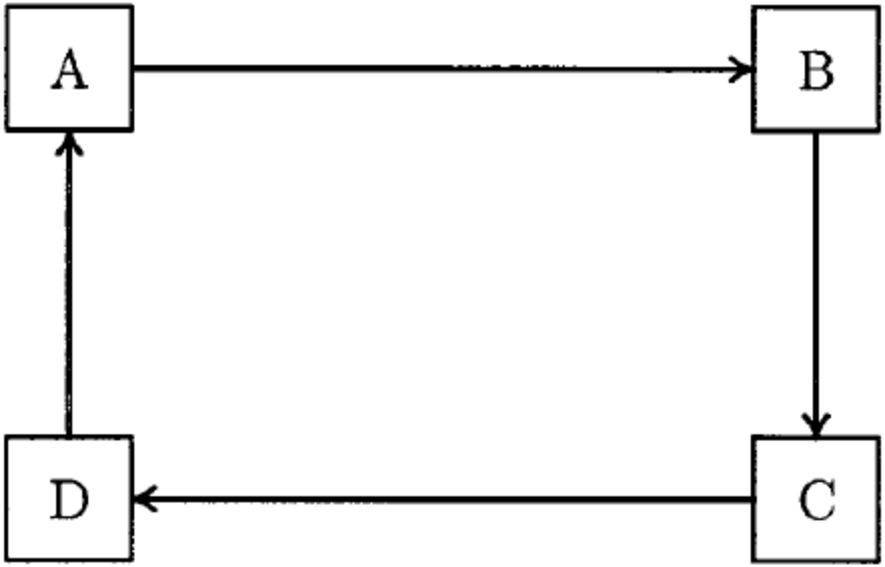

FIG. 2.—Incomplete market structure

- Each bank holds $z^i = \omega_H - \gamma$ deposits in region $i + 1$
- Constraints of the social planner problem are satisfied
- First best achieved

Question:

- How susceptible are different network structures to liquidity shocks?

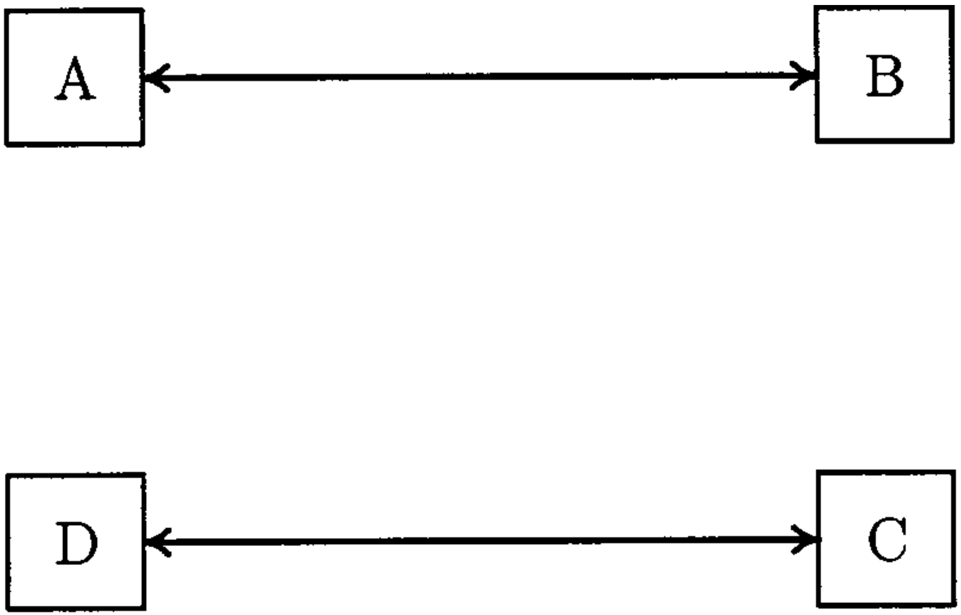

FIG. 3.—Disconnected incomplete market structure

3. Fragility

TABLE 2

REGIONAL LIQUIDITY SHOCKS WITH PERTURBATION

	A	B	C	D
S_1	ω_H	ω_L	ω_H	ω_L
S_2	ω_L	ω_H	ω_L	ω_H
$\bar{S}$	$\gamma + \epsilon$	γ	γ	γ

- $\bar{S}$ occurs with zero probability
- Does not change optimal allocation at $t = 0$

Continuation equilibrium at t = 1

- Consumers decide when to withdraw
  - Early consumers always withdraw at t = 1
  - Late consumers withdraw at t = 2 iff $c_2 \geq c_1$

- Banks pay c_1 to whoever demands liquidity at t = 1
  - Solvency, Insolvency, Bankruptcy

- Pecking order: short assets, deposits, long assets

Continuation equilibrium at t = 1, state $\bar{S}$

- $q^A < c_1 \Rightarrow$ all will withdraw from bank A
- $q^A \leq \frac{y+rx+zc_1}{1+z} \equiv \bar{q}^A$
- In state $\bar{S}$, A is insolvent
- Avoid run $\Leftrightarrow c_2 \geq c_1 \Leftrightarrow$ keep at least $\frac{(1-\omega)c_1}{R}$ of the long asset
- Buffer $b(\omega) = r\left[x - \frac{(1-\omega)c_1}{R}\right]$

- Bank A avoids a run iff

$$\epsilon c_1 \leq b(\gamma + \epsilon)$$

- Interbank deposits are liquidated
    - Cancel out if $q^A = c_1$
    - If $q^A < c_1$, D gets insolvent
- D goes bankrupt $\Rightarrow$ all banks go bankrupt
- Lower bound for spill over effect

$$z(c_1 - \bar{q}^A)$$

Proposition 2

- Under the equilibrium allocation at $t = 0$
- $\bar{S} \Rightarrow A$ is insolvent
- A bankrupt $\Leftrightarrow \epsilon c_1 < b(\gamma + \epsilon)$
- $z(c_1 - \bar{q}^A) > b(\gamma) \Rightarrow D$ goes bankrupt $\Rightarrow$ all go bankrupt

- We can find parameters such that
  - Incomplete market: all banks go bankrupt
  - Complete market: no bankruptcy

Complete Market

- Each bank hold $\frac{z}{2} = \frac{\omega_H - \gamma}{2}$ deposit in each other bank

If $\bar{S}$ occurs, under conditions of proposition two

- A goes bankrupt
- Assuming no other region is bankrupt

$$\bar{q}^{A^*} = \frac{y + rx + \frac{3z}{2} c_1}{1 + \frac{3z}{2}}$$

- Loss of other banks is $\frac{z}{2}(c_1 - \bar{q}^{A^*})$
- Banks go insolvent
- Do not go bankrupt $\Leftrightarrow \frac{z}{2}(c_1 - \bar{q}^{A^*}) \leq b(\gamma)$

In Figure 3, if $\bar{S}$ occurs

  - A goes bankrupt, B is insolvent (or bankrupt)
  - C and D are not affected

4. Conclusion

- Links expose the system to contagion
- Incomplete networks are more prone to contagion than complete structures
- Better connected networks: proportion of the losses in one bank's portfolio is transferred to more banks
- Incomplete network: failure of a bank may trigger the failure of the entire banking system