# A Macroeconomic Model with Financial Panics

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# Motivation



- Develop a model to analyze both qualitatively and quantitatively the dynamics of a financial crisis

- Incorporate banks and banking panics within a conventional macroeconomic framework - a New Keynesian model with capital accumulation

- Characterizing the sudden and discrete nature of banking panics as well as the circumstances that makes the economy vulnerable

# Result



- How crises may occur even in the absence of large exogenous shock to the economy
- Model generates the highly nonlinear contraction in economic activity associated with financial crises
- The model is broadly consistent with the recent financial crisis

# Model



- Capital at the beginning of period $t$, $K_t$
- Capital at the end of period $t$, $S_t$

$$S_t = \Gamma\left(\frac{I_t}{K_t}\right)K_t + (1-\delta)K_t$$

- Firm wishing to finance new investment as well as old capital issues a state-contingent claim on the earnings generated by the capital

$$S_t^b + S_t^h = S_t$$

- A multiplicative "capital quality" shock, $\xi_{t+1}$, that randomly transforms the units of capital available at $t$:

$$K_{t+1} = \xi_{t+1}S_t$$

# Model



- Households are less efficient than bankers in handling investments
- They suffer a management cost that depends on the share of capital they hold, $S_t^h / S_t$
- The cost is in utility terms and takes the following piece-wise form:

$$\varsigma(S_t^h, S_t) = \begin{cases} \dfrac{\chi}{2}\left(\dfrac{S_t^h}{S_t} - \gamma\right)^2 S_t, & if\ \dfrac{S_t^h}{S_t} > \gamma > 0 \\ 0, otherwise \end{cases}$$

# Model



- Saving: claims on capital, deposits at banks
- Two reasons for default: insolvency, bank run
- Gross rate of return on the deposit contract:

$$R_{t+1} = \begin{cases} \bar{R}_{t+1} \; with\; probability\; 1 - p_t \\ x_{t+1}\bar{R}_{t+1} \; with\; probability\; p_t \end{cases}$$

# Model: Households



- Household: $1 - f$ workers and $f$ bankers
- With i.i.d. probability $1 - \sigma$ , a banker exits: Gives all its accumulated earnings to the household
- Each period, $(1 - \sigma)f$ workers become bankers
- Household provides each new banker with an exogenously given initial equity stake in the form of a wealth transfer, $e_t$

# Model: Households



- Household problem:

$$Max\ U_t = E_t\left\{\sum_{\tau=t}^{\infty}\beta^{\tau-t}\left[\frac{C_\tau^{1-\gamma_h}}{1-\gamma_h}-\frac{L_\tau^{1+\varphi}}{1+\varphi}-\varsigma(S_\tau^h,S_\tau)\right]\right\}$$

$$s.t.\ \ C_t + D_t + Q_t S_t^h = w_t L_t - T_t + \Pi_t + R_t D_{t-1} + \xi_t[Z_t + (1-\delta)Q_t]S_{t-1}^h$$

- First order condition:

$$\lambda_t = (C_t)^{-\gamma_h}$$

$$w_t\lambda_t = (L_t)^{\varphi}$$

$$1 = [(1-p_t)E_t(\Lambda_{t+1}|no\ def) + p_t E_t(\Lambda_{t+1}x_{t+1}|def)].\bar{R}_{t+1}$$

$$E_t\left(\Lambda_{t+1}\xi_{t+1}\frac{Z_{t+1}+(1-\delta)Q_{t+1}}{Q_t+\frac{\partial\varsigma(S_t^h,S_t)}{\partial S_t^h}/\lambda_t}\right)$$

$$stochastic\ discount\ factor{:}\ \Lambda_{t+1} = \beta\frac{\lambda_{t+1}}{\lambda_t}$$

# Model: Banks



- Discounted expected value of net worth upon exit:

$$V_t = E_t\{\Lambda_{t+1}[(1-\sigma)n_{t+1} + \sigma V_{t+1}]\}$$

- Asset holding in each period (new or survived):

$$Q_t s_t^b = d_t + n_t$$

- Net worth of surviving bankers:

$$n_t = R_t^b Q_{t-1} s_{t-1}^b - R_t d_{t-1}$$

Where $R_t^b$ is the gross rate of return on capital intermediated by banks:

$$R_t^b = \xi_t \frac{Z_t + (1-\delta)Q_t}{Q_{t-1}}$$

- Recovery rate in default situation ($n_t < 0$):

$$x_t = \frac{R_t^b Q_{t-1} s_{t-1}^b}{\bar{R}_t d_{t-1}} < 1$$

# Model: Banks



- Moral hazard problem of banks:

$$\theta Q_t s_t^b \leq V_t$$

- So the problem for bakers is:

$$Max\ \frac{V_t}{n_t} = \mu_t \phi_t + v_t$$

$$s.t. \begin{cases} \theta \phi_t \leq \mu_t \phi_t + v_t \\ \bar{R}_{t+1} = [(1-p_t)E_t(\Lambda_{t+1} | no\ def) + p_t E_t(\Lambda_{t+1} x_{t+1} | def)]^{-1} \end{cases}$$

- Expected discounted excess return on banks assets relative to deposits:

$$\mu_t = (1-p_t)E_t\{\Omega_{t+1}\left(R_{t+1}^b - \bar{R}_{t+1}\right) | no\ def\}$$

- Expected discounted cost of a unit of deposits:

$$v_t = (1-p_t)E_t\{\Omega_{t+1}\bar{R}_{t+1} | no\ def\}$$

$$where\ \ \Omega_{t+1} = \Lambda_{t+1}(1-\sigma+\sigma\psi_{t+1}), \phi_t = \frac{Q_t s_t^b}{n_t}, \psi_{t+1} = \frac{V_{t+1}}{n_{t+1}}$$

# Model: Banks



- Expected discounted marginal return to increasing leverage multiple:

$$\mu_t^r = \frac{d\psi_t}{d\phi_t} = \mu_t - (\phi_t - 1)\frac{v_t}{\bar{R}_{t+1}}\frac{d\bar{R}_{t+1}(\phi_t)}{d\phi_t} < \mu_t$$

- Maximization results:

$$\begin{cases} \phi_t = \dfrac{v_t}{\theta - \mu_t} \; if\, \mu_t^r > 0 \\ \phi_t < \dfrac{v_t}{\theta - \mu_t} \; if\, \mu_t^r = 0 \end{cases}$$

# Model: Banks



- Aggregation of the financial sector absent default:

$$\phi_t = \frac{Q_t K_t^b}{N_t}$$

- Evolution of aggregate net worth:

$$e_t = \frac{\zeta}{(1-\sigma)f} S_{t-1}$$

$$N_t = \sigma\left[\left(R_t^b - \bar{R}_t\right)\phi_{t-1} + \bar{R}_t\right] N_{t-1} + \zeta S_{t-1}$$

# Model: Banks



- Condition for a bank run equilibrium

$$x_t^R = \frac{[\xi_t(1-\delta)Q_t^* + Z_t]S_{t-1}^b}{\bar{R}_t D_{t-1}} = \frac{R_t^{b*}}{\bar{R}_t} \cdot \frac{\phi_{t-1}}{\phi_{t-1} - 1} < 1$$

- The liquidation price

$$S_t^h = S_t$$
$$N_{t+1} = \xi S_t$$

Then:

$$Q_t^* = E_t \left\{ \sum_{\tau=t+1}^{\infty} \tilde{\Lambda}_{t,\tau} (1-\delta)^{\tau-t-1} \left( \prod_{j=t+1}^{\tau} \xi_j \right) \cdot \left[ Z_\tau - \chi \left( \frac{S_t^h}{S_t} \right) / \lambda_t \right] \right\} - \chi(1-\gamma)/\lambda_t$$

# Model



- Probability of insolvency

$$x(\xi^I_{t+1}) = \frac{\xi^I_{t+1}[Z_{t+1}(\xi^I_{t+1}) + (1-\delta)Q_{t+1}(\xi^I_{t+1})]S^b_t}{\bar{R}_t D_t} = 1$$

$$p^I_t = prob_t(\xi_{t+1} < \xi^I_{t+1})$$

- Probability of bank run: probability of a run, probability a run equilibrium exists:

$$p^R_t = w_t.\eta$$

$$x(\xi^I_{t+1}) = \frac{\xi^I_{t+1}[Z_{t+1}(\xi^I_{t+1}) + (1-\delta)Q^*\,(\xi^I_{t+1})]S^b_t}{\bar{R}_t D_t} = 1$$

$$w_t = prob_t(\xi^I_{t+1} \leq \xi_{t+1} < \xi^R_{t+1})$$

$$p_t = p^I_t + p^R_t$$

# Model: Firms



- Production

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{\epsilon-1}{\epsilon}} df\right]^{\frac{\epsilon}{\epsilon-1}}$$

$$Y_t(f) = \left[\frac{P_t(f)}{P_t}\right]^{-\epsilon} Y_t$$

$$P_t = \left[\int_0^1 P_t(f)^{1-\epsilon} df\right]^{\frac{1}{1-\epsilon}}$$

$$Y_t(f) = A_t K_t(f)^{\alpha} L_t(f)^{1-\alpha}$$

$$MC_t = \frac{1}{A_t}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{Z_t}{\alpha}\right)^{\alpha}$$

# Model: Firms



- Maximizing expected discount value of profit:

$$E_t\left\{\sum_{\tau=t}^{\infty}\Lambda_{t,\tau}\left[\left(\frac{P_\tau(f)}{P_\tau}-MC_\tau\right)Y_\tau(f)-\frac{\rho^r}{2}Y_\tau\left(\frac{P_\tau(f)}{P_{\tau-1}(f)}-1\right)^2\right]\right\}$$

$$s.t.\ \ Y_t(f)=\left[\frac{P_t(f)}{P_t}\right]^{-\epsilon}Y_t$$

- Philip's curve:

$$(\pi_t-1)\pi_t=\frac{\epsilon}{\rho^r}\left(MC_t-\frac{\epsilon-1}{\epsilon}\right)+E_t\left[\Lambda_{t,t+i}\frac{Y_{t+1}}{Y_t}(\pi_{t+1}-1)\pi_{t+1}\right]$$

# Model: Firms/Central Bank



- Capital producers:

$$\underset{I_t(j)}{\text{Max}}\, Q_t \Gamma\left(\frac{I_t(j)}{K_t}\right) K_t - I_t(j)$$

$$Q_t = \left[\Gamma'\left(\frac{I_t(j)}{K_t}\right)\right]^{-1}$$

- Monetary Policy:

$$R_t^n = \frac{1}{\beta}(\pi_t)^{\kappa_\pi}(\Theta_t)^{\kappa_y}$$

- Cyclical resource utilization:

$$\Theta_t = \frac{1+\mu}{1+\mu_t}, 1+\mu = \frac{\epsilon}{\epsilon-1}, 1+\mu_t = MC_t^{-1} = \frac{(1-\alpha)(Y_t/L_t)}{L_t^{\varphi} C_t^{\gamma_h}}$$

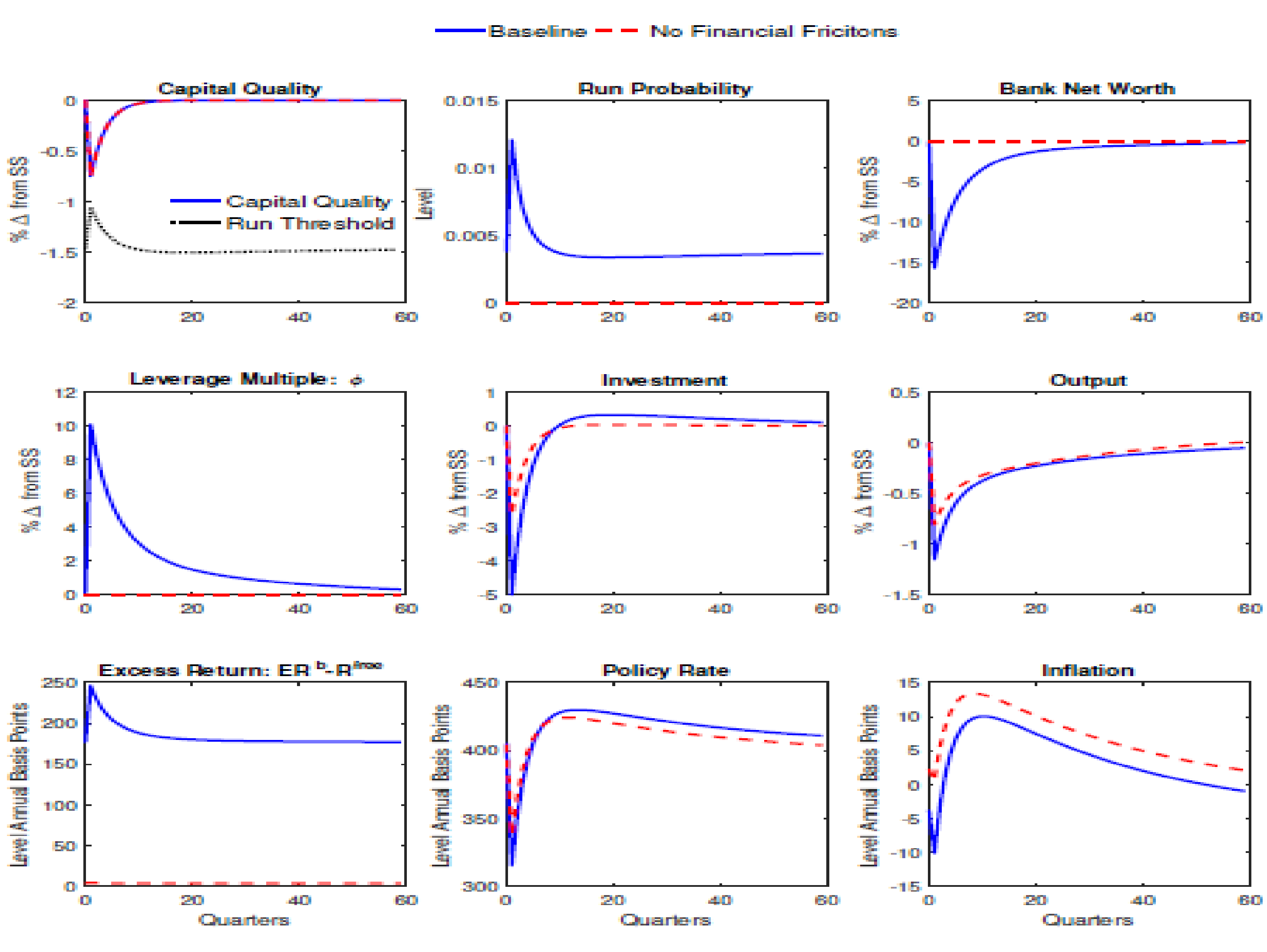
- Zero net supply of bonds! Household Euler equation to price nominal interest:

$$E_t\left(\Lambda_{t,t+1}\frac{R_t^n}{\pi_{t+1}}\right) = 1$$

# Shock Analysis

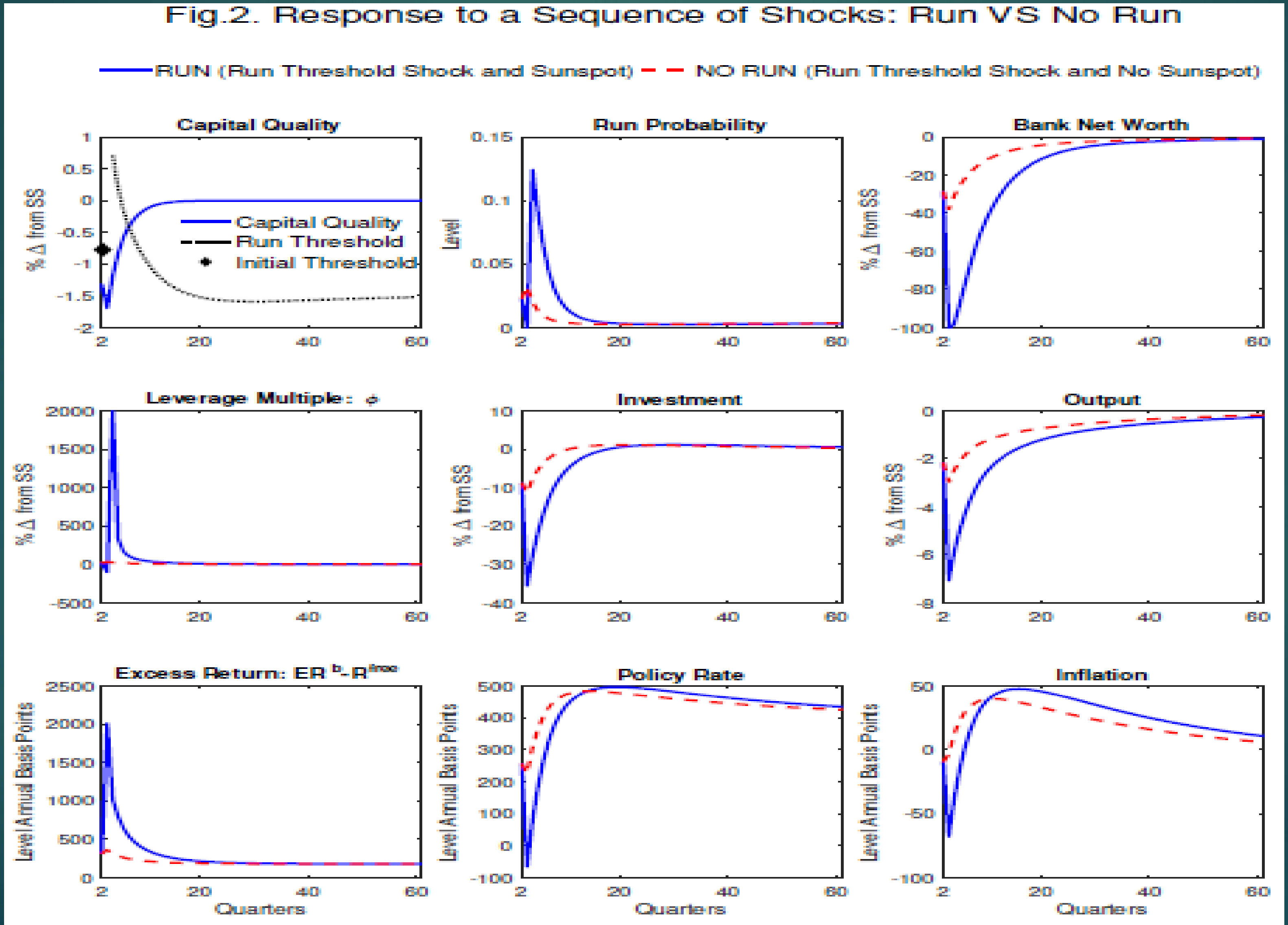


Fig. 1. Response to a Capital Quality Shock (1 std): No Run Case



# Shock Analysis

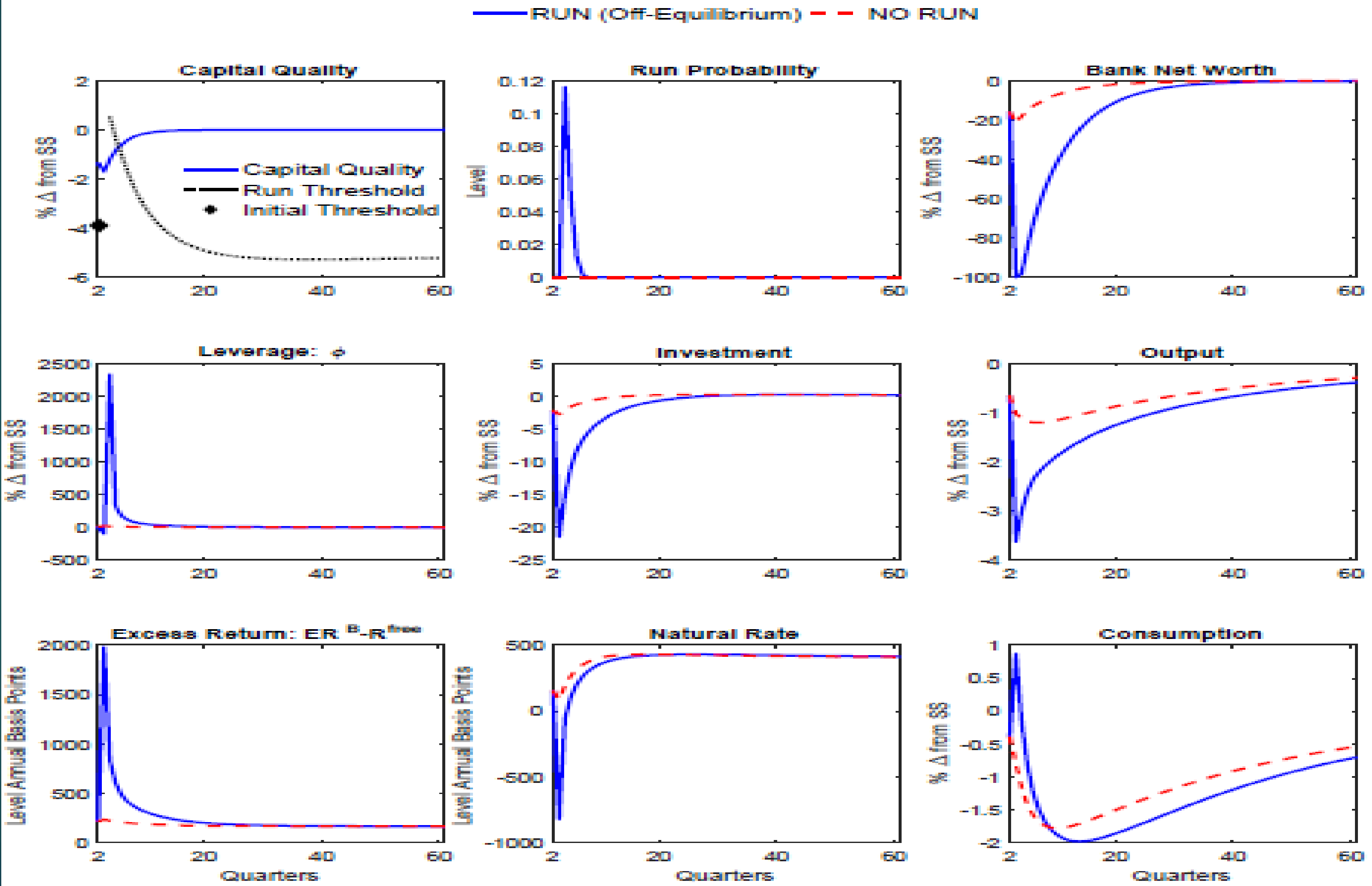




# Shock Analysis

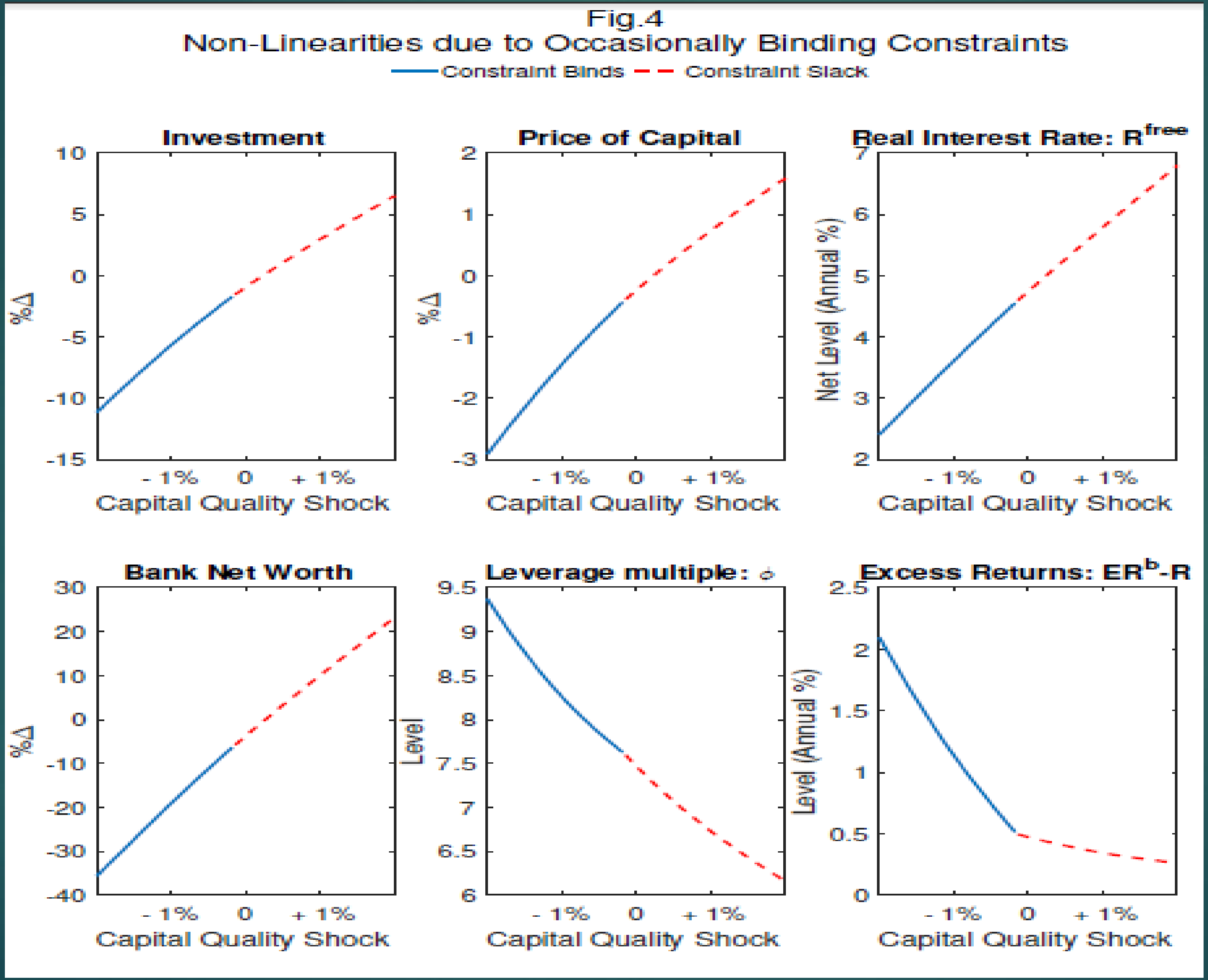


Fig.3. Response to the Same Sequence of Shocks in Flex Price Economy: Run VS No Run



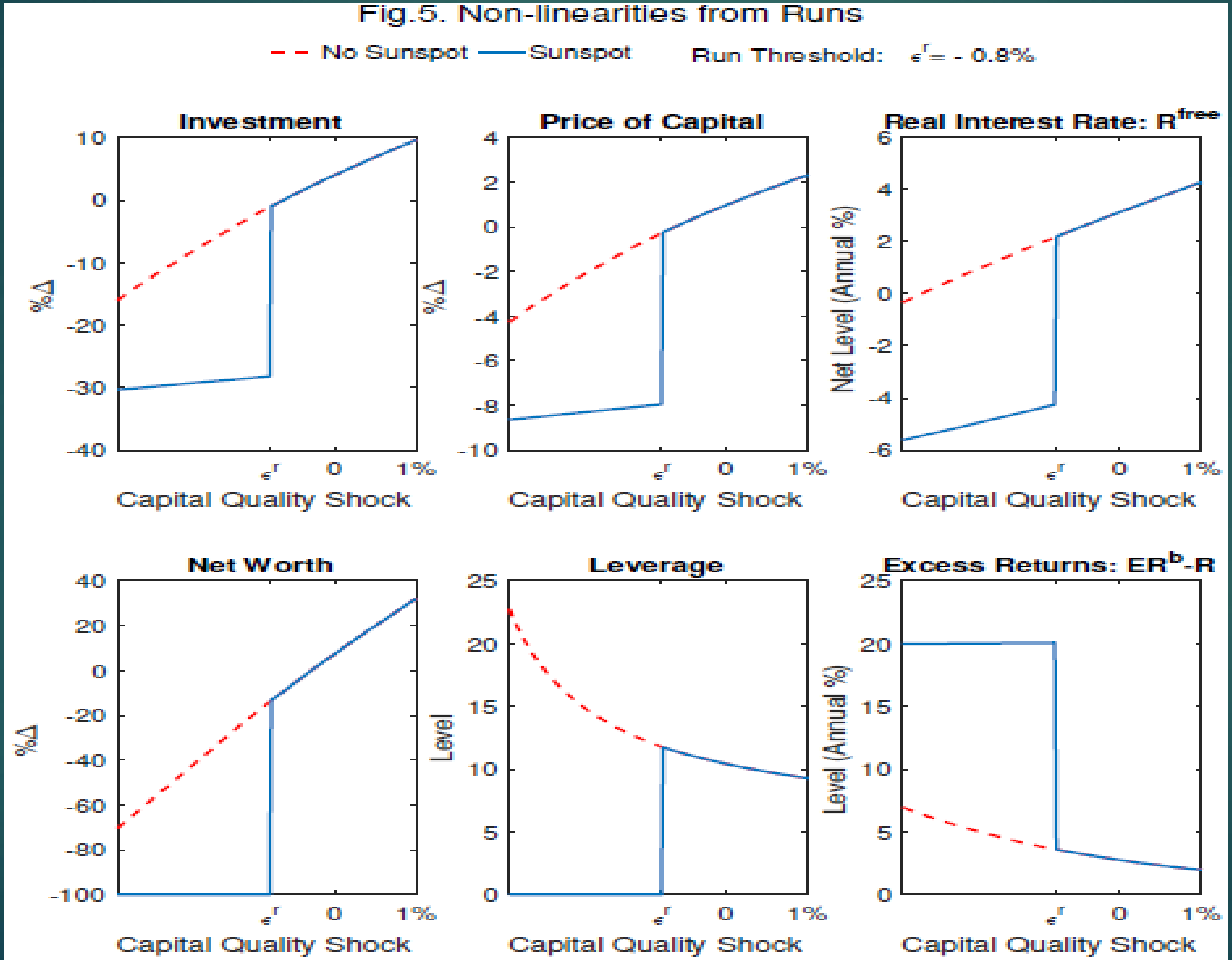
# Nonlinearity





# Nonlinearity





# Financial crisis and model