

## In the name of Allah

A Model of Reference-Dependent Preferences

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# Main claim

- We consider the **essential intuitions** in prospect theory and subsequent models of reference dependence, but **extend and modify these models** to develop a **more generally applicable theory**.
- deriving a corresponding reference-dependent model from a reference-independent model based on consumption utility
- develop a model of **reference-dependent** preferences and **loss aversion**.

## Example: The theory

- Supports the "endowment effect" found in the **laboratory**.
- Makes the less common prediction that the endowment effect among such owners and nonowners with **no predisposition to trade** will disappear among sellers and buyers **in real-world markets who expect to trade**.

# REFERENCE- DEPENDENT UTILITY

# Choice outcome evaluation

- its contrast with a reference point as intrinsic taste for the outcome.
- In **deterministic** environments, choices maximize **consumption utility**.
- **gain-loss utility** influences behavior when there is **uncertainty**.
- A person's utility for a riskless outcome depends on her K-dimensional consumption bundle  $C$  and on a reference bundle  $r$  :  $u(c | r)$

# Uncertain environment

- Model allows for both **stochastic outcomes** and **stochastic reference points**.
- Assumes that a stochastic outcome  $F$  is evaluated according to its expected utility,
- utility of each outcome = the average of how it feels relative to each possible realization of the reference point  $G$
- assumes that the reference point is beliefs about outcomes.

$$U(F|r) = \int u(c|r) dF(c)$$

$$U(F|G) = \int \int u(c|r) dG(r) dF(c)$$

- This formulation captures the notion that the sense of gain or loss from a given consumption outcome derives from comparing it with all outcomes possible under the reference lottery. The overall sensation is a mixture of these two feelings.

- While preferences are reference-dependent, **gains and losses are clearly not all that people care about.**
- The sensation of **gain or avoided loss from having more money** does significantly affect our utility, but so does the **absolute pleasure of consumption we purchase with the money.**
- In contrast to prior formulations based on a "value function" defined solely over gains and losses, our approach makes explicit the way preferences also depend on absolute levels.

# Overall utility

$c = (c_1, c_2, \dots, c_K) \in \mathbb{R}^K$  is consumption

$r = (r_1, r_2, \dots, r_K) \in \mathbb{R}^K$  is a “reference level” of consumption

$$u(c|r) \equiv m(c) + n(c|r)$$



consumption utility



Gain-loss utility

For simplicity and for further reasons discussed  
in Koszegi and Rabin [2004]

$m, n$  are separable across dimensions

$$m(c) \equiv \sum_k m_k(c_k) \text{ and } n(c|r) \equiv \sum_k n_k(c_k|r_k)$$

In combination with loss aversion, this separability is at the crux of many implications of reference-dependent utility, including the endowment effect.

- The sensation of gain or loss due to a departure from the reference point seems closely related to the consumption value attached to the goods in question.
  - *It depends in a universal way on the changes in consumption utility associated with such gains or losses:*

$$n_k(c_k|r_k) \equiv \mu(m_k(c_k) - m_k(r_k))$$

$\mu$  is a "universal gain-loss function"

- satisfies the properties of Kahneman and Tversky's value function (defined on c-r).

"gain-loss utility" is directly derived from standard "consumption utility"



Loss aversion for large stakes

Diminishing sensitivity

Loss aversion for small stakes

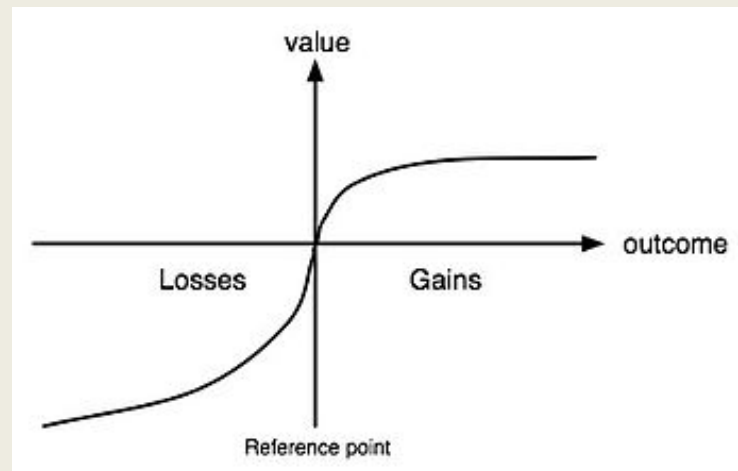
**A0.**  $\mu(x)$  is continuous for all  $x$ , twice differentiable for  $x \neq 0$ , and  $\mu(0) = 0$ .

**A1.**  $\mu(x)$  is strictly increasing.

**A2.** If  $y > x > 0$ , then  $\mu(y) + \mu(-y) < \mu(x) + \mu(-x)$ .

**A3.**  $\mu''(x) \leq 0$  for  $x > 0$ , and  $\mu''(x) \geq 0$  for  $x < 0$ .

**A4.**  $\mu'_-(0)/\mu'_+(0) \equiv \lambda > 1$ , where  $\mu'_+(0) \equiv \lim_{x \rightarrow 0} \mu'(|x|)$  and  $\mu'_-(0) \equiv \lim_{x \rightarrow 0} \mu'(-|x|)$ .



# Example

- consider a person choosing between two gambles: a 50-50 chance of gaining a paper clip or losing a paper clip, and the comparable gamble involving \$10 bills.
- It seems likely that she would risk losing the paper clip rather than the money, and do so because her **sensation of gains and losses is smaller for a good whose consumption utility is smaller.**
- Yet since  $m(.)$  is approximately **linear** for such **small stakes**, the **choice** depends almost entirely on the **comparison of  $n_k(.)$**  across dimensions,
- so that any model that does not relate gain-loss assessments to consumption utility is not equipped to provide guidance in this or related examples.

- We shall sometimes be interested in characterizing the implications of reference dependence with loss aversion but without diminishing sensitivity as a force on behavior. For doing so, we define an alternative to A3:

**A3'. For all  $x \neq 0$ ,  $\mu''(x) = 0$**

**PROPOSITION 1.** If  $\mu$  satisfies Assumptions A0–A4, then the following hold.

1. For all  $F, G, G'$  such that the marginals of  $G'$  first-order stochastically dominate the marginals of  $G$  in each dimension,  $U(F|G) \geq U(F|G')$ .
2. For any  $c, c' \in \mathbb{R}^K$ ,  $c \neq c'$ ,  $u(c|c') \geq u(c'|c') \Rightarrow u(c|c) > u(c'|c)$ .
3. Suppose that  $\mu$  satisfies A3'. Then, for any  $F, F'$  that do not generate the same distribution of outcomes in all dimensions,  $U(F|F') \geq U(F'|F') \Rightarrow U(F|F) > U(F'|F)$ .

- Part 1 means fixing the outcome, a lower reference point makes a person happier;
- Parts 2 and 3 mean that preferences exhibit a status quo bias: if a person is willing to abandon her reference point for an alternative, then she strictly prefers the alternative if that is her reference point.

**PROPOSITION 2.** If  $m$  is linear and  $\mu$  satisfies Assumptions A0–A4, then there exists  $\{v_k\}_{k=1}^K$  satisfying Assumptions A0–A4 such that, for all  $c$  and  $r$ ,

$$(3) \quad u(c|r) - u(r|r) = \sum_{k=1}^K v_k(c_k - r_k).$$

$$v_k(x) = m_k(x) - m_k(0) + \mu(m_k(x) - m_k(0))$$

- as for local changes  $m(\cdot)$  can be taken to be more closer to linear than  $\mu(\cdot)$ , Proposition 2 says that for small changes our utility function shares the **qualitative** properties of standard formulations of prospect theory.
- This equivalence does not hold when the changes are large or marginal consumption utilities change quickly.

# Example

- If, for instance, a person's reference level of water is a quart below the level needed for survival, loss aversion in  $\mu(\cdot)$  will not induce loss aversion in  $u(c | r)$ : she would be much happier about a one-quart increase in water consumption than she would be unhappy about a one-quart decrease.

# WHAT IS THE REFERENCE POINT?

In addition to the widely investigated question of how people react to departures from a **posited reference point**, predictions of reference-dependent theories also depend crucially on the understudied issue of **what the reference point is**.

# Assumptions

## Older models

- reference point is the **status quo**.

comes from contexts where people plausibly **expect to maintain the status quo**. But **when expectations and the status quo are different** --- a common situation in economic environments --- **equating the reference point with expectations** generally makes better predictions.

## Current model

- a person's reference point is the **expectations (probabilistic beliefs) she held in the recent past (between the time she first focused on the decision determining the outcome and shortly before consumption occur) about outcomes**.

Expectations have been mentioned by many researchers as a candidate for the reference point. This paper is the first to formalize the idea that expectations determine the reference point and to specify a rule for deriving them endogenously in any environment.



# Example 1

## "endowment effect"

- Merchants do not assess intended sales as loss of inventory, but do assess failed sales as loss of money;
- buyers do not assess intended expenditures as losses, but do assess failures to carry out intended purchases or paying more than expected as losses.

our theory may be useful for understanding instances where the endowment effect has not been found

## Example 2

- while an unexpected monetary windfall in the lab may be assessed as a gain, a salary of \$50,000 to an employee who expected \$60,000 will not be assessed as a large gain relative to status-quo wealth, but rather as a loss relative to expectations of wealth.
- a decrease in salary is not a reduction in the status-quo level of wealth
  - *it is a reduction from the expected rate of increase in wealth.*
- Whatever the notion of loss aversion would be, our model not only accommodates all these scenarios, but **predicts which is the appropriate notion as a function of the environment.**

## Example 3

- In nondurable consumption — where there is no object with which the person can be endowed — a status-quo-based theory cannot capture the role of reference dependence at all:
  - *a person who misses a concert she expected*
  - *a person who expects to undergo a painful dental procedure*

so irrespective of expectations a status quo theory would always predict the same gain-loss utility of zero from this experience.

- Our theory posits that preferences depend on **lagged expectations**, rather than expectations contemporaneous with the time of consumption.
- This does not assume that beliefs are slow to adjust to new information or that people are unaware of the choices that they have just made,
- but that **preferences do not instantaneously change when beliefs do.**
- When somebody finds out five minutes ahead of time that she will for sure not receive a long-expected \$100, she would presumably immediately adjust her expectations to the new situation, but she will still five minutes later assess not getting the money as a loss

# Expectations formation

- While alternative theories of expectations formation could be used, in this paper we complete our model by **assuming rational expectations**,
  - *a person correctly predicts the environment she faces (e.g. market distribution of prices)*
  - *The person correctly predicts her own reaction to this environment (her behavior in reaction to market prices)*

# Personal Equilibrium

Using the framework of Koszegi [2005] to determine rational expectations when preferences depend on expectations, we define

## "personal equilibrium" (PE):

- a situation where the **stochastic outcome implied by optimal behavior conditional on expectations** coincides with **expectations**.

## "preferred personal equilibrium (PPE)"

- which selects the (typically unique) personal equilibrium with highest expected utility.

# PE definition

Formally, suppose that the decision-maker has probabilistic beliefs described by the distribution  $Q$  over  $\mathbb{R}$  capturing a distribution over possible choice sets  $\{D_l\}_{l \in \mathbb{R}}$  she might face, where each  $D_l \in \Delta(\mathbb{R}^K)$ . In the first and weaker of two solution concepts we consider, rational expectations is the only restriction we impose:

**DEFINITION 1.** A selection  $\{F_l \in D_l\}_{l \in \mathbb{R}}$  is a *personal equilibrium (PE)* if for all  $l \in \mathbb{R}$  and  $F'_l \in D_l$ ,  $U(F_l | \int F_l dQ(l)) \geq U(F'_l | \int F_l dQ(l))$ .

- If the person expects to choose  $F_l$  from choice set  $D_l$  then given her expectations over possible choice sets, she expects the distribution of outcomes  $\int F_l dQ(l)$
- Definition 1 says that with those expectations as her reference point, she should indeed be willing to choose  $F_l$  from choice set  $D_l$ .
- There may be multiple PE

# PPE

**DEFINITION 2.** A selection  $\{F_l \in D_l\}_{l \in \mathbb{R}}$  is a *preferred personal equilibrium* (PPE) if it is a PE, and  $U(\int F_l dQ(l) | \int F_l dQ(l)) \geq U(\int F'_l dQ(l) | \int F'_l dQ(l))$  for all PE selections  $\{F'_l \in D_l\}_{l \in \mathbb{R}}$ .



# A central implication of our theory

- In **deterministic** environments PPE predicts that **decision-makers maximize consumption utility**, replicating the predictions of classical reference-independent utility theory.
- When there is **uncertainty**, a **decision-maker's preferences** over consumption bundles will be **influenced by her environment**.

# Applying the model to consumer behavior

- Willingness to pay for a good is increasing in the expected probability of purchase and in the expected prices conditional on purchase.
- a consumer's willingness to pay a given price for shoes depends on the **probability with which she expected to buy them** and **the price she expected to pay**.
- **"attachment effect"** : On the one hand, an increase in the likelihood of buying increases a consumer's sense of loss of shoes if she does not buy, thus increases her willingness to pay. Hence, the greater the likelihood she thought prices would be low enough to induce purchase, the greater is her willingness to buy at higher prices.
- **"comparison effect"** : On the other hand, holding the probability of getting the shoes fixed, a decrease in the price a consumer expected to pay makes paying a higher price feel like more of a loss, thus lowers her willingness to pay the high price. Hence, the lower the prices she expected among those prices that induce purchase, the lower is her willingness to buy at higher prices.

# Applying the model to within-day labor-supply decisions

- We develop a model where, **after earning income in the morning and learning her afternoon wage**, a taxi driver decides whether to continue driving in the afternoon.
- In line with the empirical results of the target-income literature (some workers seem to have a daily "target" income), our model predicts that when drivers experience unexpectedly high wages in the morning, for any given afternoon wage they are less likely to continue work.
- Yet expected wage increases will tend to increase both willingness to show up to work, and to drive in the afternoon once there.
- Our model therefore replicates the key insight of the literature that exceeding a target income might reduce effort.
- In addition, it both provides a theory of what these income targets will be, and \_ through the fundamental distinction between unexpected and expected wages \_ **avoids the unrealistic prediction that generically higher wages will lower effort.**
- a worker is **less likely to continue work** if **income earned thus far is unexpectedly high**, but **more likely to show up** as well as continue work **if expected income is high**.

# conclusion

- develop a model of **reference-dependent** preferences and **loss aversion**
  - *"gain-loss utility"* is derived from standard "consumption utility"
  - *reference point* is determined *endogenously* by the economic environment; rational expectations held in recent past about outcomes.

because a full specification of  $\mu(\cdot)$  allows us to derive both gain-loss utility and the reference point itself from consumption utility and the economic environment, it moves us closer to:

- a universally applicable, zero-degrees-of-freedom way to translate any existing reference-independent model into the corresponding reference.

- Although straightforward to apply in most cases, our model falls short of providing a recipe for entirely formulaic application of the principles of reference-dependent.
- Psychological and economic judgment is needed, for instance, in choosing the appropriate notion of "recent expectations."
- And there are also settings where the same principles motivating our approach suggest an alternative to our reduced-form model.

Thank you